

**REINFORCED CONCRETE
DESIGNERS' HANDBOOK**

REINFORCED CONCRETE DESIGNERS' HANDBOOK

BY

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A reference book giving the essential
data and method of designing any
type of reinforced concrete structure



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PREFACE

THE principal object of this volume is to provide a Handbook that will give the average reinforced concrete designing engineer the data that he may normally require in the ordinary course of his duties. The book is divided into three principal Parts, the second of which comprises a series of tables that endeavour to present in a concise form, and in accordance with modern practice, the necessary information and formulæ. The first Part gives explanatory notes upon, and subject matter additional to, that given in the tables, and thus by not interspersing the text among the tables the latter are supplemented without sacrificing completeness or compactness.

The text deals with the factors of design in the following sequence : (a) loads and pressures ; (b) calculations of moments and forces ; (c) determination of stresses and design of sections. There is no pretence of dealing with the wide subject of the construction of reinforced concrete structures except in so far as the work on the site affects that of the engineer at the drawing board, and in the endeavour to avoid duplicating much material that is fully and admirably dealt with elsewhere the mathematical processes by which the various formulæ are derived have been omitted. The third Part takes the form of a descriptive bibliography to which the reader is referred for fuller information on those parts of the subject that are briefly dealt with in this volume.

With this as his aim, the author feels that no apology is needed for adding this volume to the literature of reinforced concrete design. Although it is hoped that the presentation of some aspects of the subject may strike a new note, information has had to be obtained from many sources, and acknowledgment must be made to those engineers who are mentioned in the text.

The author's thanks are also due to Mr. R. A. Manthei, A.C.G.I., D.I.C., and Mr. R. Hicks for their assistance in preparing the work for publication.

Although this volume is primarily addressed to concrete designers, it is hoped that it may be of some value to those engineers whose practice only occasionally brings them in touch with reinforced concrete, and that even the expert may find the tables interesting and of assistance.

C. E. R.

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CONTENTS

PART I

	PAGE
CHAPTER I REINFORCED CONCRETE ENGINEERING	3
1 DESIGN IDEALS	3
2 DESIGN ECONOMICS	5
CHAPTER II LOADS	8
1 DEAD LOADS	8
2 LIVE LOADS ON FLOORS, ROOFS, AND STAIRS	8
3 STORES	9
4 WIND PRESSURE	10
5 MOVING LOADS	11
6 MODIFICATION TO LOADING	13
7 DISPERSION OF POINT LOADS	14
CHAPTER III HORIZONTAL PRESSURE DUE TO EARTH AND CONTAINED MATERIALS	17
1 LIQUIDS AND GRANULAR MATERIALS	17
2 VERTICAL WALLS	19
3 SURCHARGE	20
4 EXPERIMENTAL DATA	21
5 DEEP CONTAINERS	23
6 PASSIVE RESISTANCE	25
CHAPTER IV BENDING MOMENT AND SHEARING FORCE	26
1 MOMENT, SHEAR, AND DEFLECTION	26
2 CANTILEVERS AND SINGLE-SPAN BEAMS	26
3 THEORETICAL CONSIDERATION OF CONTINUOUS BEAMS	29
4 CONTINUOUS BEAMS WITH VARYING MOMENTS OF INERTIA	35
5 APPROXIMATE BENDING MOMENTS IN CONTINUOUS BEAMS	35
6 GRAPHICAL DETERMINATION OF BENDING MOMENTS IN CONTINUOUS BEAMS	36
7 ROLLING LOADS	37
8 BENDING MOMENTS IN SLABS	37
9 RECTANGULAR SLABS	38
10 POLYGONAL AND CIRCULAR SLABS	40
11 PLAT SLABS	41
CHAPTER V FRAMED STRUCTURES	45
1. THEORETICAL CONSIDERATIONS	45
2. EXTERIOR COLUMNS	48
3. INTERIOR COLUMNS	51
4. APPROXIMATE TREATMENT	51
5. EARTHQUAKE-RESISTING STRUCTURES	52
6. END CONDITIONS	52
7. MOMENTS OF INERTIA OF REINFORCED CONCRETE SEC- TIONS	53

CHAPTER VI. FOUNDATIONS	57
1. SITE INSPECTION	57
2. SAFE BEARING PRESSURES	59
3. TYPES OF FOUNDATIONS	60
4. PRESSURE UNDER FOUNDATIONS	63
5. DESIGN OF INDEPENDENT BASES	63
6. DESIGN OF COMBINED FOUNDATIONS	64
7. SAFE LOAD ON PILES	67
8. DESIGN OF PILES	70
CHAPTER VII. RETAINING WALLS AND CONTAINERS	74
1. TYPES OF CONTAINERS	74
2. TYPES OF RETAINING WALLS	74
3. SHEET PILE WALLS	77
4. DESIGN OF CANTILEVER RETAINING WALLS	80
5. CYLINDRICAL TANKS	81
6. BOTTOMS OF ELEVATED TANKS	82
7. BUNKERS AND SILOS	83
8. HOPPER BOTTOMS	86
CHAPTER VIII. BRIDGES, BUILDINGS, AND OTHER STRUCTURES	87
1. STRUCTURAL DESIGN	87
2. ARCH DESIGN	87
3. FIXED ARCHES	88
4. GIRDER BRIDGES	97
5. BRIDGE PIERS AND ABUTMENTS	98
6. BUILDING DETAILS	98
7. INDUSTRIAL STRUCTURES	101
8. CHIMNEYS	102
9. CULVERTS	103
10. ROADS	105
11. JOINTS IN CONCRETE CONSTRUCTION	107
CHAPTER IX. CONCRETE MATERIALS, MIXES, AND STRESSES	109
1. QUALITY OF CEMENT	109
2. QUALITY OF AGGREGATE	110
3. CONCRETE MIXES	110
4. PROPORTION OF CEMENT	111
5. PROPORTIONS OF FINE AND COARSE AGGREGATE	111
6. GRADING OF AGGREGATE	112
7. WATER CONTENT	113
8. SPECIAL MIXES AND MORTARS	114
9. PROPERTIES OF CONCRETE	116
10. CONCRETE STRENGTHS AND STANDARD STRESSES	117
11. MODIFICATIONS TO STANDARD STRESSES	118
12. TENSILE AND SHEAR STRESSES	119
13. MODULUS OF ELASTICITY	120
14. RAPID-HARDENING CEMENTS	120
CHAPTER X. REINFORCEMENT	122
1. QUALITY OF MATERIAL	122
2. WORKING STRESSES	123
3. ADHESION	124
4. DETAILING REINFORCEMENT	125
5. COVER OF CONCRETE	126
6. STEEL AREAS AND WEIGHTS	127

CONTENTS

ix

	PAGE
CHAPTER XI. BEAMS AND SLABS	128
1. FORMULÆ FOR RECTANGULAR SECTIONS REINFORCED FOR TENSION ONLY	128
2. TABULATED FACTORS	129
3. DESIGN OF RECTANGULAR BEAMS	130
4. RECTANGULAR BEAMS WITH COMPRESSIVE REINFORCE- MENT	131
5. DESIGN OF SLABS	133
6. FLANGED BEAMS	134
CHAPTER XII. SHEAR RESISTANCE	136
1. LIMITING UNIT SHEAR STRESSES	136
2. SHEAR TAKEN BY CONCRETE	137
3. SHEAR TAKEN BY BINDERS	137
4. SHEAR TAKEN BY DIAGONAL BARS	138
5. DESIGN PROCEDURE	140
CHAPTER XIII. COLUMNS	141
1. COLUMN LOADS	141
2. COLUMNS WITH INDEPENDENT BINDERS	142
3. SLENDER COLUMNS	144
4. HELICALLY-BOUND COLUMNS	146
5. ECONOMICAL COLUMN DESIGN	147
CHAPTER XIV. COMBINED STRESSES	148
1. GENERAL PRINCIPLES	148
2. RECTANGULAR SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS LESS THAN $0.167d$	151
3. RECTANGULAR SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS GREATER THAN $0.167d$ AND LESS THAN $0.5d$	151
4. DETERMINATION OF STRESSES FOR A RECTANGULAR SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS GREATER THAN $0.5d$ AND LESS THAN $1.5d$	151
5. DESIGN OF RECTANGULAR SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS GREATER THAN $0.5d$	152
6. RECTANGULAR SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS GREATER THAN $1.5d$	153
7. ANY SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS LESS THAN $\frac{Z}{A}$	151
8. ANY SECTION SUBJECT TO BENDING AND COMPRESSION WHERE e IS GREATER THAN $\frac{Z}{A}$	154
9. ANY SECTION SUBJECT TO BENDING AND TENSION WHERE e IS LESS THAN $0.5f$	157
10. RECTANGULAR SECTION SUBJECT TO BENDING AND TENSION WHERE e IS GREATER THAN $0.5f$ AND LESS THAN $1.5d$	158
11. RECTANGULAR SECTION SUBJECT TO BENDING AND TENSION WHERE e IS GREATER THAN $1.5d$	159
12. ANY SECTION SUBJECT TO BENDING AND TENSION WHERE COMPRESSIVE AND TENSILE STRESSES ARE SIMULTANEOUSLY PRODUCED	160
13. POSITION OF NEUTRAL AXIS	160

CONTENTS

CHAPTER XIV. SPECIFICATIONS, QUANTITIES, AND COST ESTI-	PAGE
• MATING	162
1. ASSOCIATED MATERIALS	162
2. TIMBER: SHUTTERING, JOINERY, AND PILES	162
3. BRICKWORK AND MASONRY	164
4. IRON, STEEL, AND OTHER METALS	165
5. SPECIFICATIONS	166
6. QUANTITIES	167
7. COST ESTIMATING	168
8. MISCELLANEOUS DATA	171

PART II

TABLES AND EXAMPLES

No.	1. LOADINGS (DEAD LOADS AND LIVE LOADS)	175
	2. LOADINGS (ROLLING LOADS)	177
	3. LOADINGS (RAIL VEHICLES, CRANES, AND STORED MATERIALS)	179
	4. LOADINGS (IMPACT, DISPERSION, AND WIND PRESSURES)	181
	5. PRESSURE OF CONTAINED MATERIALS	183
	6. DEEP SILOS	185
	7. CANTILEVERS AND SINGLE-SPAN BEAMS	187
	8. CONTINUOUS BEAMS (EQUAL LOADING ON EQUAL SPANS)	189
	9. CONTINUOUS BEAMS (ANY SPAN AND LOAD)	191
	10. CONTINUOUS BEAMS (B.M. COEFFICIENTS AND INERTIA VARIATION)	193
	11. CONTINUOUS BEAMS (FIXED POINTS)	195
	12. INFLUENCE LINES (TWO AND THREE SPANS)	197
	13. INFLUENCE LINES (FOUR, FIVE, AND MORE SPANS)	199
	14. SLABS	201
	15. FRAMED STRUCTURES	203
	16. CONTINUITY BETWEEN COLUMNS AND BEAMS	205
	17. FOUNDATIONS (PRESSURES)	207
	18. FOUNDATIONS (INDEPENDENT BASES)	209
	19. PILES	211
	20. RETAINING WALLS AND TANK WALLS	213
	21. MOMENTS IN BIN WALLS	215
	22. BOTTOMS OF BUNKERS AND TANKS	217
	23. CONCRETE	219
	24. REINFORCEMENT (ADHESION)	221
	25. REINFORCEMENT (AREAS OF BARS)	223
	26. REINFORCEMENT (WEIGHTS OF BARS)	225
	27. RECTANGULAR BEAM FACTORS	227
	28. RESISTANCE MOMENTS: RECTANGULAR BEAMS	229
	29. RESISTANCE MOMENTS: RECTANGULAR BEAM"	231
	30. SLAB DESIGN FACTORS	233
	31. RESISTANCE MOMENTS: SLABS	235
	32. BEAM DATA	237
	33. SHEAR REINFORCEMENT	239
	34. COLUMNS	241
	35. COLUMNS (SAFE LOADS)	243
	36. BENDING AND DIRECT FORCE	245
	37. METRIC EQUIVALENTS	247
	38. DECIMAL EQUIVALENTS	249
	39. MISCELLANEOUS DATA	251
	40. TRIGONOMETRICAL RATIOS	253

ADDITIONAL EXAMPLES

(i) DESIGN OF FLOOR PANEL	258
(ii) DESIGN OF FLAT SLAB	261
(iii) DESIGN OF HOPPER BOTTOM	265
(iv) QUANTITIES AND COST ESTIMATING	268

PART III

DESCRIPTIVE BIBLIOGRAPHY

1. REINFORCED CONCRETE ENGINEERING :	
• GENERAL AUTHORITIES	272
ARCHITECTURE AND ECONOMY	272
2. LOADS :	
DEAD AND LIVE LOADS	273
WIND PRESSURE	273
MOVING LOADS	273
IMPACT	273
POINT LOADS	273
3. HORIZONTAL PRESSURES :	
THEORETICAL PRESSURES	274
• GRAPHICAL DETERMINATION	274
EXPERIMENTAL DATA	274
FLEXIBLE WALLS	274
DEEP SILOS	274
4. BENDING MOMENTS AND SHEARING FORCE :	
THEORY	274
RECTANGULAR SLABS	275
FLAT SLABS	275
5. FRAMED STRUCTURES :	
THEORY (i) SLOPE-DEFLECTION METHOD	275
(ii) PRINCIPLE OF LEAST WORK	275
PRACTICAL FORMULAE	276
EARTHQUAKE-RESISTING STRUCTURES	276
• GENERAL	276
6. FOUNDATIONS :	
GENERAL	276
PILLS	276
7. RETAINING WALLS AND CONTAINERS	277
8. BRIDGES, BUILDINGS, AND OTHER STRUCTURES :	
ARCHES	277
GIRDER BRIDGES	277
CULVERTS	278
BUILDINGS	278
ROADS	278
GENERAL	278
9. CONCRETE :	
MATERIALS	278
PROPERTIES OF CONCRETE	278

	PAGE
10. REINFORCEMENT	279
11. BEAMS AND SLABS	279
12. SHEAR.	279
13. COLUMNS	280
14. COMBINED STRESSES	280
15. SPECIFICATIONS, QUANTITIES, AND COSTING :	
SPECIFICATIONS	280
SPECIAL MATERIALS	280
QUANTITIES	281
COSTING	281
INDEX	282

LIST OF DIAGRAMS

	PAGE
FIG 1. MINISTRY OF TRANSPORT EQUIVALENT LOADING . . .	12
" 2. PRESSURE BEHIND VERTICAL WALLS	18
" 3. SURCHARGES AND SUPERLOADS BEHIND RETAINING WALLS . . .	20
" 4. DISPERSION OF POINT LOAD THROUGH EARTH	22
" 5. SILOS	23
" 6. PROPPED CANTILEVERS	27
" 7. TYPICAL BENDING MOMENT AND SHEAR DIAGRAMS	28
" 8. CONTINUOUS BEAMS	30
" 9. CONTINUOUS BEAM EXAMPLE	31
" 10. INCIDENCE OF LOADING TO PRODUCE MAXIMUM MOMENTS . . .	33
" 11. FIXED POINTS	37
" 12. FLAT SLAB WITH DROPPED PANEL	42
" 13. FLAT SLAB WITHOUT DROPPED PANEL	43
" 14. FRAME NOTATION	46
" 15. ELL FRAMES	49
" 16. TEE FRAMES	49
" 17. B.M. IN COLUMNS WHEN RESTRAINT MOMENT IN BEAM IS NEGLECTED	50
" 18. TYPICAL DETAIL OF HINGE	53
" 19. TYPICAL SECTIONS FOR COMPARISON OF MOMENTS OF INERTIA . .	55
" 20. RAFT FOUNDATIONS	61
" 21. COMBINED BASES	65
" 22. BALANCED FOUNDATIONS	66
" 23. PILE SHOES	71
" 24. REINFORCED CONCRETE PILES	72
" 25. TYPICAL DETAILS OF PILE CAPS	73
" 26. TYPICAL DETAILS OF SINGLE CANTILEVER RETAINING WALLS	75
" 27. COUNTERFORT RETAINING WALLS	76
" 28. SHEET PILE WALLS	76
" 29. SHEET PILES	77
" 30. WATER TOWER BOTTOMS	83
" 31. TYPICAL DETAILS OF BUNKERS	85
" 32. APPROXIMATE METHOD OF DETERMINING THRUSTS (ARCHES) . . .	88
" 33. ARCH TYPES	89
" 34. DETAILS OF STAIRS	100
" 35. LOADING ON CULVERTS	104

	PAGE
FIG. 36. TYPICAL JOINTS	108
„ 37. RECTANGULAR BEAMS AND FLANGED BEAMS	129
„ 38. SLAB DETAILS	134
„ 39. BEAM BINDERS	138
„ 40. SHEAR BARS	139
„ 41. COLUMN SECTION	142
„ 42. TYPICAL COLUMN BINDERS	143
„ 43. ALTERNATIVE DETAILS OF COLUMN JOINTS	144
„ 44. RECTANGULAR SECTION: COMBINED STRESSES	149
„ 45. IRREGULAR SECTION: COMBINED STRESSES	150
„ 46. IRREGULAR SECTION: COMBINED STRESSES	150
„ 47. MOMENT AND THRUST ON ANNULAR SECTION	156
„ 48. TYPICAL DETAILS OF FLOOR PANEL--BEAM AND SLAB DESIGN	256-7
„ 49. TYPICAL DETAIL OF FLAT-SLAB DESIGN WITH DROPPED PANEL	262
„ 50. TYPICAL DETAILS OF FLAT-SLAB DESIGN WITHOUT DROPPED PANELS	263
„ 51. TYPICAL DETAIL OF HOPPER BOTTOM DESIGN	266

PART I
DESCRIPTIVE TEXT

CHAPTER I

REINFORCED CONCRETE ENGINEERING

1.—Design Ideals.

IN his everyday practice the reinforced concrete designing engineer is beset by many varied problems. Unlike the structural steel designer, who has but one standard material, and that supplied in standardised sections, the concrete designer has to think simultaneously in terms of five materials, namely, stone, sand, cement, steel and timber. These materials have to be combined to fulfil efficiently the designer's requirements. When he makes his calculations, instead of being guided along the simple lines of elementary mechanics he is assailed by the apparently abstruse mathematics of the elastic theory. The concrete engineer's realises that his detail design is as important as his main planning. Efficiency and economy—related but not quite identical—must always be uppermost in his mind. Efficient detailing is of paramount importance, and by efficient detailing is meant much more than providing in an economical manner the computed area of reinforcements; it implies that this calculated amount of steel can be practically placed so that it is given a chance to do the work it will be called upon to perform, and that reinforcement is provided to take those many incalculable secondary tensile forces that are inherent in a monolithic concrete structure.

Towards the production of safe and economical structures good judgment will do nearly as much as calculations, and especially in the matter of the assumption of loads to be carried does the shrewd designer have scope for the display of this judgment. It is of little use to make calculations with the effective depth of a section carried to two places of decimals, if the loads are 25 per cent. under- or over-estimated.

Competent designing must be associated with efficient and relentless supervision of the work on the site if, in the interests of his profession and his client, the reinforced concrete engineer is to take full advantage of the latest research results and refinements in calculation, and of the excellency and uniformity of modern cements.

The trend of modern design is such that, where proper precautions are taken to ensure that the assumed loading and specified quality of concrete are realised, high working stresses can be employed with safety, and the more factors allowed for in the calculations the higher the working stresses may be, and vice versa. At the same time complicated and pseudo-exact mathematics should not be allowed to confuse the sense of what is good engineering. If concrete engineers continue to compute with stresses in the order of 16,000 and 600 lb.

per square inch, then approximate bending moment factors and roughly calculated resistance moments are good enough for all normal beam calculations.

The object of all technical advances should be to introduce refinements and simplifications into the methods of computation that the engineer must make to aid his judgment. The refinements should render the stress survey more comprehensive and therefore more satisfactory, and the simplifications should surely commend themselves to the ever-growing company of those who are termed reinforced concrete engineers.

It would seem that the concrete designer should be a versatile engineer, so many and varied are the duties he may be called upon to perform. From the initial stages of site inspection and general planning of a structure through the intermediate stages involving the preparation of the detail designs, cost estimate, bills of quantities and specification, to the final stage of inspection of works in progress, measuring-up, amendments to designs due to unforeseen circumstances arising during the course of the job, he is continually putting to use, and at the same time augmenting, his store of knowledge and experience of structural materials and design, as well as of construction methods and costs.

The ideal reinforced concrete engineer is the one who can blend sufficient of theory and practice. At one end of the scale is the mathematician who can calculate precisely the stresses and strains produced by an assumed imposition of loads; he is a very useful man when his theoretical knowledge of mathematical intricacies is coupled with experience of what can be done and what are the limitations of the materials he is hypothetically handling. At the other end of the scale is the essentially practical man with little advanced technical knowledge but with the knack of knowing what can be done with a given material; a man who can skilfully set out an intricate ground plan, who can make good concrete, who can efficiently handle the labour-saving constructor's plant of to-day, who can organise, and who can economically cause the mathematician's dreams to materialise.

And what of the man between these two extremes? At his best he must be a man with sufficient technical knowledge to control and interpret the mathematician's conceptions and to translate them into projects that the constructor can erect; he must know where he can safely sacrifice theoretical accuracy for economy of construction; he must be able to comprehend the requirements of his non-technical clients and to draw up a practical specification that will defend his client against unscrupulous contractors but that will not be too severe on conscientious builders. He must know sufficient of the practical man's ways and of the materials' market to estimate with precision the probable cost of a project, and he must have sufficient acquaintance with the business side of engineering to realise from where he can best gather his work—in short he must be one-third part technician, one-third part constructor, and one-third part lawyer, commercial man, and organiser.

Admitting that this versatility constitutes the make-up of an engineer, the question arises as to what is the training (education) the aspirant should undergo. Limiting our consideration to the reinforced concrete engineer who intends to make designing his principal business, it is reasonable that in his blending of practical mathematics and field experience his study of theory should predominate, and in his early years he should endeavour to proceed along both

roads at once in order that in an intensive study of one side he does not lose sight of the importance of the other. His studies should obviously include the usual theory of structures and strength of materials, and such other subjects as are usually embraced by the university degrees and professional examinations. These can be valuably supplemented by the less abstract studies of building construction, geology, and the elements of architecture. Nor should languages be overlooked, since much that is of value in engineering literature is often presented in the German or French language.

On the practical side it should be borne in mind that clear draughtsmanship is all-important in any branch of engineering. The outdoor studies of the concrete designer should embrace surveying, and familiarising himself with such details as timber sizes, the amount of concrete that can be made and placed in a given time, and, even more important, what difficulties are experienced in the bending and placing of reinforcing bars. If he is wise he will realise that a personal insight into such lowly occupations as tracing and the keeping of site and office records is useful to any man who, whatever technical diplomas he may possess, desires to qualify as an engineer.

2. Design Economics.

There are few, if any, reliable criteria of economy in reinforced concrete design, since besides the obvious effect of the relative prices of concrete, steel, and timber on the economic proportions of the quantity of concrete, reinforcement, and shuttering, there are possibly other and just as important factors at the back of the individual designer's mind. For example, a designer engaged by a contracting-designing organisation may have to bear in mind that a particular piece of special plant, such as steel forms of a certain size, will be available for use on a particular job, and therefore it is economy so to dispose the structural arrangement that such plant can be used instead of leaving it idle in the yard. Or again, if labour troubles or a rise in wage rates are forecast in the joiners' trade, then economy may accrue by saving shuttering at the expense of concrete.

Such considerations as whether less concrete of a rich mix is cheaper than a greater volume of a leaner mix; whether the use of higher-priced bars of over-long lengths will offset the cost of the extra weight used in lapping cheaper bars of normal length; whether, consistent with efficient detailing, a few bars of large diameter can replace a larger number of bars of smaller diameter; whether the extra cost of rapid hardening cement justifies the saving made by a greater number of uses of the shuttering; or whether uniformity in the sizes of members saves in shuttering what it may cost in extra concrete—these are a few among the many problems that beset and are solved by the individual designer of every job of importance.

Then there is the wider aspect of the economical problem, such as whether the anticipated life and use of a proposed structure warrants the use of a high or low factor of safety, or whether the extra cost of adopting an expensive type of construction will be commensurate with the improvement in internal facilities. A common example of the latter consideration occurs when the extra cost of helically-bound columns is compared with the rental value of the floor space saved.

The wording of a contract and the experience of the contractor, the locality of the site and the nature of the available materials, and even the method of measuring up the quantities, together with numerous other points, all have their effect, consciously or not, on the designer's attitude towards a contract. So many and varied are the collateral factors to consider that only experience and the study of the trend of design can give the designer any reliable guide. Attempts at determining the most economical proportions for a given member, basing that determination only on inclusive prices for concrete, steel, and shuttering, generally lead to dimensions that are usually unconventional and sometimes absurd. It is nevertheless possible to lay down certain broad principles that, coupled with the observations made in subsequent chapters, should be of some guidance.

Weight for weight the material and labour costs on small diameter bars are greater than those on large diameter bars, and within wide limits long bars are cheaper than short bars if there is a sufficient tonnage to justify special transport charges and facilities.

The less cement in a cubic yard of concrete the cheaper the concrete, but, other factors being equal, the lower the strength. Taking strength and cost into account a rich mix is more efficient than a leaner mix, and what is usually known as a 1 : 1 : 2 mix may be 20 per cent. more efficient than the more usual 1 : 2 : 4 mix. In beams and slabs, however, where much of the concrete is in tension and therefore neglected in design calculations, a lean mix is cheaper than a rich mix. In columns, where all the concrete is working in compression, the adoption of a rich mix leads to economy, since besides the more efficient concrete there is the saving in shuttering consequent upon the reduction in size of the member. The relative economy of various column designs is discussed in Chapter XIII.

The use of compression steel is always uneconomical when the cost of a single member is being considered, but advantages accruing therefrom by increasing headroom under beams or by reducing the size of columns may offset the extra cost of the individual member.

Bent-up bars are more economical than binders for resisting shear in beams.

Tee beams are cheaper if the rib is made as deep as possible ; here again the increase in headroom consequent on reducing the depth may offset the small extra cost of the beam. An increase in overall depth of $33\frac{1}{3}$ per cent. may result in a 20 per cent. decrease in cost.

Shuttering is obviously cheaper if all angles are right angles, if surfaces are plane, and if there is a fair amount of repetition. Thus splays, fillets, chamfers, etc., should be dispensed with unless essential to structural strength, and wherever possible architectural features in cast-in-situ work should be formed in straight lines with right-angle breaks. When shuttering costs are considered in conjunction with concrete and steel costs, the introduction of certain complications in the shuttering may lead to more economic construction. For instance, heavy continuous beams are more economical if haunched at the supports, and circular tanks of medium capacity are cheaper than rectangular tanks of the same capacity. In some cases domed roofs and tank bottoms are more economical than flat beam and slab construction, although the shuttering may cost 100 per cent. more for spherical work. When shuttering can be used several times over the advisability of employing steel forms should be considered,

and, owing to the less adaptability of steel as compared with wood, the design should be modified to allow of their use. Generally steel forms for beam and slab or column construction are only cheaper than timber shuttering if twenty or more uses can be assured, but for circular work half this number of uses may warrant the adoption of steel forms. Timber forms for slabs, walls, beams, column sides, etc., are usually good for four uses before repair, and six to eight times before the cost of repair equals the cost of remaking. Beam-bottom boards can be used at least twice this number of times.

A very essential aspect of the reinforced concrete engineer's work is an appreciation of the economical qualities of materials other than concrete. The judicious incorporation of such materials into his designs may lead to economies on a large scale, and because his structure is primarily built in his specialised material he should not be insensible to the applicability of other building materials in appropriate parts of his structure. Just as there is little excuse for facing reinforced concrete bridges with stone, so there is no economic reason for constructing 4-in. reinforced concrete panel walls when a $4\frac{1}{2}$ -in. or 9-in. brick wall will serve the same purpose. A 4-in. reinforced concrete wall has almost the same resistance to bending as a 9-in. brick wall and is about equal in cost. The direct load-carrying capacity of a 9-in. brick wall is almost equal to that of a $5\frac{1}{2}$ -in. concrete wall, whereas a 14-in. brick wall has the same bending strength as a 6-in. concrete wall and the same load carrying capacity as an 8-in. concrete wall, but in all cases the concrete wall is cheaper than the 14-in. brick wall.

Other common cases of the consideration of the claims of other materials are that of asphalt road surfaces on concrete foundations compared with all-concrete roads, the installation of timber or steel bunkers in place of concrete structures when short life only is required, the erection of light steel framing for the superstructures of industrial buildings, the provision of pitched steel trusses with suitable roof coverings, compared with concrete roof slab and beam construction, suspension cables as against long-span concrete arches, masonry blocks as against concrete monoliths in marine work, and timber piles compared with reinforced concrete piles. Included in such economic comparisons should be such factors as fire-proofing, deterioration, depreciation, insurance, appearance, and constructional time, besides such structural considerations as foundations, convenience of construction, and availability of materials.

CHAPTER II

LOADS

1.—Dead Loads.

THE careful and judicious assessment of the loads for which a whole structure or any particular structural member should be designed is as important a factor in the production of an efficient engineering design as is the general economic planning of the scheme, or the precise estimation of the dimensions of the various members. Loads are conveniently divided into (1) dead (or permanent) loads and (2) live (or incidental) loads. The primary dead load is the weight of the structure or member itself, which, if of reinforced concrete, is conveniently calculated at 144 lb. per cubic foot. Other dead loads include the permanent weights carried by the member, such as surfacings; walls and superstructure of either concrete, masonry, brickwork, or steelwork; asphaltting; floor finishes; roof screedings; fixed machinery; and tanks. On *Table No. 1* are tabulated some values for guidance in assessing the total dead loading that has to be carried.

In addition to the weights of various structural materials, this Table gives the unit weights for glazed, sheeted, and slated roofs, together with the average equivalent loads per square foot of horizontal projection for steel trusses of various spans. These figures should be of value in estimating the loads carried by a concrete substructure having a pitched steel roof. For ordinary steelwork made up of standard joists or compound sections it is generally sufficient if the weight is estimated by adding to the nominal weight of the section an allowance of 10 per cent. for connections and rivets, although extra should be allowed for stanchion bases and caps.

Where concrete lintels support brick walls it is not necessary to consider the lintel as carrying all the wall above it, but it is sufficient to allow for a triangular portion as indicated in the diagram on *Table No. 1*, the remainder of the wall being carried by arch action.

2.—Live Loads on Floors, Roofs, and Stairs.

Any external loading that may come on to a structural member when the structure is serving its normal purposes should be allowed for as incidental or live loading. Such loading would include stored materials, furniture and movable equipment, cranes, vehicles, snow, wind and people. The actual value of these live loads depends on the individual requirements of each part of the structure.

For buildings in cities, the live loads to be taken by floors, staircases and

roofs are laid down in the regulations controlling building construction in any particular district. The floor loads for various classes of buildings given on *Table No. 1* are generally suitable for structures that do not come under any particular controlling authority. In addition to a uniformly distributed live load, some authorities require the floor to be sufficient to carry an alternative point load that may occupy any position on the otherwise unloaded floor. The magnitude of this load varies with the type of floor as given on *Table No. 1*; the load can be considered as acting over an area of 2 ft. 6 in. square.

A live load of 120 lb. per square foot on stairs and landings should cover all possible loadings, but it is suggested that these parts of a structure should be designed to carry the same loads as those carried by the floors served by the staircase, except in the case of a warehouse type of building where the possibility of heavy goods being transported by means of the stairways is remote. In no case should a stair be designed for less than $\frac{3}{4}$ cwt. per square foot, and each step should be designed to be capable of carrying independently a point load of 300 lb.

The superloads for roofs given on *Table No. 1* are additional to all surfacing materials and they include allowances for wind, ice, snow, and other incidental loads. With regard to the weight of snow, it should be borne in mind that although freshly fallen snow weighs about 5 lb. per cubic foot, compacted snow may reach 20 lb. per cubic foot, and this fact may require consideration when designing structures in districts subject to heavy snowfall. For pitched roofs the snow load decreases with an increase in steepness, but the wind load increases. If a flat roof is used for any purpose such as a café, roof-garden, etc., an additional loading must be allowed for and a minimum superload of $\frac{1}{2}$ cwt. per square foot should be taken. The possibility of converting a roof into a future floor should also be anticipated.

3.—Stores.

Buildings of the warehouse class should be designed for a minimum live load of 200 lb. per square foot, but the actual purposes for which the building is to be used should be considered, and, if thought necessary, a higher figure should be taken into account.

• Wherever possible the weights of the stored material should be determined and designed for; especially does this apply to such structures as tanks, reservoirs, silos, bunkers, etc., the primary object of which is storage. Weights of some of the materials more commonly stored in concrete containers are given on *Table No. 3*. Where the material is floating in water the loads and pressures due to the head of water are the maximum forces for which the structure should be designed. When a granular material is heavier than water and completely immersed, the load carried by the bottom of the container is the sum of the weight of the dry material measured in bulk, the weight of water filling the voids, and the weight of water, if any, above the top of the material.

Where containers may have to hold a heavy material in either a dry or wet state, the maximum load would be when the material is fully saturated. For example, coal-draining bunkers should be calculated for the loads due to the

contained coal and water. A solid block of coal may have a specific gravity of about 1.3 and coal in lumps a specific gravity of approximately 0.7. Hence coal in water has a specific gravity of 1.16; that is, the combined weight of one cubic foot of coal and water is $72\frac{1}{2}$ lb.

The horizontal pressure due to liquids, earth, and other contained materials is discussed in the next Chapter.

4.—Wind Pressure.

The magnitude of the wind pressure on a structure depends on the position and height of the structure. For a comparatively sheltered structure in Britain a pressure of 15 lb. per square foot on the upper two-thirds of the structure is adequate, but some authorities recommend that each panel of walling should be designed for a pressure of 25 lb. per square foot acting from either side of the panel. For exposed structures less than 80 ft. in height, the wind pressure should be taken as 25 lb. per square foot on the whole structure and 40 lb. per square foot on panels. For exposed structures over 80 ft. in height a pressure of 40 lb. per square foot should be taken on the upper part. These pressures are given on *Table No. 4*.

When special structures are to be built in exposed positions where the probable wind velocities are known from records, it is preferable to design for the pressures calculated from the maximum velocity. The pressure of the wind varies as the square of the velocity, and various formulæ have been derived by different investigators. Amongst the best known results are the following:

$$\text{Stanton} \quad P = 0.0027 V^2 \text{ to } 0.0032 V^2$$

$$\text{Dines} \quad P = 0.003 V^2$$

$$\text{Froude} \quad P = 0.0037 V^2$$

where P = pressure in lb. per square foot,

V = velocity in miles per hour.

For general purposes a value of $P = 0.003 V^2$ seems a reliable mean, and pressures calculated in accordance therewith are given on *Table No. 4* for various strengths of wind. For a steady wind of a hundred miles per hour, as occurs at rare intervals, the pressure would be 30 lb. per square foot but the impact from gusts of high wind may exert greater pressures. Thus for tall structures subject to the full force of the wind a pressure of 40 lb. per square foot of the total area seems to be a suitable value well on the safe side even allowing for suction on the leeward side for small areas of such structures or for high and narrow exposed towers.

The principal factor in the design of chimneys is wind pressure, and every problem of isolated exposed stacks should be considered carefully to avoid an unduly high assessment of this factor leading to wasteful design. Where records of winds in the locality of the site are available an estimate of possible wind pressures can be made. Due account should be taken of the susceptibility to gust impacts on the comparatively thin stalk, and the following pressures should be considered as absolute minimum values allowing for pressure on the windward side and suction on the leeward side and for impact:

For length of stack between tops of adjacent buildings and 80 ft. from ground level.	25 lb. per square foot
For length of stack above 80 ft. from ground	40 lb. per square foot

These values are less than those usually considered in the design of brick chimneys, but the margin of safety is greater for concrete structures than for brickwork or masonry.

For circular chimneys these pressures can be reduced by 40 per cent. to 15 lb. and 24 lb. respectively owing to the relieving effect of the curved surface. This reduction would also apply to the pressure on circular tanks of water-towers if the diameter is 20 ft. or less. For larger diameters and for tanks polygonal in plan less reduction should be made. A reduction of $33\frac{1}{3}$ per cent. can be taken when the wind blows normally to a diagonal of a square tower.

The values of wind pressures on steel bridges are covered by the British Standard Specification, and for concrete bridges not designed to this specification a similar consideration should be applicable. Thus the values of the wind pressure in accordance with good practice could be as follows:

Unloaded bridge: 40 lb. per square foot

Loaded bridge: 20 lb. per square foot

These values are slightly lower than those given by the B.S.S. and apply per square foot of exposed surface, which for a loaded bridge includes the vertical area of the structure and any portions of the load projecting above the parapet. For an unloaded bridge where the width between the parapets exceeds twice the combined depths of the parapet and outer girder, the exposed area should be calculated as twice the area of the parapet and outer girder. If the width is less than this the exposed area should be reckoned as three-quarters of the value for a wide bridge.

5.—Moving Loads.

On *Table No. 2* are illustrated standard trains of loads for the design of road bridges. Both the Ministry of Transport Loading and that specified by the British Standards Institution are given with self-explanatory notes. It should be observed that the Ministry of Transport loading includes for an impact allowance of 50 per cent., but the B.S.S. loading is subject to an increase due to impact (see Paragraph 6). Most main road bridges in this country are designed for the Ministry of Transport loading, and for such major structures as do not come under the Ministry's jurisdiction either of the prescribed loadings should be sufficient. To simplify calculations, the Ministry of Transport has issued a memorandum, reproduced in *Fig. 1*, which indicates uniformly distributed loads that can be taken as equivalent to the specified point loads.

In addition to this loading, or where such loading would be out of all proportion to the requirements of the traffic using the bridge (having due regard to future developments), consideration should be given to the loading from normal and special types of vehicles using the structure. The effect of the occasional passage of team rollers, lorries conveying boilers or heavy machinery, etc., should be studied, and in the case of railway bridges allowance should be made for the rolling loads indicated on the diagrams supplied by the railway company.

STANDARD LOAD FOR HIGHWAY BRIDGES. EQUIVALENT LOADING CURVE.

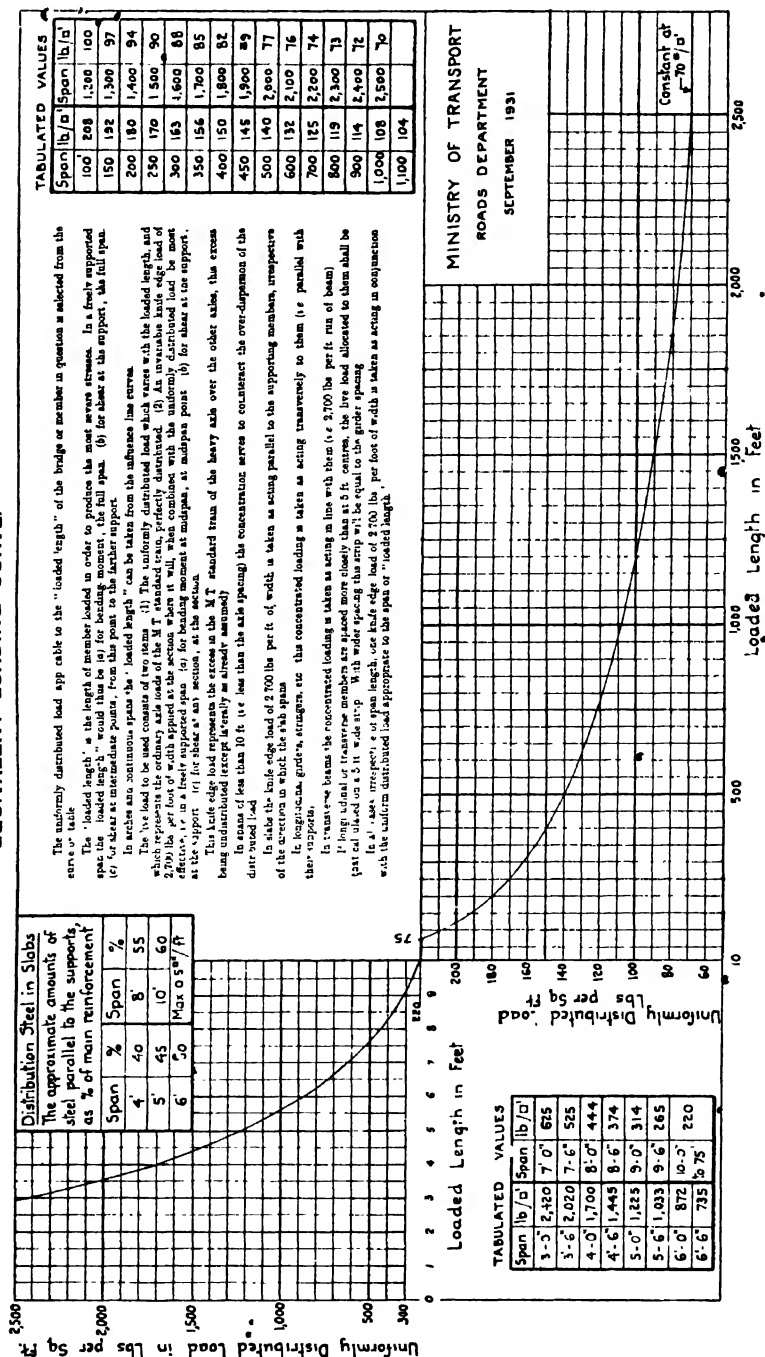


Fig. 1.

The wheel loads (without impact allowance) and other data for various types of road vehicles, including lorries, steam wagons, rollers, buses, etc., and the Ministry of Transport limiting axle loads for buses and lorries, are given on *Table No. 2*, and on *Table No. 3* some weights of rail vehicles are given. It will be appreciated that wheel loads and dimensions of road and rail vehicles and cranes vary with different types and manufacturers, and any data specified in the tables are an indication of magnitude rather than of precise values.

The raised footpaths of road bridges should be designed for crowding and for the occasional mounting of a vehicle upon the pavement. A load of $\frac{3}{4}$ to 1 cwt. per square foot is sufficient to allow for pedestrian traffic, and a single wheel load of 3 or 5 tons including impact should be adequate for accidental loading. If there is any possibility of the footpath being converted into a roadway in the future, the pavement slab and supporting beams should be designed for the same loads as the primary roadway.

A rolling load commonly met with in reinforced concrete structures is that from overhead cranes. On *Table No. 3* are listed the maximum wheel loads that have to be carried by the crane rail beam for travelling overhead cranes of normal design. To allow for vibration and impact the maximum wheel load should be augmented by 25 per cent. of the total weight of the crane and its load. Braking induces a longitudinal thrust in the railbeam, and the magnitude of this thrust can be taken as 20 per cent. of the wheel loads. The traversing of the crab subjects the railbeam to a transverse horizontal thrust equal to about 20 per cent. of the total weight of the crane.

Jib cranes running on rails produce loading problems of their own which may be solved from the particulars applicable to the particular machine. The dynamic effects on the wheel loads of lifting, discharging, slewing, etc., need consideration. The maximum wheel load would occur when, at the same time, the jib is oblique to the track with the line of the jib vertically over one of the wheels, when the crane is travelling, lifting the load, and a wind is blowing. Such severe conditions rarely occur simultaneously and therefore the maximum wheel load can be considered as occurring when the crane is stationary with the line of the jib over one wheel and the jib slewing with load. The variations of wheel loading for a 3-ton steam crane under various conditions are given on *Table No. 3*.

6. -Modification to Loading.

Under certain conditions the standard loadings specified can be reduced and in others they must be augmented. For example, in the case of buildings that are not warehouses or stores and that have more than two floors, for the purpose of calculating the maximum loads on the columns the live loads can be reduced in accordance with the following rules :

(1) Consider the roof and top floor as fully loaded ; reduce the load on the second floor from the top by 10 per cent., the third floor by 20 per cent., and so on in 10-per cent. increments down to the sixth floor from the top, where the load is reduced by 50 per cent. ; the design load on all floors from the sixth downwards can also be reduced by 50 per cent. :

(2) Where any floor in a building not of the warehouse class exceeds say 300 sq. ft. in area, the live load on the main beams can be reduced by 25 per

cent. This reduction can also be made in the calculation of the column loading if the rule for multi-floor buildings has not been applied.

(3) For floors subject to vibration from such causes as dancing, drilling, etc., a live load of 100 lb. per square foot is adequate to allow for the static live load plus the dynamic effect. For structural members subjected to incessant vibration due to machinery, etc., it is usual to make an allowance by reducing the working stresses in the steel and concrete by 15 per cent. to 20 per cent., or by increasing the total dead and live load by this amount. The advantage of the latter method is that standard stresses are worked to and thus standard tables and curves are applicable.

(4) The impact effect of rolling loads is usually allowed for by increasing the static load by 10 per cent. to 75 per cent. The Ministry of Transport loading allows for 50 per cent. increase, but for more general cases the allowance should be decided after consideration of the type of vehicle, the nature of the road or rail surface, the type of wheel (whether rubber or steel-tired), and the speed and frequency of crossing the structure. An allowance of 25 per cent. is sufficient for road bridges if the maximum load for which the structure is designed occurs infrequently, and such an allowance is sufficient for electric crane loads. Road structures not designed for the maximum loads that are common in any district should be protected by a permanent notification of the maximum loads permitted to use the structure, and a limitation in both weight and speed should be enforced on all concrete bridges during the first few months after completion of the concrete work. For concrete bridges over roads or railways a speed limitation should be enforced on all traffic passing under the bridge during the period of construction and for a few weeks afterwards.

The British Standard Specification for girder bridges gives the following comprehensive formula for calculating the impact allowance:

$$I = \frac{80}{90 + \frac{(n+1)L}{2}}$$

where I = impact factor

L = the loaded length in feet of the track or tracks producing the maximum stress in the girder or member considered.

n = the number of lines of rolling load traffic which the girder or member is designed to support or assist in supporting.

The formula in this form applies to road bridges only, and for railway bridges the impact allowance should be 50 per cent. greater. Values of I for road bridges and for various values of n and L are given on *Table No. 4*. The maximum value of I can be taken as 0.70 for road bridges and 1.15 for railway bridges.

7.—Dispersion of Point Loads.

Although specified as a point load, a wheel load is considered as bearing on a definite area of the road surface and is then further dispersed over an area dependent on the combined thickness of the road surfacing, filling, and concrete slab. Upon this latter area is based the estimation of the breadth of slab that assists in carrying the wheel load.

The width of the contact area of the wheel on the slab is equal to the width of the tyre, which can be taken as 1 ft. 6 in. for the heaviest loaded wheels of the tractor in the standard loadings, and 9 in. for the trailer wheels. For other vehicles a minimum of 6 in. is usually taken. The length of the contact area depends on the type of tyre and the nature of the road surface, and would be zero for iron tyres on a steel-plate or concrete surface. The maximum contact would probably be obtained with an iron wheel on loose metalling or a pneumatic tyre on a tar-macadam surface. A maximum figure of 12 in. is suggested.

The further dispersal of the load through the total thickness of the road formation and concrete slab is usually considered as acting at an angle of 45 deg. from the edge of the contact area to the centre of the bottom reinforcement, as is shown in the diagram on *Table No. 4*. If " a " equals the length or breadth of the contact area and D equals the depth from the road surface to the centre of the reinforcement, the corresponding length or breadth of the dispersal plane is given by

$$A = a + 2D$$

Mr. W. L. Scott, in his treatment of the wheel loads in accordance with Pigeaud's theory, employs the expression

$$A = \sqrt{(a + 2d)^2 + H^2}$$

• where H = slab thickness and d = thickness of metalling.

This formula gives smaller values than the 45-deg. expression.

In the case of a pair of wheel loads on two rails, if the latter are carried on sleepers the load in any position can be considered as distributed longitudinally over two effective sleepers and transversely over the length of a sleeper. The dispersal is then taken at 45 deg. through the ballast and decking, as indicated on *Table No. 4*. When a rail bears directly on a concrete beam the longitudinal dispersion of the load can be taken at from 4 to 6 times the depth of the rail.

When the concrete slab spans between longitudinal beams, the span of the slab being less than half the span of the beams (as in normal slab and beam bridge construction), authorities differ concerning what width of slab S should be considered as effective in supporting a point load and transmitting this to the beams. All authorities allow dispersion as described to be taken down to the steel, and the most conservative present-day practice is to consider S equal to A . United States practice is to disperse the load horizontally at an angle the tangent of which is $\frac{2}{3}$ that is, $S = 0.67(L - B) + A$ (as indicated on *Table*

No. 4). Mr. E. A. Scott advocates an expression of the form $S = \frac{L}{4} + A$, and Mr. W. L. Scott considers the moment in the slab directly by the Pigeaud method, without computing the equivalent width of slab; this method is further discussed for uniform loads in Chapter IV, and is the basis of the Ministry of Transport's stipulation for the amount of distribution steel given in *Fig. 1*. Sir E. O. Williams has given as an approximate formula $S = 0.67L + 6$ in. with a maximum value of 7 feet. These various formulæ give rather differing results, and, when applied to a wheel contact area of 12 in. by 9 in., an effective

depth $D = 14$ in. and a span of slab $L = 7$ ft., the various values of S are as follows :

	S
Minimum width	3 ft. 1 in.
E. A. Scott	4 ft. 10 in.
Sir E. O. Williams.	5 ft. 2 in.
United States.	5 ft. 6 in.

For general purposes, the United States method gives economical results and is secure if ample longitudinal steel, say one-half of the area of principal transverse reinforcement, is provided.

HORIZONTAL PRESSURE DUE TO EARTH AND CONTAINED MATERIALS

The calculation of the exact value of the horizontal pressure exerted by contained materials on the walls of the container is an uncertain operation except when the material is a liquid. In the latter case the intensity of pressure (p) is equal to the intensity of vertical pressure at any depth (h) from the free surface of the liquid, and is given by the simple hydrostatic expression:

where w = unit weight of the liquid. The units of w and h determine those of p , that is, with w in lb. per cubic foot and h in feet, p would be in lb. per square foot.

$$p = k\omega h \quad (2)$$

The value of k can be determined either graphically or by calculations that are usually based on the wedge theories or the developments of Rankine and Cain. If the total pressure normal to the back of a wall of height H as shown in Fig. 2 equals P , the value of P is given by

where

in which

$$n = \sqrt{\frac{\sin(\theta + \mu) \sin(\theta - \phi)}{\sin(\mu + \beta) \sin(\beta - \phi)}}$$

Referring to Fig. 2 the notation is as follows:

θ = angle of repose of the material.

μ = angle of friction between the wall and the contained material.

ϕ = angle of surcharge of the surface of the material.

β = angle between wall and horizontal.

The total pressure on the back of the wall is given by

$$P_1 = \frac{P}{\cos \mu} \quad \dots \dots \dots (5)$$

and the line of application of P_1 makes an angle of $(90 \text{ deg.} - \mu)$ with the back of the wall. The corresponding force parallel to the wall is

$$F = P_1 \sin \mu = P \tan \mu \quad \dots \dots \dots (6)$$

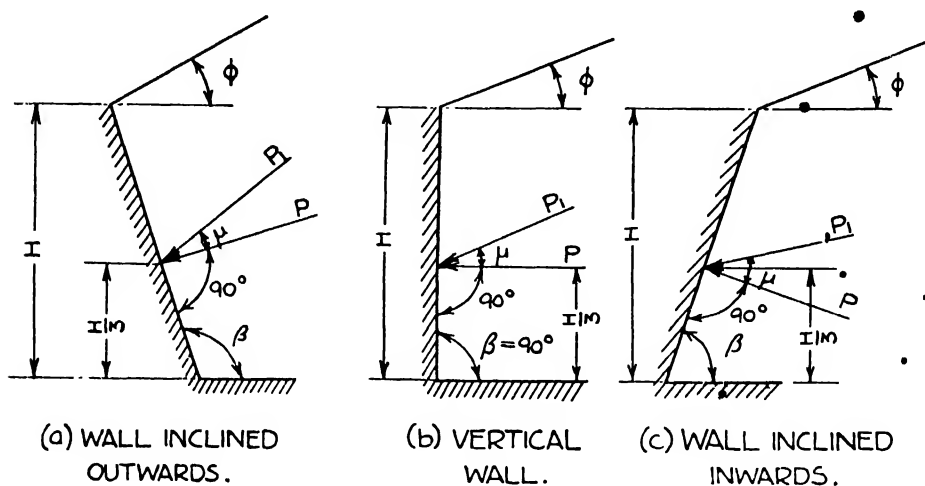


Fig. 2.- Pressure behind Walls.

These general formulæ can be modified to suit the various conditions met with in practice, when usually the friction between the wall and the material is neglected. This modification results in a higher normal pressure and is therefore on the safe side. It is advocated that for general cases the effect of friction should be neglected, and imperatively so in all cases where the material in contact with the wall can become saturated and thus reduce the friction practically to zero or to an uncertain figure. Only in the cases where dry coal, dry sand, grain, cement, or other materials of well-known properties, are being stored, should friction be accounted as relieving the wall of pressure.

When friction is neglected (i.e. $\mu = \text{zero}$) the previous formula reduces to

$$K = \left[\frac{\sin(\beta - \theta)}{(n + 1) \sin \beta} \right]^2 \cdot \frac{I}{\sin \beta} \quad \dots \dots \dots (4a)$$

where

$$n = \sqrt{\frac{\sin \theta \sin(\theta - \phi)}{\sin \beta \sin(\beta - \phi)}}$$

and

$$P_1 = P \text{ and } F = 0 \quad \dots \dots \dots (5a) \text{ and } (6a)$$

For sloping walls the pressures are determined by substituting the known factors in either formula (4) or (4a) depending on whether friction is included or not.

2.—Vertical Walls.

Generally in the case of earth-retaining walls and bunker walls the wall is vertical and the introduction of $\beta = 90$ deg. into formula (4a) in which friction is neglected results in the expressions (7), (8), (9), and (10) which give the factors for determining the intensity of horizontal pressure at any depth h for the limiting conditions of maximum positive surcharge ($\phi = \theta$), level fill ($\phi = \text{zero}$), and maximum negative surcharge ($\phi = -\theta$), and for the general condition of any surcharge ϕ .

VERTICAL WALL, FRICTION NEGLECTED:

Maximum positive surcharge

$$k_1 = \cos^2 \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Level file

$$k_2 = \frac{1 - \sin \theta}{1 + \sin \theta} \quad (8)$$

Maximum negative surcharge

$$k_3 = \left[\frac{\cos \theta}{1 + \sqrt{2} \sin \theta} \right]^2 \quad . \quad . \quad . \quad . \quad (9)$$

- Any surcharge

$$k = \left(\frac{\cos \theta}{n - 1} \right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where

$$n = \sqrt{\sin^2 \theta - \frac{1}{3} \tan \phi \sin 2\theta}$$

Values of k_1 , k_2 , and k_3 for various angles of repose (expressed in degrees and as gradients) are given in *Table No. 5*, together with the various soils, grains, coals, etc., that correspond to these angles. For ordinary earth retaining wall problems with level fill, the value of k_2 is usually taken as 0.27, and, considering the weight of earth as 100 lb. per cubic foot, the corresponding intensity of horizontal pressure is 27 lb. per square foot per foot of height.

When friction between the contained material and the back of the wall is allowed for, the formulæ for the factors for determining the pressure intensity normal to a vertical wall are as follows :

VERTICAL WALL, FRICTION INCLUDED:

Maximum positive surcharge

$$k_1 = \cos^2 \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Level fill

$$k_2 = \left[\frac{\cos \theta}{n + 1} \right]^2 . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8a)$$

where

$$n = \sqrt{\sin \theta (\sin \theta + \cos \theta \tan \mu)}$$

Maximum negative surcharge

$$k_3 = \left[\frac{\cos \theta}{n\sqrt{2} + 1} \right]^2 \quad (9a)$$

where n is as in (8a).

Any surcharge

$$k = \left[\frac{\cos \theta}{n + 1} \right]^2 \quad (10a)$$

where

$$n = \sqrt{\frac{\sin(\theta + \mu) \sin(\theta - \phi)}{\cos \mu \cos \phi}}$$

Generally accepted values of μ , θ , and w for such dry materials as finely-ground cement, coal, grain, sand, and ashes are given on *Table No. 5*, and for these materials the values of K_1 , K_2 , and $K_3 (= k_1 w$, etc.) are tabulated, having been calculated in accordance with formulæ (7), (8a) and (9a). From these values of K can be computed the intensity of horizontal pressure: $p = Kh$. If H is the total height of the wall, the total horizontal pressure is given by $P = p \frac{H}{2}$, and the total thrust on the wall acting at an angle of μ to the horizontal is given by $P_1 = \frac{P}{\cos \mu}$.

3.—Surcharge.

As will be seen from the tabulated values of the pressure factors, the amount of surcharge of the surface of the filling behind the wall has a marked effect on the magnitude of the pressure. For all practical purposes any alteration of the profile of the fill beyond the point *B*, *Fig. 3(a)*, or any additional loading

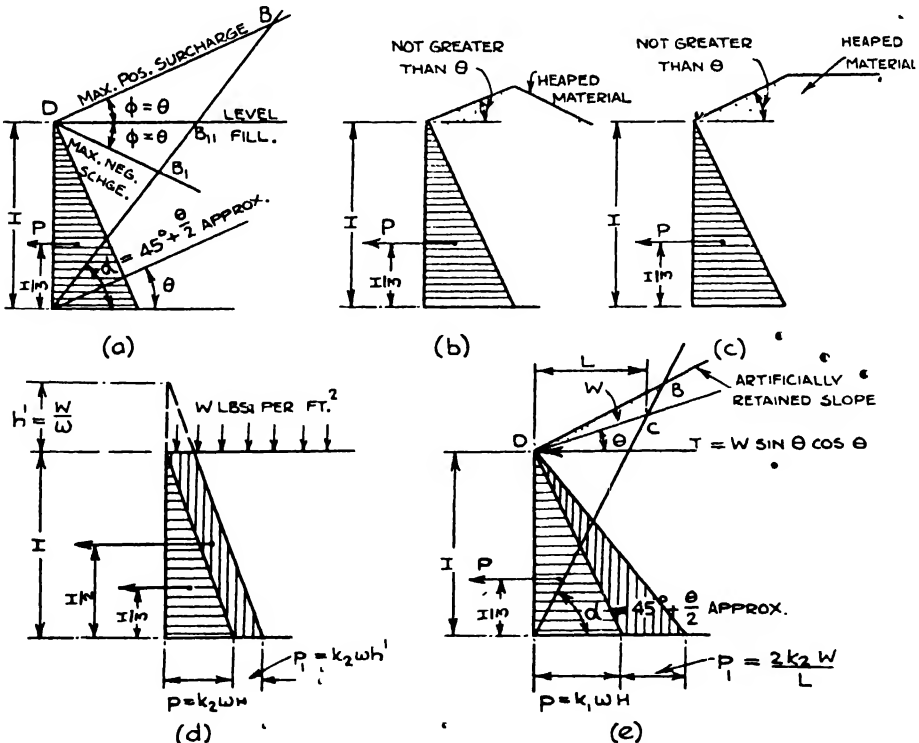


Fig. 3.—Surcharges.

beyond this point, has little or no effect upon the pressure on the wall. The general formula takes account of the surcharge due to any fill profile lying between the limits DB and DB_1 .

When the surface of the filling is not uniformly sloped, special treatment is required. In the common cases depicted in *Figs. 3(b)* and *3(c)* the magnitude of the pressure on the wall is somewhere between that due to level fill and maximum positive natural surcharge. On *Table No. 5* empirical expressions are suggested that allow for the increase of pressure due to these types of intermediate surcharges. The total pressure on the wall is augmented above that for a level fill by an amount proportional to the mean increase in head of material. Such surcharges give zero pressure at the top of the wall, and the centre of total pressure will be at one-third of the total height.

When the fill behind a wall is level but liable to subjection to self-retaining superloads, such as stacked materials, traffic, or buildings, the total superload should be converted into an equivalent head of the same material as that retained by the wall, and the pressure intensity on the back of the wall augmented uniformly throughout the depth of the wall. Thus there will be a definite intensity of pressure at the top of the wall; these conditions are illustrated in *Fig. 3(d)* and indicated on *Table No. 5*.

A type of surcharge not covered by the foregoing observations and formulae is that shown in *Fig. 3(e)*. In this case the angle of surcharge exceeds the natural angle of repose, as may occur by artificially protecting a bank of earth by turfing or stone pitching. For such a case it has been suggested that the weight of earth W above the angle of repose, represented by the triangle BCD , should be considered as a point load operating at the top of the wall. The magnitude of the resultant horizontal thrust T and increase in pressure would be as shown in *Fig. 3(e)*, and each section of the wall should be designed for the extra moment due to T .

A single point superload on the filling behind a wall presents a problem in dispersion, and, considering 45 deg. as a reasonable angle of dispersion, the pressure intensity additional to the pressure due to the filling alone is indicated in *Fig. 4*, from a study of which the method of arriving at the final result should be clear.

4.—Experimental Data.

In general practice in Britain, earth pressures are determined by the purely theoretical formulae of Rankine, Cain, and Coulomb. Many investigators, working with a sand filling, have experimented on model walls in order to determine what relation actual pressures bear to the theoretical pressures. The unanimous conclusion is that the Rankine formula gives too great a value for the pressure exerted, and thus retaining walls designed on this formula err on the side of safety. The theoretical deduction assumes that the angle of internal friction and the surface angle of repose are identical, whereas Crosthwaite and other investigators have found that the friction angle is less than the angle of repose and depends on the consolidation of the material. The ratio between the internal friction angle and the natural angle is approximately 0.9 to 1 according to experiments carried out at the University of Cincinnati.

For level fills the horizontal pressure given by Coulomb's formula (a special case of the general formula) is in the form

$$p = wh \cdot \tan^2\left(45^\circ - \frac{\theta}{2}\right). \text{ This is formula No. 8 in another form and agrees}$$

very closely with actual pressures if the angle of internal friction is substituted for the natural angle of repose. The maximum pressure seems to occur immediately after back-filling has been completed, and the pressure decreases as settling proceeds.

The measured vertical component of the pressure on the back of the wall appears to conform to the theoretical relationship $V = P \tan \mu$ where P is horizontal pressure and μ is the independently determined angle of friction

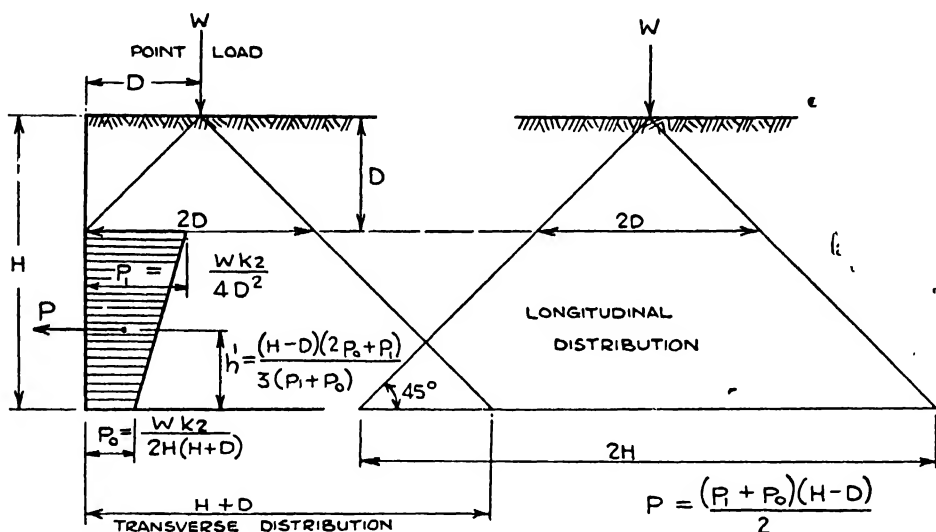


Fig. 4.—Dispersion of Point Load through Earth.

between the wall and filling. A rise in temperature produces measurable increases in pressure, the increase being of the order of 1 per cent. per 10 deg. F.

The point of application of the resultant thrust on walls with level fills appears to be at the theoretical height of $\frac{\text{total height}}{3}$ for shallow fills, and according to the Cincinnati tests rises with time and with increased depth of fill. According to the Feld-Moncrieff investigations the height of application is at the third-point for negative surcharge and level fills, but rises with increase of angle of surcharge.

Some investigators report that superloading beyond the plane of rupture increases the pressure on the wall, and others come to the conclusion that loads outside the wedge ordinarily considered can be neglected. The increase of pressure due to incidental superloading remains after the load is removed. For walls surcharged by the slope of earth behind them the wedge theory seems to give results almost 30 per cent. in excess of measured values.

Mr. Bell has shown that Rankine's formula is inadmissible for pressures due to clay, and gives the following expression for the intensity of pressure at any depth h :

$$p = wh \tan^2\left(\frac{\pi}{4} - \frac{a}{2}\right) - 2k \tan\left(\frac{\pi}{4} - \frac{a}{2}\right)$$

where the factors k and a have the following values:

Very soft puddled clay:	$k = 450$ lbs. per square foot	$a =$ zero
Soft puddled clay:	670	3 degrees.
Moderately firm clay:	1120	5 "
Stiff clay:	1570	7 "
Very stiff boulder clay:	3600	16 "

The view has been put forward that the pressure behind a flexible wall adjusts itself as the wall deflects, and this adjustment is such as substantially to decrease the bending moments. The magnitude of the reduction is discussed in Chapter VII in the consideration of sheet pile walls.

5. Deep Containers.

The foregoing observations and formulæ for the pressures on walls of containers refer only to retaining walls and shallow containers. In deep containers

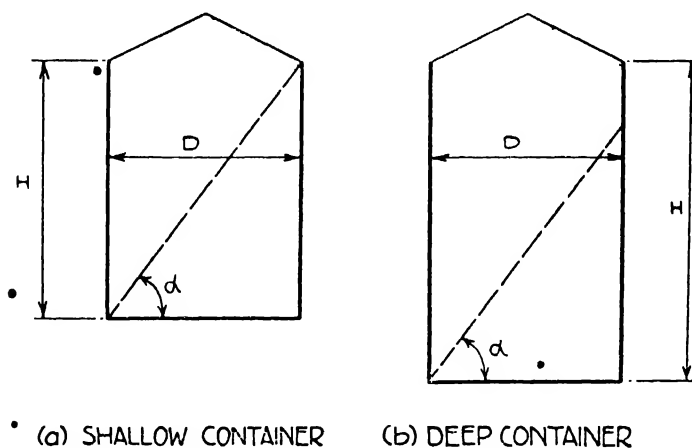


Fig. 5.—Silos.

such as tall grain silos, the arching effect of the contained material considerably reduces the horizontal pressure, so that the principle that the pressure is proportional to depth of filling no longer holds good. The limiting condition that determines whether any particular container shall be considered as "shallow" or "deep" (that is, as a "bunker" or as a "silo") is that if the plane of rupture strikes the opposite wall before reaching the free surface, as in Fig. 5(b), the con-

tainer can be treated as "deep." If otherwise, as in *Fig. 5(a)*, the container must be considered as "shallow."

The theoretical minimum depth of a "deep" container is given by

$$H = D \left(\tan \theta + \sqrt{\tan \theta \frac{1 + \tan^2 \theta}{\tan \theta + \tan \mu}} \right)$$

where D is the least breadth of the container.

For grain the least depth is theoretically approximately equal to $1\frac{1}{4}$ times the breadth of the container, and for cement approximately equal to one-and-a-half times the breadth. During the emptying process the arching action on which pressure reduction depends is completely destroyed, and pressures appreciably in excess of the theoretically determined pressures are obtained. It would seem therefore that for absolute security no container that is not at least twice as deep as it is wide should be treated as a "silo."

The value and variation of value of the pressure in silos is usually computed by either Janssen's formula or Airy's formula. The former gives results slightly less than the latter and in its general form is as follows:

$$p = \frac{wQ}{\tan \mu} \left(1 - \frac{1}{N} \right)$$

where

p = horizontal pressure in lb. per square foot.

w = weight in lb. per cubic foot of material.

Q = ratio of plan area in sq. ft. to perimeter in ft. of bin $\frac{A}{L}$

μ = angle of friction between wall and material.

N = the number whose common log. $\frac{K \tan \mu H}{2.303 Q}$

where

K = ratio of horizontal to vertical pressures

H = depth of filling in feet at point considered.

The vertical pressure (lb. square foot) on any horizontal section of the material is given by

$$V = \frac{p}{K}$$

Therefore the total pressure on any horizontal plane is $\frac{pA}{K}$, and the load transferred to the walls of the container by friction is $\left(wH - \frac{p}{K} \right) Q$ lb. per foot run of wall.

For granular materials such as grain and cement a value of $K = 0.5$ is usually adopted, and a value of $\tan \mu = 0.444$. *Table No. 6* gives the magnitude of

$C = \frac{p}{w}$ for various values of Q and H , incorporating these values of K and $\tan \mu$.

For circular, square, or regular polygonal containers, $Q = \frac{D}{4}$ where $D =$

diameter or distance between opposite faces of the container, and the following expressions apply for grain at 48 lb. per cubic foot :

$$N = \text{the number whose common log.} = \frac{0.385H}{D}$$

$$p = 27D \left(1 - \frac{1}{N} \right) \text{ lb. per square foot}$$

$$V = 2p \text{ lb. per square foot}$$

and the load carried by the walls at any depth H

$$= (12H - 0.5p)D \text{ lb. per foot run.}$$

From an inspection of the data on *Table No. 6* it will be seen that the pressure increases very little below a depth of three times the least width of the container. Tests have indicated that the maximum pressures occur in silos during filling, and are somewhat greater for a bin filled rapidly than one filled slowly.

6.— Passive Resistance.

The whole of the foregoing remarks on the horizontal pressure exerted by contained or retained filling concern the active pressures due to such filling. If a horizontal pressure in excess of this active pressure is applied to the vertical face of a retained bulk of granular material, the passive resistance of the material is brought into action. Up to a limit, determined by the characteristics of the particular material, the passive resistance will be equal to the applied pressure ; the maximum value the resistance can attain for a granular material with a level surface is given theoretically by the reciprocal of the Rankine active pressure coefficient, that is,

$$p_r = \left(\frac{1 + \sin \theta}{1 - \sin \theta} \right) wh = \frac{wh}{k_2}$$

where p_r = maximum passive resistance at any depth

w = weight per cubic foot of material.

θ = angle of repose.

k_2 = coefficient as given on *Table No. 5*.

It is not easy to assess a practicable working value for the passive resistance when the surface of the material is other than level, and it is advisable never to assume a value exceeding the value given, and to use discretion when the surface has a negative surcharge in which case the passive resistance may be very small.

The passive resistance of earth has to be taken into account when considering the resistance to sliding in retaining wall design, when dealing with the forces acting on sheet piling, and when designing earth anchorages, but in these cases careful consideration must be given to those factors that may reduce the probable passive resistance to a value approaching more nearly the active pressure. The chief of these factors is wetness. Abnormal dryness may cause clayey soils to shrink away from the surface of the structure, thus necessitating a small but most undesirable movement of the structure before the passive resistance is brought into play.

CHAPTER IV

BENDING MOMENT AND SHEARING FORCE

1.—Moment, Shear, and Deflection.

THE moment at any section in a member subject to flexure is defined as the algebraic sum of the moments of all loads and reactions to the left (or right) of the section considered, and the shear force at any section is the algebraic sum of all the loads and reactions to the left (or right) of the section. The shear force at any section can thus be expressed as the rate of change of bending moment at the section; expressed mathematically, bending moment is the first integral of shear, which in turn is the first integral of the load. It follows from the definition of shear that the point of zero shear is the point of maximum bending moment.

From considerations of the theory of elastic flexural deformations it is shown that the rate of change of the slope at any point in a beam is expressed by the value of the $\frac{M}{EI}$ factor for that point, where M = bending moment, I = moment of inertia, and E = elastic modulus, all factors relating to the point under consideration. Thus slope is the first integral of $\frac{M}{EI}$, and the first integral of the expression for the slope at any point in the beam represents the expression for the elastic deflection.

2.—Cantilevers and Single-Span Beams.

For a cantilever or for a single-span beam simply supported (that is without restraint) at its two extremities, the practical interpretation of the foregoing mathematical definition of bending moment and shear force at any section is a simple matter. On *Table No. 7* are tabulated the maximum bending moments and end shears (that is, reactions on the supports) produced by various commonly occurring loads carried on cantilevers and single-span freely-supported beams.

A beam with one end fixed and one end freely supported can be considered as a cantilever subject to two distinct systems of loading:

- (i) the imposed load
- (ii) a point load (the reaction on the prop)

the magnitude of which is such as would give the same end deflection upwards on a cantilever as the downward deflection produced by the imposed load. For the loaded cantilevers tabulated on *Table No. 7* the maximum deflection coefficients are given, as are also the moments and reactions produced by similar loads on single-span beams fixed at one end and simply supported at the other.

For cases of loading other than those tabulated the deflections can be found from the following expressions :

Type of Load.	End Deflection of Cantilever. (multiply coefficient by $\frac{WL^3}{EI}$)
(a) Point load at aL from support :	$\frac{a^2(3-a)}{6}$
(b) Point load at end of cantilever :	$\frac{1}{3}$
(c) Uniformly distributed load ; loaded length = bL ; load starts at point aL from support (see diagram on Table No. 7) :	$A + B(1-a+b)$ where $A = \frac{1}{24}[8a^3 + 18a^2b + 12ab^2 + 3b^3]$ and $B = \frac{1}{6}[(a+b)^3 - a^3] \frac{1}{b}$
(d) Uniformly distributed load throughout span	
(e) Triangularly distributed load decreasing from maximum at support to zero at a point aL from support	$\frac{1}{60}(5-a)$
(f) Ditto when $aL = L$	$\frac{1}{15}$
(g) Triangularly distributed load decreasing from maximum at free end to zero at a point aL from support	$\frac{1}{60}[20 - 10a + a^3]$
(h) Ditto when aL equals zero	$\frac{1}{3}$
(j) Triangularly distributed load with apex at centre of span	$\frac{11}{90}$
(k) Parabolic load symmetrical on span	$\frac{7}{60}$

The values of E and I that should be adopted are discussed in Chapters V and IX, but when comparative deflections are required the exact numerical values of these factors do not enter into the calculation. The method of applying these deflection formulæ is illustrated in the following example.

To find the reaction in the prop at C and the bending moment at A for a beam loaded as shown in Fig. 6.

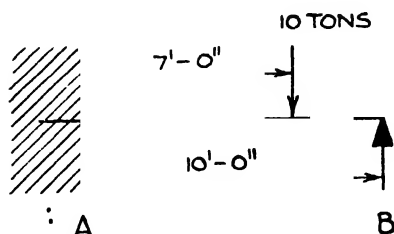


Fig. 6. —Propped Cantilever.

The expression (a) given on page 27 is applicable for determining the end deflection of a 10-ft. cantilever supporting a load placed 7 ft. from the support, that is, $a = \frac{7}{10} = 0.7$

$$\therefore d = \frac{0.7^2(3 - 0.7)WL^3}{6EI} = 0.188 \frac{WL^3}{EI}.$$

From Table No. 7 or from (b) the deflection at the end of a cantilever subjected to a point load R located at its extremity is given by $\frac{RL^3}{3EI}$, and since this deflection must equal d

$$0.188 \frac{WL^3}{EI} = \frac{RL^3}{3EI}; \text{ therefore } R = 0.188 \times 3W = 0.564W$$

hence $R = 5.64$ tons, which is the reaction on the prop at B .

The bending moment at A will be the algebraic sum of the moments of R and W about A , or

$$M = (5.64 \times 10) - (10 \times 7) = -13.6 \text{ foot-tons.}$$

The bending moment diagram for an encastré beam, that is, one fixed at both ends, is derived from the principle that the area of the bending moment diagram

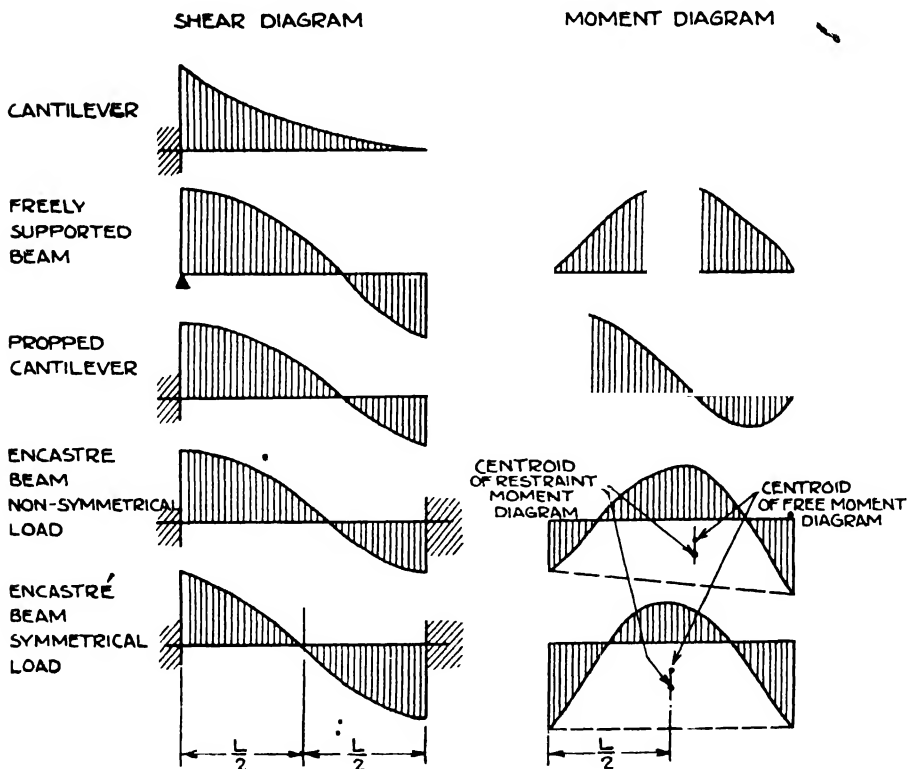


Fig. 7.—Typical Bending Moment and Shear Diagrams.

due to the same imposed load on a freely-supported beam of equal span (the "free-moment" diagram) is equal to the area of the restraint moment diagram; the centroids of the two diagrams should be vertically one above the other. The shape of the free-moment diagram depends upon the characteristics of the imposed load, but the restraint-moment diagram is a trapezium. For loads symmetrically disposed on the beam the centroid of the free-moment diagram will be above the mid-point of the span, and thus the restraint-moment diagram will be a rectangle, giving at each support identical restraint moments each equal to the mean height of the free B.M. diagram. Typical bending moment and shear force diagrams for cantilevers, propped cantilevers, freely-supported beams, and encastré beams are illustrated in *Fig. 7*.

The shear force diagram for a beam with one or both ends fixed can be derived from consideration of the variation in bending moment. The shear due to the restraint moment alone is constant throughout the length of the beam, and equals the difference between the two end moments divided by the span. This shear should be algebraically added to the shear due to the imposed load, the latter shear being determined as if the beam were simply supported; that is the resultant reactions at either support will be the sum or difference of the restraint-moment shear and the free-moment shear.

With a symmetrically loaded beam with both ends fixed, the restraint moments at each end being the same, the shear due to these moments considered alone is zero; therefore the resultant shear force variation is identical with that for the same beam freely supported, and the end reactions are both equal to half the total load on the beam. The reactions and maximum moments in encastré beams carrying ordinary types of loading are given on *Table No. 7*.

3. —Theoretical Consideration of Continuous Beams.

The bending moment in a beam continuous over two or more spans is ordinarily calculated by the Three Moment Theorem (or can be derived from the general formulæ for members subject to flexure given in the following Chapter). In its general form, Clapeyron's Theorem of Three Moments for any two successive continuous spans is expressed by

$$M_A \frac{I_1}{I_1} + 2M_B \left(\frac{I_1}{I_1} + \frac{I_2}{I_2} \right) + M_C \frac{I_2}{I_2} = 6E \left[\frac{d_B - d_A}{L_1} - \frac{d_C - d_B}{L_2} \right] - 6 \left[\frac{A_1 z_1}{I_1 L_1} + \frac{A_2 (L_2 - z_2)}{I_2 L_2} \right]$$

where, referring to *Fig. 8*,

M_A , M_B , and M_C = bending moments at supports A , B , and C .

I_1 and I_2 = moment of inertia of spans L_1 and L_2 respectively.

d_A , d_B , and d_C = deflection of each support from original position of beam.

A_1 and A_2 = area of free moment diagrams for loads on span L_1 and L_2 respectively.

z_1 and z_2 = position of centroid of A_1 and A_2 respectively.

For the usual consideration of level supports this general expression becomes

$$M_A \frac{I_1}{I_1} + 2M_B \left(\frac{I_1}{I_1} + \frac{I_2}{I_2} \right) + M_C \frac{I_2}{I_2} = -6 \left[\frac{A_1 z_1}{I_1 L_1} + \frac{A_2 (L_2 - z_2)}{I_2 L_2} \right]$$

If the moment of inertia of the beam throughout the two spans is assumed to be constant, the formula reduces to

$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2$$

$$= 6EI \left[\frac{d_B - d_A}{L_1} - \frac{d_C - d_B}{L_2} \right] - 6 \left[\frac{A_1 z_1}{L_1} + \frac{A_2 (L_2 - z_2)}{L_2} \right]$$

and with level supports and constant moment of inertia (the conditions usually approximated to in normal design) the Three Moment Theorem becomes

$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = -6 \left[\frac{A_1 z_1}{L_1} + \frac{A_2 (L_2 - z_2)}{L_2} \right]$$

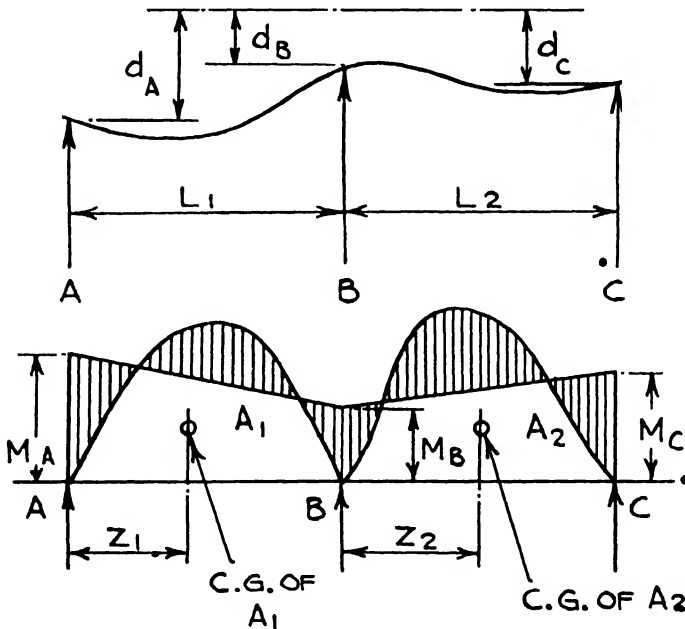


Fig. 8.— Continuous Beams.

If the load is uniformly distributed and equal throughout the two spans considered, and assuming level supports and constant moment of inertia, the following formula applies :

$$M_A L_1 + 2M_B (L_1 + L_2) + M_C L_2 = -\frac{w}{4} (L_1^3 + L_2^3)$$

where w = load per foot run of beam.

The shear force variation can be determined by first considering each span as freely supported and algebraically adding the rate of change of restraint moment for the span considered. For example, if W_1 is the total load on span AB , the "free beam" shear at A is $\frac{W_1(L_1 - z_0)}{L_1}$ and at B is $-\frac{W_1 z_0}{L_1}$, where z_0 = distance from A to centre of gravity of the load. The restraint moment shear is constant throughout the span and equals $\frac{M_A - M_B}{L_1}$. The resultant shear at A is given by

$$V_A = \frac{W_1(L_1 - z_0)}{L_1} + \frac{M_A - M_B}{L_1}$$

• and at B is $V_B = -\frac{W_1 z_0}{L_1} + \frac{M_A - M_B}{L_1}$

When the known factors relative to load, span, moment of inertia, and relative levels of the supports are substituted in the appropriate general moment

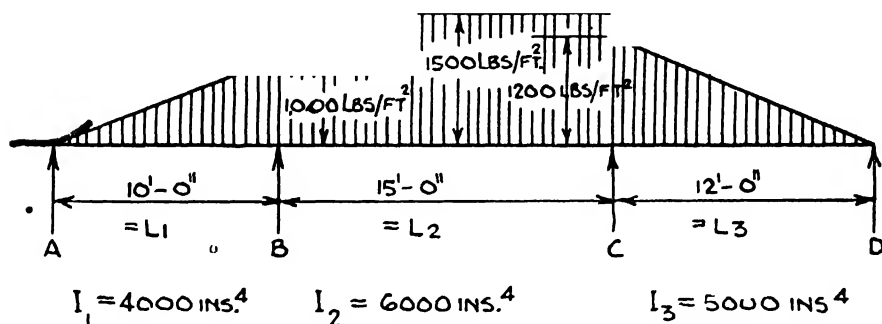


Fig. 9. Continuous Beam Example.

formula, an expression for three unknown moments is obtained for any pair of spans. That is, for n spans, $n - 1$ equations can be obtained containing $n + 1$ unknowns (the moments at $n + 1$ supports). The two excess unknowns represent the moments at the end supports, and if these moments are known or can be assumed the moments at the intermediate supports can be determined. Thus for a freely-supported end the moment would be zero, and the unknown representing the moment at a free support would automatically disappear. For a perfectly fixed end the moments can be determined if an additional span is considered continuous at the fixed end. This additional span should be identical in length, loading, etc., with the original end span except that the loading is so arranged as to produce symmetry about the original end support with the loading on the original end span. The moment at the new end support should be considered equal to that at the original penultimate support, and thus an additional equation is obtained without introducing a further unknown.

A numerical example will illustrate the application of the Three Moment Theorem in determining the support moments for the beam system indicated in

Fig. 9. Assuming level supports, the appropriate formula for spans AB and BC is

$$M_A \frac{L_1}{I_1} + 2M_B \left(\frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + \frac{M_C L_2}{I_2} = - \left[\frac{A_1 z_1}{I_1 L_1} + \frac{A_2 (L_2 - z_2)}{I_2 L_2} \right] 6. \quad (1)$$

and for spans BC and CD ,

$$M_B \frac{L_2}{I_2} + 2M_C \left(\frac{L_2}{I_2} + \frac{L_3}{I_3} \right) + \frac{M_D L_3}{I_3} = - \left[\frac{A_2 z_2}{I_2 L_2} + \frac{A_3 (L_3 - z_3)}{I_3 L_3} \right] 6. \quad (2)$$

For span AB , $\frac{L_1}{I_1} = \frac{10}{4000} = 0.0025 \left(\frac{\text{ft.}}{\text{ins.}^4} \right)$

$$\frac{A_1}{L_1} = \frac{w_1 L_1^2}{24} = \frac{1000 \times 10^2}{24} = 4,170 \text{ ft. lb.}$$

$$z_1 = \frac{8}{15} L_1; \text{ thus } \frac{z_1}{I_1} = \frac{8}{15} \times 0.0025 = 0.00133.$$

$$\text{Hence } \frac{A_1 z_1}{I_1 L_1} = 4,170 \times 0.00133 = 5.55.$$

For span BC , $\frac{L_2}{I_2} = \frac{15}{6000} = 0.0025$

$$\frac{A_2}{L_2} = \frac{w_2 L_2^2}{12} \text{ (from Table No. 15)} = \frac{1500 \times 15^2}{12} = 28,125 \text{ ft. lb.}$$

$$\frac{z_2}{I_2} = \frac{L_2}{2I_2} = \frac{L_2 - z_2}{I_2} = 0.5 \times 0.0025 = 0.00125.$$

$$\text{Hence } \frac{A_2 z_2}{I_2 L_2} = \frac{A_2 (L_2 - z_2)}{I_2 L_2} = 28,125 \times 0.00125 = 35.156.$$

For span CD , $\frac{L_3}{I_3} = \frac{12}{5000} = 0.0024$

$$\frac{A_3}{L_3} = \frac{w_3 L_3^2}{24} = \frac{1200 \times 12^2}{24} = 7,200 \text{ ft. lb.}$$

$$z_3 = \frac{7}{15} L_3; \text{ thus } \frac{L_3 - z_3}{I_3} = \frac{8}{15} \times 0.0024 = 0.00128.$$

$$\text{Hence } \frac{A_3 (L_3 - z_3)}{I_3 L_3} = 7,200 \times 0.00128 = 9.21.$$

If the beam is simply supported at A and D , $M_A = M_D = \text{zero}$, and substituting known values in (1) and (2), we obtain

$$2M_B(0.0025 + 0.0025) + 0.0025M_C = -(5.55 + 35.16)6 \quad (3)$$

$$0.0025M_B + 2M_C(0.0025 + 0.0024) = -(35.16 + 9.21)6 \quad (4)$$

Thus

$$0.0100M_B + 0.0025M_C = -244 \quad (5)$$

$$0.0025M_B + 0.0098M_C = -266 \quad (6)$$

Multiplying (6) by $\frac{0.0100M_B}{0.0025M_B} = 4$

$$0.0100M_B + 0.0392M_C = -1064 \quad (7)$$

Subtracting (5) from (7)

$$0.0367M_C = 820$$

$$\text{Hence } M_C = -22,400 \text{ ft. lb.}$$

By substituting in (5)

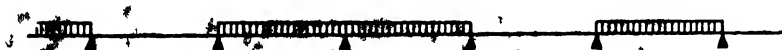
$$0.0100M_B = -244 - 55 = -189$$

$$\text{Hence } M_B = -18,900 \text{ ft lb}$$

When the negative (or positive) support moments have been calculated, the diagram of the continuity moments is combined with the diagram of "free moments" and the resulting bending moments on the beam system are obtained. The value of the bending moments at the support and in the spans depends upon the incidence of live loading, and generally for equal spans or with spans approximately equal the disposition of live loading given in Fig. 10(a) gives the maximum



(a) MAXIMUM POSITIVE B.M. IN SPAN A B.



(b) MAXIMUM NEGATIVE B.M. AT SUPPORT A

Fig. 10.—Incidence of Loading to Produce Maximum Moments.

positive bending moment at midspan, and that given in Fig. 10(b) results in the maximum negative bending moment at a support. On Table No. 8 are given the values of the coefficients $\left(\frac{\text{moment}}{\text{total load and span}} \right)$ for the bending moments at the middle of each span and at each support for two, three, four, and five or more continuous equal spans carrying identical loading on each span, which is the usual disposition of the dead loading on a beam system. The coefficients for the maximum bending moments at midspan and support for any incidence of loading are also tabulated, the types of loading considered being a uniformly distributed load, a single point load, and double point loads.

The shear force produced by a uniformly distributed loading when all spans are loaded, together with the maximum shears due to any incidence of such loading, are also tabulated on Table No. 8.

Table No. 9 has been prepared to facilitate the calculation of the support moments in beams with constant moment of inertia and continuous over two, three, or four equal or unequal spans, and carrying almost any type or incidence of live and dead loads. The basis of the method is that the loading on any span

of a given system of continuous beams can be divided into one or more of the following types of loading

- (a) Uniformly distributed load over the whole of a span.
- (b) Triangularly distributed load with maximum value at the centre of the span.
- (c) Triangularly distributed load with maximum value at one end of span.
- (d) Point load at centre of span.
- (e) A point load at each of the third-points of span.

It is not practicable to cover every probable type of loading, but other types of loads could be allowed for as follows:

Partially distributed load at one end of span would give coefficients having values between those for (a) and (c).

Three or more equal point loads would give coefficients having values between (a) and (e).

A beam system with several spans loaded can be divided into a series of beam systems having the same number of spans, only one of which is loaded with any one particular type of load, and each of these loads gives certain bending moments at each support, the ultimate bending moment at each support being the algebraic sum of the bending moments at that support due to each type of load. Although the table only gives data to enable support moments to be calculated, the midspan moments can be readily calculated or determined graphically therefrom.

For equal spans the moment at any support due to any one span loaded with any one type of load is given by the expression

$$\begin{aligned} \text{Support moment} &= \text{load factor} \times \text{support moment} \\ &\quad \text{coefficient} \times \text{total load} \times \text{span} \\ \text{or} \quad M_s &= FQ.WL \end{aligned}$$

and the total moment at any one support = ΣM_s .

If any one type of load extends equally over all the spans of a given system of equal spans, the loading need not be split up into single-span loads, but the values of Q tabulated for this case should be used.

For unequal spans the method is similar to that for equal spans, except for the introduction of the further factor U , termed the "moment multiplier," which varies with the particular support being treated, and with the particular span considered loaded. The value of U depends only on the ratio that the several spans bear to the "base span," which is chosen to give the simplest expressions for the moment multiplier. These expressions are given on Table No. 9. For unequal spans a support moment would be expressed by

$$\begin{aligned} \text{Support moment} &= \text{support moment for equal spans} \\ &\quad \times \text{moment multiplier.} \\ \text{or} \quad M_s &= UFQ.WL. \end{aligned}$$

When all spans of an unequal span system are loaded with an identical type of load it is not possible to use the moment coefficients tabulated for such a case, as can be done for equal spans, as the moment multiplier differs for each span and for each support.

4.—Continuous Beams with Varying Moments of Inertia.

The foregoing considerations have particular application to beams that have a constant moment of inertia throughout all spans, and, although strictly speaking the moment of inertia of a reinforced concrete beam of constant depth may vary not inconsiderably at various sections, it is usual to neglect these variations in practice both for beams having equal depth throughout and for beams having normally proportioned haunches at the supports.

Where the depth of a beam system varies beyond these limits, neglect of the variation of moment of inertia in the bending moment calculations leads to results that differ considerably from the probable moments. When the moment of inertia is practically constant throughout each span of a system of continuous beams, but differs in one span relative to another, the general formula given at the beginning of the preceding paragraph for these conditions is applicable, but when the moment of inertia varies within the length of each span the semi-graphical method given on *Table No. 10* should be employed.

If the moment of inertia varies throughout the span in such a way that it can be represented by a simple equation, the general Three Moment Theorem can be used if $\frac{M}{I}$ is substituted for M and if the area of the $\frac{M}{I}$ diagram is taken instead of the area of the free moment diagram. The solution of the derived simultaneous equations will then give values of $\frac{\text{support moment}}{I}$ enabling a complete $\frac{M}{I}$ diagram for the beam system to be constructed, for which the moment at any section can be readily obtained by multiplying the appropriate ordinate of the $\frac{M}{I}$ diagram by the moment of inertia at the section.

When time or other circumstances do not permit these methods to be followed, and the bending moments are calculated upon the assumption of constant moment of inertia, some adjustment can be made for the effect of the neglected factor if it is borne in mind that an increase in the moment of inertia in the vicinity of the support will cause an increase in the bending moment at that support and a consequent decrease in the positive bending moments in the adjacent spans, and vice versa. As a guide in making the adjustment the approximate factors given on *Table No. 10*, representing the percentage addition to or deduction from the calculated moments, can be employed.

5.—Approximate Bending Moments in Continuous Beams.

The precise determination of the theoretical moments in continuous beams involves much mathematical labour, except in certain cases that occur often enough to warrant tabulation. Having regard to the assumptions of unyielding knife-edge supports and probably constant moment of inertia that are involved in the theoretical elastic analysis, the probability of the mathematically obtained moments being greater or less than those actually realised should be considered. The effect of variation of moment of inertia has been discussed in the preceding paragraph, and of equal importance is the consideration of the characteristics of the support.

The following factors cause a decrease in the negative moment at a support and consequently an increase in the positive moment in the adjacent spans:

- (a) Yielding of the support considered relative to adjacent supports.
- (b) Supports of considerable width.
- (c) Support and beam monolithic.

Likewise sinking of one or both of the adjacent supports can cause an increase in the negative moment at a support and consequently a decrease in the positive moment in the adjacent span. Yielding of one support relative to the neighbouring ones may be sufficient to convert the moment at that support into a positive moment.

When the moments have been calculated with spans equal to the distance between centres of supports, the maximum moment can be considered as that occurring at the edge of the support. When the supports are of considerable width the span can be considered as the clear distance between the supports plus the depth of beam, and an additional span introduced equal to the width of the support minus the depth of the beam. The loading on this additional intermediate span can be considered as the reaction on the support spread uniformly along the part of the beam over the support. When a beam is constructed monolithic with a very wide and massive support the continuity effect of the span or spans beyond the support may be negligible, in which case the beam should be treated as fixed at the support.

Other cases of beams being constructed monolithic with a support of moderate size, such as in normal beam and column construction, are dealt with in the following Chapter, but in these cases the moment at the support section of the beam is never more than the moment calculated by the Three Moment Theorem.

The indeterminate nature of the actual moments produced leads in practice to the adoption of approximate bending moment coefficients for continuous beams of approximately equal spans and with uniformly distributed loads, and the commonly accepted coefficients are given on *Table No. 10*. These coefficients can be adopted to the total load on the span, or can be used to determine the moments due to dead and live loading separately. In the table the coefficients have been extended to give the values for normal ratios of dead to live uniformly distributed loads.

6.—Graphical Determination of Bending Moments in Continuous Beams.

A commonly adopted method of finding the distribution of bending moments on a continuous beam is by a graphical method based on the determination of "fixed points." The basis of the method, which is fully described on *Table No. 11*, is that there is a point (termed the "fixed point") adjacent to the left-hand support of any span of a continuous system at which the moment is unaffected by any alteration in the bending moment at the right-hand support. A similar point occurs near the right-hand support, the moment at this point being unaffected by alteration in the moment at the left-hand support. When a beam is rigidly fixed at a support the "fixed point" is one-third of the span from that support, and when freely supported the fixed point coincides with the support. For intermediate conditions of fixity the "fixed point" will be located between these extremes, and the relative position of the left-hand (or right-

hand) fixed points in two adjacent continuous spans is given by the expression

$$l_2 = \frac{L_2^2(L_1 - l_1)}{3(L_1 + L_2)(L_1 - l_1) - L_1^2}$$

referring to *Fig. 11*; or l_2 can be found from l_1 by graphics.

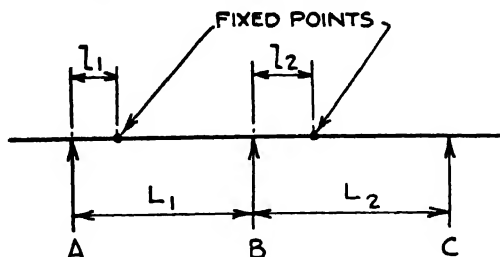


Fig. 11.—Fixed Points.

Upon combining the free moment diagram with the position of the fixed points for any given span, as described on *Table No. 11*, the resultant negative and positive moments throughout the beam system, due to this loaded span, can be determined, and by treating each span separately the envelopes of the maximum possible moments throughout the system can be readily developed.

7.—Rolling Loads.

• Moving (or rolling) loads, such as vehicular loads, that can traverse a system of continuous spans and can be in any position thereon, are best treated by influence line methods. An influence line is usually a curve with the beam span as a base, the ordinates of the curve at any selected point giving the value of the bending moment produced at some particular section of the beam when a unit load is acting at the selected point. Influence lines can be drawn for any section of the beam, and sufficient data are given on *Tables Nos. 12 and 13* to enable the influence lines for the critical sections of beams continuous over two, three, four, or more spans to be constructed. By plotting the relative position of the load on the beam (drawn to scale) the moments at the section being considered are derived as explained in the examples given on the pages facing the appropriate tables.

8.—Bending Moments in Slabs.

Although the bending moments in slabs supported on two opposite sides are usually calculated in the same way as are beam moments, due account being taken of continuity where necessary, it is generally acknowledged that there is a greater margin of strength in a slab than in a beam when both members are designed on the same principles and for the same limiting stresses. There is, therefore, some reason for adopting smaller bending moment coefficients in the design of continuous slabs than are adopted for beams, and for this purpose mid-span moment coefficients similar to those given in the German regulations are advocated for slabs carrying uniformly distributed loads and continuous over

approximately equal spans. Such coefficients are given on *Table No. 14*, and allow for the effect on the moments of the provision of haunches at each end of the span. The coefficients given on *Table No. 10* for beams continuous over approximately equal spans, with various ratios of live and dead load, can also be applied to slabs, and in the cases of small live to dead load ratios the adoption of these coefficients will prove more economical than the German values.

Slabs in special work, such as bridge decks, where the ratio of live to dead load exceeds two, should be designed for a possible negative bending moment at midspan equal to

$$M = \frac{L^2}{24} (0.5W_L - W_D)$$

where W_L = the intensity of live loading in lb. per square foot

W_D = the intensity of dead loading in lb. per square foot

L = span (ft.)

M = negative moment in foot-lb.

9.—Rectangular Slabs.

When a rectangular slab is supported on more than two sides a certain amount of load is transmitted to each of the bearers. It is almost impossible to determine theoretically, with assumptions resembling practical conditions, the precise amount of load taken by each support, and therefore the bending moment in the two directions of slabs supported on four sides is equally indeterminable exactly. Various empirical formulæ and formulæ based on approximate theory are in everyday use, the most common in Great Britain being the Grashof and Rankine formula and the French Government formula. These are as follows:

Grashof and Rankine, $W_B = \frac{Wk^4}{k^4 + 1}$; $W_L = W - W_B$.

French Government, $W_B = \frac{Wk^4}{k^4 + 2}$; $W_L = \frac{W}{1 + 2k^4}$

where W = total load on slab.

W_B = load carried in direction of short span.

W_L = load carried in direction of long span.

k = ratio of long span to short span.

The usual limit of application of both formulæ is when $k = 2$ (that is, when the length of the slab equals or exceeds twice the breadth), in which case the slab is spanning across the short span only. The formula advocated by the American Society of Civil Engineers and adopted in the Australian (Sydney) regulations applies to slabs in which the length does not exceed $1\frac{1}{2}$ times the breadth and takes the form

$$\begin{aligned} W_B &= W(k - 0.5) \\ W_L &= W - W_B \end{aligned}$$

The more modern French formula derived by M. Pigeaud and given in English texts by Mr. W. L. Scott is based on the exact theory as applied to thin plates and is unrestricted by the value of k , thus being applicable to determining the

amount of longitudinal steel required in a long rectangular slab. Whereas the formulæ previously given are strictly speaking only applicable to slabs totally covered by uniformly distributed loads, the Pigeaud method can be adapted to any partially or totally distributed loading.

For a square slab ($k = 1$) the amount of load carried in each direction equals one-half the total load in accordance with both the Grashof and Rankine formula and the American formula, whereas by the French Government formula the load in each direction is only one-third of the total, and by the Pigeaud method about three-tenths. On Table No. 14 the values of $\frac{W_B}{W}$ and $\frac{W_L}{W}$ for k varying from 1.0 to 2.0 are tabulated in accordance with the four methods mentioned. It will be observed that the Pigeaud method would give the smallest moments while the American formula is the most conservative. Although the French Government formula is adopted by many practitioners, the even less conservative Pigeaud method has been sufficiently tested to warrant its use by those engineers who wish to blend economy with scientific methods. To those requiring a more conservative method for work not restrained by the regulations, the following disposition of the reinforcement is suggested. In the whole length (and breadth) of the slab provide for the total resistance required by the French Government formula; but in the centre half-strip provide the resistance required by Grashof and Rankine, making up the difference equally in the outer quarter-strips.

In deriving the four methods just discussed consideration is only given to slabs freely supported on four edges, and in practice the same subdivision of the loading is assumed when the slab is continuous over all four supports. The simplest way to allow for such conditions as freely supported on two sides, continuous on two sides, or freely supported on three sides, continuous on one side, etc., is by substituting for $k = \frac{\text{long span}}{\text{short span}}$ the expression

$$k = \frac{\text{longer equivalent span}}{\text{shorter equivalent span}}$$

where the equivalent span equals f times the actual span. Suggested values of f for various conditions of supports are given on Table No. 14 and vary from 0.67 for a span continuous over both supports to 3.0 for a cantilever, with unity for a span freely supported at both ends. This method approximates very closely to attempts to calculate theoretically the effect of continuity over some of the supports, and in its adoption it should be observed that the shorter actual span may not be the shorter equivalent span.

All the foregoing discussion on rectangular slabs is concerned with the subdivision of the load in order to determine the moments, but, except for the American and the Grashof and Rankine methods, W_B and W_L do not represent the loads carried by the supports parallel to the long and short axis respectively, and in those cases where the French Government or the Pigeaud methods are adopted for moment determination the load carried by the various supports must be calculated independently. This can be done by taking for this purpose only Grashof and Rankine coefficients for the proportions in question, or more logically the total load should be divided into two parts, W_b , the load carried on the supports parallel to the equivalent longer sides, and W_l , that carried on

the supports parallel to the equivalent shorter sides; and $W_l + W_b = W$; $W_b = \frac{W_B}{W_L + W_B} \cdot W$ and $W_l = \frac{W_L}{W_L + W_B} \cdot W$ where W_L and W_B are determined by the method adopted for the moment calculation. It is usually sufficient to consider that each parallel support takes an equal share of the load.

10.—Polygonal and Circular Slabs.

When a non-rectangular slab is supported on four edges and is of such proportions that reinforcement in two directions is suggested, the moments can be determined by the following approximate considerations.

If the slab is triangular (or very nearly so) the inscribed circle should be drawn touching the three sides. If D = diameter of this circle, the bending moment in each of two directions at right angles to each other, assuming the slab is freely supported on all sides, is given by

$$M = \frac{WD^2}{16}$$

where W = the intensity of loading.

If the slab is fixed at one or more edges, suitable allowance should be made for the reduction of the positive moments and production of negative moments.

When a slab is in the form of a regular polygon or approximates to a square slab, the size of a square slab equal in area to the given slab should be determined and the moments determined for such a square slab.

For a trapezoidal slab, compute the size of an equivalent rectangle having the same area with the sides in the same ratio as the mean longitudinal and transverse axes of the given slab.

A circular slab of diameter D carrying a uniformly distributed load and freely supported around the circumference, should be designed for a moment of $\frac{WD^2}{12}$ in two directions at right angles. Actually the bending moment varies

from zero at the edges to a maximum of $\frac{WD^2}{12}$ at the centre, and this variation should be borne in mind if the reinforcement is to be economically arranged. For a circular slab carrying a concentrated load at the centre, the bending moment that should be provided for in two directions at right angles is given by

$$M = \frac{WD}{2\pi} \left(1 - \frac{2d}{3D} \right)$$

where d = the diameter of the loaded area.

For circular slabs rigidly fixed at the supports the following moments should be provided for:

Uniform load: At centre $\frac{WD^2}{24}$

At support $\frac{WD^2}{18}$

Concentrated load:

At centre and support $\frac{WdD}{4\pi} \left(1 - \frac{2d}{3D} \right)$

11.—Flat Slabs.

The design of beamless slabs ("flat slabs" or "mushroom floors") is based on empirical considerations. The formulæ and principles which follow, and are summarised on *Table No. 14*, are practically in accordance with the stipulations of the American Society of Civil Engineers and agree generally with German and Australian practice. The design of this type of floor can either incorporate drop panels at the column heads (as in *Fig. 12*) or the slab can be of uniform thickness throughout as in *Fig. 13*.

The following formulæ assume all dimensions in inches and the notation is

L = length of panel side.

B = breadth of panel side.

w = intensity of loading (lb. per square foot).

D = diameter of column capital.

S_L = effective span in direction of L .

S_B = effective span in direction of B .

The diameter of the column shaft should not be less than 12 in., nor less than one-fifteenth of the height between floors, nor less than one-twentieth of the mean of L and B . The diameter of the column capital should equal $0.225L$ (or $0.225B$ whichever is greater), and the minimum length of side of the drop panel, if provided, should be four-tenths of the corresponding panel side.

The thickness of the slab should be such as is required by the bending moments and in no case less than 6 in. nor less than one-thirty-second of the mean panel length for floors or one-fortieth for roof slabs, nor less than

$$[0.001(L + B)\sqrt{w} + 1.5] \text{ without dropped panels,}$$

or $[0.0008(L + B)\sqrt{w} + 1.0]$ with dropped panels the width of which exceeds 0.4 times the panel length.

For the purpose of moment determination the effective spans are given by

$$S_L = L - 0.67D$$

$$S_B = B - 0.67D.$$

The total bending moments in in.-lb. to be provided for along the principal lines of the slabs are:

$$\text{Along } XY + \frac{wBS_L^2}{3600} = M_x$$

$$VW + \frac{wLS_B^2}{3600} = M_y$$

$$RS - \frac{wLS_B^2}{2160} = M_s$$

$$UT \quad \text{Ditto}$$

$$RU - \frac{wBS_L^2}{2160} = M_u$$

$$ST \quad \text{Ditto.}$$

These moments should be provided for as follows.

Section.	With dropped panels.	Without dropped panels.
cd	30 per cent. of M_x	40 per cent. of M_x
Xc and dY	35 per cent. of M_x in each	30 per cent. of M_x in each
ab	30 per cent. of M_y	40 per cent. of M_y
Va and bW	35 per cent. of M_y in each	30 per cent. of M_y in each
fe	20 per cent. of M_u	30 per cent. of M_u
cU, JR, Sf and cT	40 per cent. of M_u in each	35 per cent. of M_u in each
gh	20 per cent. of M_s	30 per cent. of M_s
$Rg, hS, U'g$, and hT	40 per cent. of M_s in each	35 per cent. of M_s in each

In end spans and penultimate rows of columns the moments should be increased by 20 per cent. All walls and other concentrated loads must be carried on beams, and beams should be provided to trim around openings.

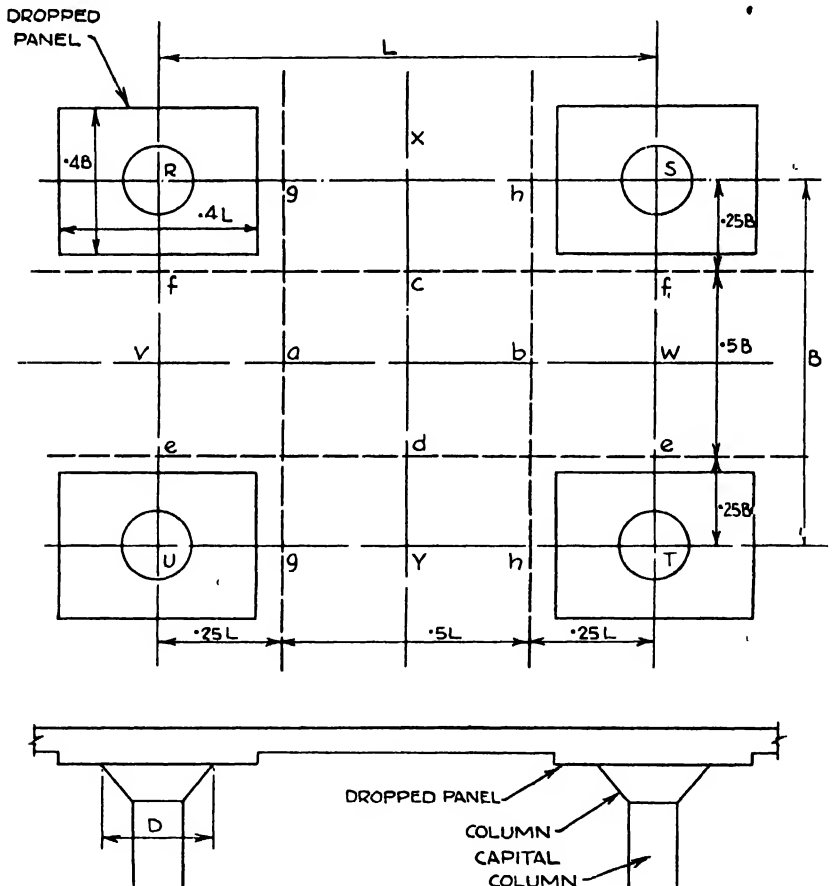


Fig. 12.—Flat Slab with Dropped Panel.

The reinforcement can be arranged to be parallel to L and B , and bands of diagonal bars can also be introduced extending between columns in opposite corners; the area value of these diagonal bars is their area multiplied by the sine of the angle between the diagonal and section considered, that is, area of

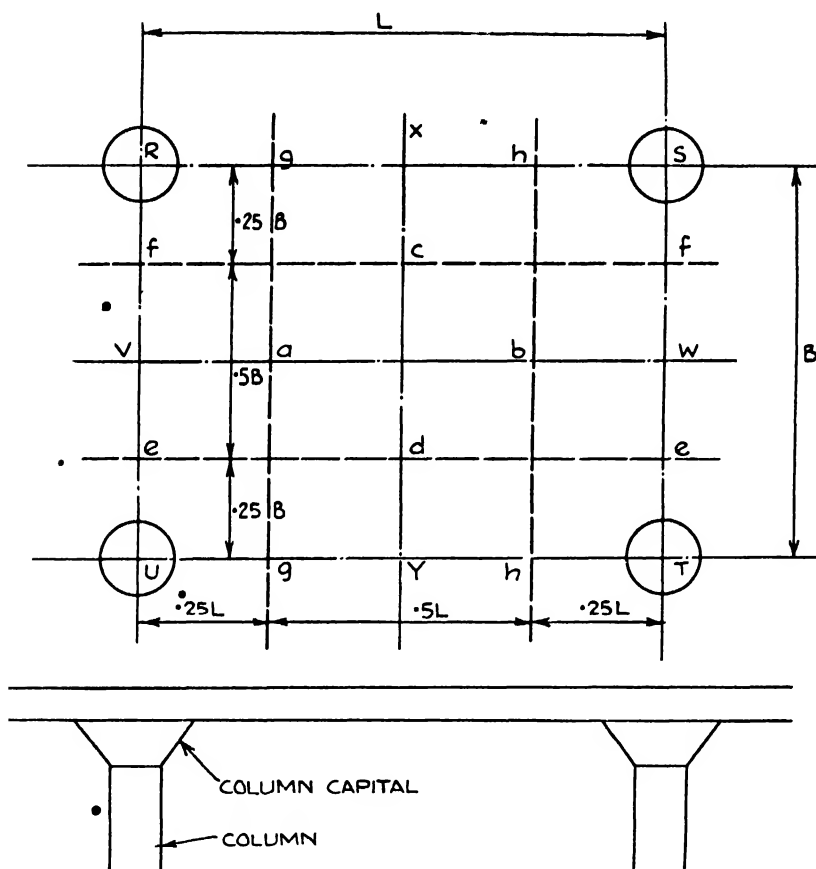


Fig. 13.—Flat Slab without Dropped Panel.

bars $\times \frac{B}{\sqrt{L^2 + B^2}}$ for a section parallel to L , and area of bars $\times \frac{L}{\sqrt{L^2 + B^2}}$ for a section parallel to B .

For purposes of determining the shear force it is usual to consider that the total shear on such a line as frg (see Figs. 12 and 13) is one-quarter of the total load on the slab for a slab without dropped panels and three-tenths of the total load for a slab with dropped panels.

An example of "flat slab" floor design is given on the pages following Table No. 40, together with an alternative beam and slab design. These alternative

designs have been priced out on a common basis and the costs determined are as follows:

Beam and slab construction . . .	30s.	per sq. yd.
Flat slab without dropped panel . . .	28s. 9d.	" "
Flat slab with dropped panel . . .	27s. 6d.	" "

Although there is less than 10 per cent. difference between these three rates it should be remembered that the total depth of construction required for the beam and slab construction is 1 ft. 11 in. from floor finish to soffit of main beam, excluding haunches, and for the flat slab construction with dropped panel the depth is 12 in. excluding the splayed column head. That is to say, in a warehouse 121 ft. high from basement to roof, with a prescribed headroom of 10 ft. for each floor, the total number of floors with beam and slab construction would be one less than with flat-slab construction.

CHAPTER V

FRAMED STRUCTURES

1.—Theoretical Considerations.

THE process of solving problems dealing with indeterminate framed structures is generally based on one of two fundamental principles :

- (a) The Principle of Least Work, and
- (b) The Slope Deflection Theory.

The consideration that will here be given to this aspect of structural design will be based on the latter of these two principles.

The two axioms in the slope deflection method are

- (a) That the difference in slope between any two points in the length of a member subject to bending is equal to the area of the $\frac{M}{EI}$ diagram between these two points.
- (b) That the distance of any point from a line drawn tangential to the elastic curve at any other point, the distance being measured normal to the initial position of the member, is equal to the moment (taken about the first point) of the $\frac{M}{EI}$ diagram between these two points.

In these considerations M represents bending moment, E the elastic modulus of the material, and I the moment of inertia of the member. Combining these axioms leads to the following general expressions for the moments at the ends of a member subject to bending as shown in *Fig. 14*.

$$M_{AB} = 2EK \left(2\theta_A + \theta_B - \frac{3d}{L} \right) - P \quad (1)$$

$$M_{BA} = -2EK \left(2\theta_B + \theta_A - \frac{3d}{L} \right) - Q \quad (2)$$

where M_{AB} = bending moment at A towards B
 M_{BA} = bending moment at B towards A
 L , θ_A , θ_B , and d are shown in *Fig. 14*

$$K = \text{the stiffness factor} = \frac{I}{L}$$

$$P = \frac{2A}{L^2}(2L - 3Z)$$

$$Q = \frac{2A}{L^2}(3Z - L)$$

where A = area of free bending moment diagram due to the external loads acting between A and B .

Z = distance of centre of area of the free moment diagram measured from A .

The signs of the various components in these general formulæ are in accordance with the following conventions :

Bending moment : concave upwards—positive.

Deflection : d as indicated in *Fig. 14*—positive.

Slope : θ_1 as indicated in *Fig. 14*—negative.

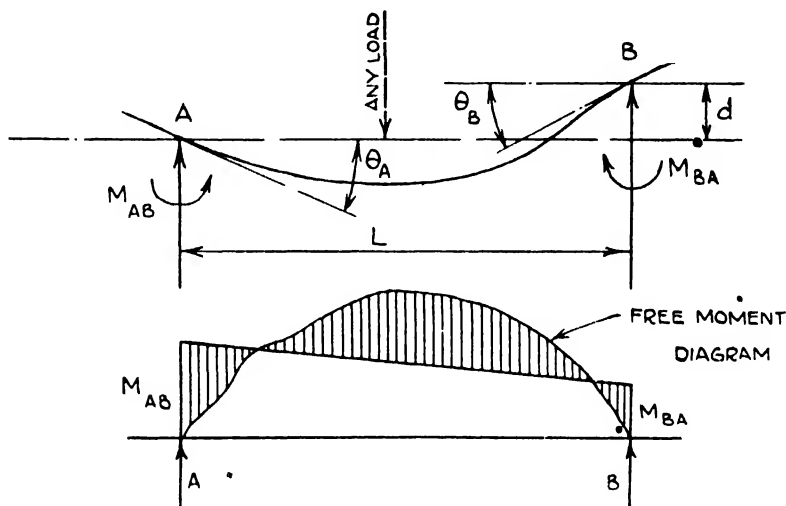


Fig. 14.—Frame Notation.

When there is no external load the factors P and Q become zero, and when the load is symmetrically disposed on the member $Z = \frac{L}{2}$ and $P = Q = \frac{A}{L}$. Values of $\frac{A}{L}$ for various cases of symmetrical loading are given on *Table No. 15*.

When there is no elastic deflection of one end of the member relative to the other (for example, when non-elastic supports are assumed), that is, when $d = 0$ the general expressions (1) and (2) become

$$M_{AB} = 2EK(2\theta_A + \theta_B) - P \quad (1a)$$

$$M_{BA} = 2EK(2\theta_B + \theta_A) - Q \quad (2a)$$

Further simplifications are introduced when the end conditions of the member are known. Thus when the end B of the member AB is hinged, $M_{BA} = 0$, and the general expression becomes

$$M_{AB} = EK\left(3\theta_A - \frac{3d}{L}\right) - \left(P + \frac{Q}{2}\right) \quad (1b)$$

Similarly if end A were hinged and not B , $M_{AB} = 0$, and the general expression becomes

$$M_{BA} = -EK\left(3\theta_B - \frac{3d}{L}\right) - \left(Q + \frac{P}{2}\right) \quad (2b)$$

Hence for conditions of one end hinged, level supports, and symmetrical loading, the formulæ become

$$\text{For end } B \text{ hinged, } M_{AB} = 3EK\theta_A - 1.5\frac{A}{L} \quad (1c)$$

$$\text{For end } A \text{ hinged, } M_{BA} = -3EK\theta_B - 1.5\frac{A}{L} \quad (2c)$$

When the restraint at end A is equivalent to perfect fixity, $\theta_A = 0$, or when end B is fixed, $\theta_B = 0$, i.e.

$$\text{For end } A \text{ fixed, } M_{AB} = 2EK\left(\theta_B - \frac{3d}{L}\right) - P \quad (1d)$$

$$M_{BA} = -2EK\left(2\theta_B - \frac{3d}{L}\right) - Q \quad (2d)$$

Hence for conditions of one end fixed, level supports, and symmetrical loading, the formulæ become

$$\text{For end } A \text{ fixed, } M_{AB} = 2EK\theta_B - \frac{A}{L} \quad (1e)$$

$$M_{BA} = -4EK\theta_B - \frac{A}{L} \quad (2e)$$

$$\text{For end } B \text{ fixed, } M_{AB} = 4EK\theta_A - \frac{A}{L} \quad (1f)$$

$$M_{BA} = -2EK\theta_A - \frac{A}{L} \quad (2f)$$

The modified general formulæ for members having various conditions of end restraint are given on *Table No. 15*.

The general problem of determining the moments in a framed structure is solved by applying the appropriate general formulæ to each member successively. The algebraic sum of the moments at any joint is equal to zero.

As a simple illustration of the application of the formulæ, consider the problem of determining the bending moment at A when the beam AB , indicated in *Fig. 16(a)*, is loaded.

$$\text{From formula (1c), } M_{AB} = 3EK_B\theta_A - 1.5\frac{A}{L}$$

$$\text{From formula (1f), } M_{AC} = 4EK_C\theta_A$$

$$\text{From formula (1f), } M_{AD} = 4EK_D\theta_A$$

$$\text{By addition, } 0 = E\theta_A(3K_B + 4K_C + 4K_D) - 1.5\frac{A}{L}$$

$$\text{hence } E\theta_A = \frac{1.5L}{3K_B + 4K_C + 4K_D}$$

$$\text{By substitution, } M_{AB} = \frac{6(K_D + K_C)}{3K_B + 4K_C + 4K_D} \cdot \frac{A}{L} \text{ (negative).}$$

The other moments could be found in a similar manner.

Apart from the consideration of continuity between columns and beams in a building skeleton, simple framed structures most commonly occur in the form of portal frames. On *Table No. 15* the formulæ for the bending moments at the ends of the columns are given for such frames having any type of vertical loading on the top beam or any type of horizontal loading on the columns.

The shear forces on any member forming part of a frame can be readily determined when the moments have been found by consideration of the rate of change of bending moment. The shear or reaction at the ends of a member *AB* that carries no external load is given by

$$F_{AB} = \frac{M_{AB} - M_{BA}}{L}$$

For a member with end *B* hinged

$$F_{BA} = M_{AB}$$

2.—Exterior Columns.

The importance of computing and designing for the secondary moments that are produced in monolithic beam and column construction due to the rigidity of the joints is recognised by all modern authorities on reinforced concrete design, and, although existing British regulations do not particularise the method of moment calculation to be adopted, most foreign regulations no longer treat this aspect of design as a refinement but as an essential factor.

Exterior columns in a framed structure are subject to a greater bending moment than interior columns (other conditions being equal) and the magnitude of the moment depends on the relative stiffness of the column and beam and on the end conditions of the members. There are two principal cases to consider for exterior columns:

Case (i). Beam supported on topmost point of the column, as in *Fig. 15*.

Case (ii). Beam fixed to column at some intermediate point as in *Fig. 16*.

Since either end of the column or beam can be considered as hinged, fixed, or subject to some intermediate degree of restraint, it follows that there are nine individual conditions of Case (i) and twenty-seven conditions of Case (ii).

For Case (i) the maximum reverse moment at point 4, the junction of the beam and column, occurs when the end *B* of the beam is hinged and the foot of the column is fixed at *C*, as in *Fig. 15(a)*. The minimum reverse moments at *A* occur when the beam is rigidly fixed at *B* and the column is hinged at *C* as shown in *Fig. 15(b)*. Conditions met with in practice lie between these two extremes, and with any condition of fixity of the column foot *C* the moment at *A* decreases as the degree of fixity at *B* increases. With any degree of fixity at end *B* of the beam, the moment at *A* increases very slightly as the degree of fixity at *C* increases.

The maximum reverse moment in the beam at the junction with the column for Case (ii) is experienced when the beam is hinged at *B* and the column perfectly fixed at both *C* and *D* as indicated in *Fig. 16(a)*. With perfect fixity at *B* and hinges at *C* and *D*, as in *Fig. 16(b)*, the reverse moment in the beam at *A* is a minimum.

Intermediate cases of fixity follow the following rules:

Increase in fixity at B decreases the beam moment at A .

Decrease in fixity at either C or D decreases the beam moment at A and vice versa.

Values of the maximum and minimum bending moment in terms of the $\frac{A}{L}$ factor for symmetrical loads on the beam are given on Table No. 16 for various relative stiffness factors for the column and beam arrangement illustrated in

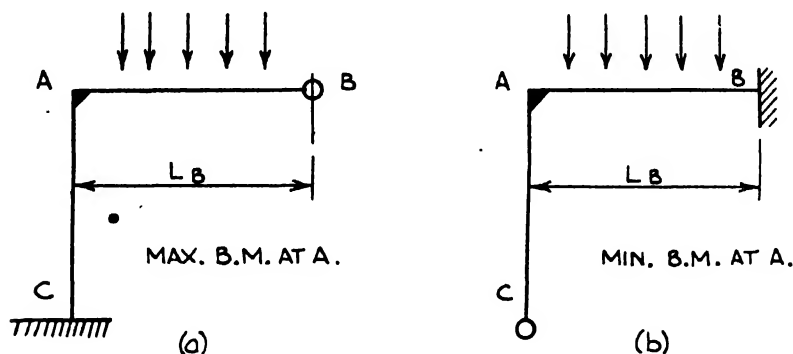


Fig. 15.—EII Frames.

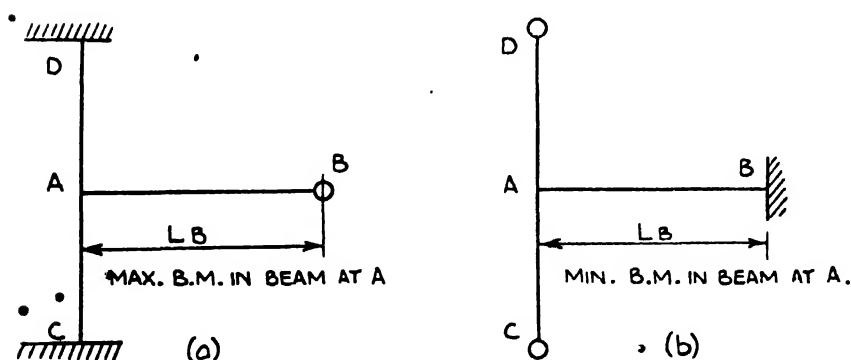


Fig. 16.—Tee Frames.

Fig. 16. These coefficients are applicable to the arrangement shown in Fig. 15 by considering that the upper column AD does not exist and therefore $\frac{K_D}{K_B} = \text{zero}$.

The beam moment at A is divided between the upper and lower columns in the ratio of their stiffness factors when their end conditions are identical. When one column is hinged and the other fixed the ratio of the two moments is in accordance with

$$\frac{\text{Moment in hinged portion}}{\text{Moment in fixed portion}} = \frac{0.75K \text{ for hinged portion}}{K \text{ for fixed portion}}.$$

The foregoing considerations are applicable to any symmetrical loading on the beam. When the loading is uniformly distributed, the formulæ given by the German Regulations may be adopted. These (see Fig. 16) are as follows.

$$\text{Moment at } A \text{ in } AD = M_{AD} = -\frac{WL_n}{12} \left(\frac{K_D}{K_B + K_C + K_D} \right)$$

$$\text{Moment at } A \text{ in } AC = M_{AC} = -\frac{WL_n}{12} \left(\frac{K_{AC}}{K_B + K_C + K_D} \right)$$

$$\text{Moment at } A \text{ in } AB \quad (M_{AD} + M_{AC})$$

where K_B , K_C and K_D are the stiffness factors of AB , AC , and AD respectively, and W is the total uniformly distributed live and dead load on the span AB . These formulæ neglect any non-uniformity in the end conditions of either the

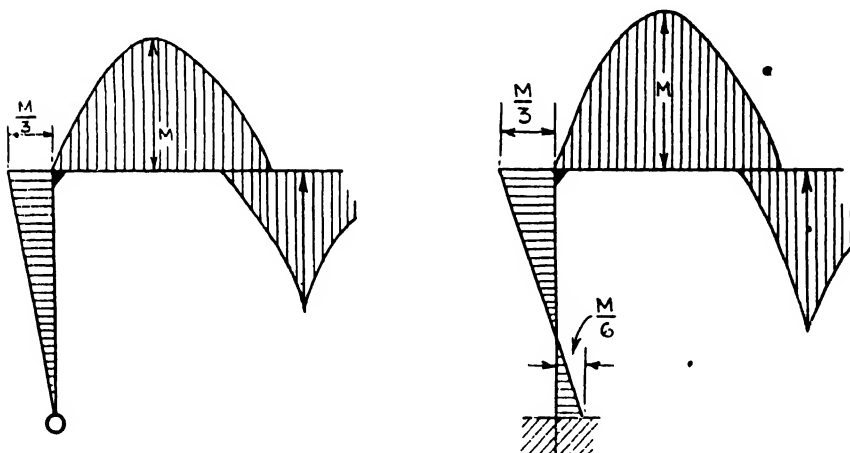


Fig. 17. B.M. in Columns when Restraint Moment in Beam at End Support is neglected.

beams or columns, but are suitable for use in ordinary building structures. Where end conditions vary considerably the effective lengths for various end conditions given on Table No. 34 can be used in calculating the slenderness factors.

When there is no upper column (as in Fig. 15) these formulæ apply if K_D is written zero.

For various ratios of stiffness factors, the moment coefficients in accordance with the German formulæ are tabulated on Table No. 16.

The top lift of all columns should be designed for bending in addition to direct load, and even when the beam has been designed as if freely supported at the column or as continuous over and independent of the column (as is usual), some moment should be taken on the column. Certain reinforced concrete regulations give reasonable values for these bending moments as shown in the diagrams in Fig. 17. When the base of the column is considered hinged, the moment to be taken at the top should be one-third of the maximum positive moment in the adjacent beam span. When the base of the column is fixed, the

moment at the top should be taken as one-third of the maximum positive moment in the adjacent beam span, and the moment at the base as one-sixth of this positive moment.

When the midspan moment is taken as $\frac{WL^2}{10}$ the bending moment at the top of the column would thus be $\frac{WL^2}{30}$ and $\frac{WL^2}{60}$ at the base of fixed columns. For midspan moments of $\frac{WL^2}{8}$ the corresponding moments would be $\frac{WL^2}{24}$ and $\frac{WL^2}{48}$.

3.—Interior Columns.

There is less variation in the moments due to continuity between beams and columns in the case of the interior columns of a building frame than in the case of exterior columns. End fixing conditions of the various members do not affect the moments so seriously, and it should be remembered that the moment in the beams at the column junction is in all cases less than the moment at this support when computed by the Three Moment Theorem neglecting fixity with the column. On *Table No. 16* are tabulated moment coefficients from which the moment in the upper or lower column can be calculated by multiplying by the $\frac{A}{L}$ factor for the "live" load only on one of the adjacent beam spans.

4.—Approximate Treatment.

The methods hitherto enumerated for evaluating the bending moments in column and beam constructions with rigid joints involve a fair amount of calculation. Except for the approximate method of computing the fixing moment at the top end of a column, they all involve the calculation of the moment of inertia of the members concerned. In general practice it is not always possible to devote the time required to make these calculations, and therefore an approximate method is of value. In order to allow a margin for the bending stresses on the column, the column may be designed for the static reaction increased by the amounts shown on *Table No. 16* for the appropriate arrangement of beams supported by the column. When the ratio of the length to the least width of the column (the slenderness factor) is less than 15 the load factor for Case 4 can be decreased to unity. The value of the working stress for columns where the slenderness factor exceeds 18 has to be decreased as explained in Chapter XIII.

For braced columns subject to horizontal thrusts, such as the legs of water towers, a simplified method of calculating the moments and forces in the columns and braces can be evolved by considering the rational deformation of the structure. The formulæ for the direct load and shears on each column and the moments at any joint both in the columns and braces are given on *Table No. 15* for the case of two practically vertical columns braced together by horizontal braces and subjected to a horizontal thrust at the topmost joint.

Columns supporting silos and bunkers are subject to a horizontal thrust

at the column head due to wind on the structure above. If P equals the total wind pressure and N equals the number of columns, the shear at the head of each column = $\frac{P}{N}$. The maximum bending moment in the columns is given by the following expressions, if L equals the column height :

$$\text{Column hinged at base, } M = \frac{PL}{N}.$$

$$\text{Column fixed at base, } M = \frac{PL}{2N}.$$

5.—Earthquake-Resisting Structures.

The design of structures to withstand the disruptive forces induced by earth tremors and quakes presents a problem in frame design. Authoritative opinions differ on the question whether such structures should be designed as rigid or flexible structures. A semi-flexible type has been advocated in the United States, but generally engineers seem to favour the rigid-frame construction. The effect of earth tremors is considered as being equivalent to a horizontal thrust that is additional to the loads and wind pressures for which the building would ordinarily be designed. The usual dead and live loads should be increased by 20 per cent. to allow for vertical movement. The magnitude of the horizontal thrust would depend on the probable acceleration of the earth tremor to which the building is likely to be subjected, and the value of this acceleration varies from less than 3 feet per second per second in firm compact ground to over 12 feet per second per second in alluvial soils and filling. A horizontal thrust equal to about one-tenth the mass of the building would seem to be adequate for all but major shocks when the building does not exceed 20 ft. in height, increasing to one-eighth of the mass when the building is of greater height. The horizontal shear on the building at any level should be considered as one-eighth or one-tenth of the total weight of the structure (including live loads) above this level.

In order that the structure shall act as a single unit, all parts of the building should be effectively bonded together. Panel walls, finishes, and ornaments should be permanently attached to the framework, so that in the event of a shock they will not collapse independently of the main structure. Isolated column footings should be connected by ties designed to take a thrust or a pull equal to one-tenth of the load on the heavier of the two footings connected.

6.—End Conditions.

Since the results given by the more accurate frame formulæ vary considerably with different degrees of fixity of the ends of the component members, it is essential that the end conditions assumed should be reasonably obtained in the actual construction. Absolute fixity is difficult to attain unless the beam or column is embedded monolithically in a comparatively huge mass of concrete. Embedment in a brick or masonry wall is more nearly of the nature of a hinge, and should be assumed as such. The standard type of isolated footing should also be considered as a hinge at the foot of the column. A continuous beam supported on a beam or column is only partially fixed, and where an end span

is supported at the outer end on another beam hinged conditions should be assumed. A column built out of a pile cap supported by two, three, or four piles is not absolutely fixed, but can be considered thus if it is built out of a thick raft whether the raft is supported on piles or not.

In special structures such as arches, large-capacity bunkers, etc., where it is necessary that the assumption of a hinged joint is fully maintained, it is generally worth while and always safer to make a definite hinge in the construction. This can be done either by inserting an appropriately designed all-steel hinge, or more commonly by forming a hinge monolithic with the column as shown in *Fig. 18*. The hinge bars "a" should be calculated to take the whole of the

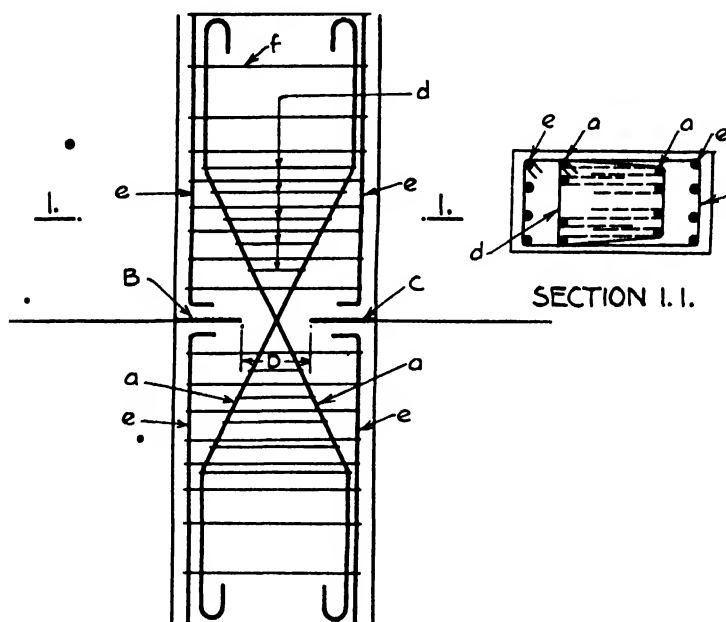


Fig. 18.—Typical Detail of Hinge.

horizontal shear at this section, and the area of concrete at "D" should be sufficient to transfer all the compressive force from the upper to the lower part of the column. The hinge bars should be well bound together by suitable binders "d" and the main column bars "e" should be terminated on each side of the slots B and C. These slots should be filled with felt or tarred paper, or preferably with asphalt, although sometimes these slots are not left at all, the concrete being allowed to crack when moments producing stresses in excess of the ultimate tensile strength of the concrete occur.

7.—Moments of Inertia of Reinforced Concrete Sections.

The moment of inertia of a reinforced concrete section is theoretically the equivalent moment of inertia of the stressed section about the neutral axis

expressed in concrete or in steel units. The concrete in tension is neglected and the steel is considered as being equivalent to m times its area of concrete (see notes on the modular ratio in Chapter IX). Until the complete section is designed (or assumed) and the percentage of steel known, such a calculation cannot be made with any precision. Moreover, the moment of inertia of an ordinary beam changes considerably throughout its length, especially with a flanged beam which acts as a tee beam at midspan but has to be treated as a simple rectangular beam at its ends where reverse moments occur.

Since only comparisons of moments of inertia are required in frame design, the errors due to approximations are of minor importance; it is therefore usually sufficient to compare the moments of inertia of the concrete sections alone for members that have somewhat similar percentages of steel. Thus for comparing a rectangular column with a rectangular beam (or with a tee beam subject to reverse moment) the ratio of the moments of inertia would be

$$\frac{I_C}{I_B} = \frac{b_c d_c^3}{b_b d_b^3}$$

This same method of calculation is sufficient for those cases where one of the sections to be compared is a true tee beam with breadth of rib = b_b while the other is reinforced in both tension and compression, since the neglect of the extra effective value of the flange is approximately balanced by the neglect of the compressive steel in the other section.

On Table No. 39 are given values of I for octagonal, diamond-shape, and other non-rectangular sections.

The effect of the approximation suggested can best be seen by considering one or two examples, and the various sections illustrated in Fig. 19 will be compared.

(a) To compare the moments of inertia of the rectangular beam shown in Fig. 19(a) with the square column in Fig. 19(b):

(i) *Approximate method:*

$$\frac{I_C}{I_B} = \frac{15^4}{8 \times 18^3} = 1.08.$$

(ii) *Accurate Method:*

Column: Concrete = $0.083 \times 15^4 = 4,210$ concrete units

Steel = $14 \times 3.14 \times 5.5^2 = 1,330$ " "

$$I_C = 5,540 \quad \text{" "}$$

Beam: Concrete = $0.33 \times 8 \times 8^3 = 1,360$ concrete units

Compression steel = $14 \times 1.33 \times 6.5^2 = 790$ " "

Tension = $15 \times 2.36 \times 8.5^2 = 2,550$ " "

$$I_B = 4,700 \quad \text{" "}$$

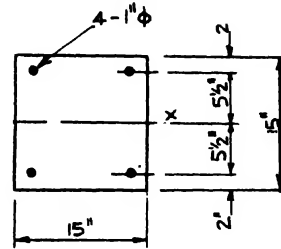
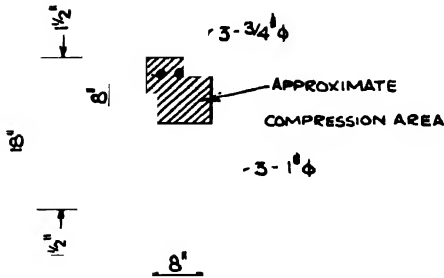
$$\frac{I_C}{I_B} = \frac{5,540}{4,700} = 1.18 \text{ as compared with } 1.08.$$

(b) To compare the rectangular beam in Fig. 19 (a) with the octagonal column in Fig. 19(c):

(i) *Approximate method:*

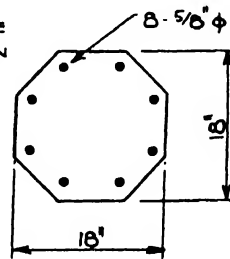
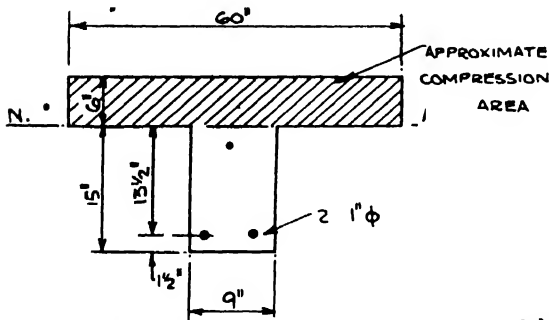
From Table No. 39 the M. of I. of an octagonal section = $0.055D^4$

$$\therefore \frac{I_C}{I_B} = \frac{0.055 \times 18^4}{0.083 \times 8 \times 18^3} = 1.49.$$



(a) RECTANGULAR BEAM

(b) SQUARE COLUMN BENDING
ABOUT X-X (COMPRESSION ALL
OVER SECTION)



(c) TEE BEAM

(d) OCTAGONAL COLUMN
(COMPRESSION ALL OVER SECTION)

Fig. 19.—Typical Sections for Comparison of Moments of Inertia.

(ii) *Accurate method:*

I_C by calculation similar to (a) = 6,780 concrete units

I_B as already calculated = 4,700 " "

$$\frac{I_C}{I_B} = \frac{6,780}{4,700} = 1.44 \text{ as compared with } 1.49.$$

(c) To compare the tee-beam in Fig. 19(d) with the square column in Fig. 19(b).

(i) *Approximate method:*

$$\frac{I_C}{I_B} = \frac{15^4}{9 \times 21^3} = 0.58.$$

(ii) *Accurate method:* I_o as in (a).

= 5,540 concrete units

• Beam: Concrete in flange = $0.33 \times 60 \times 6^3 = 4,320$ „ „
 Tension steel = $15 \times 1.57 \times 13.5^2 = 4,280$ „ „

8,600 „ „

$\frac{I_o}{I_B} = 0.64$ as compared with 0.58.

From these examples it will be seen that the approximate method readily gives comparative values that are accurate enough for trial calculations and close enough for general design purposes. Another useful approximation that can be employed when the position of the neutral axis is known or can be estimated, is that the moment of inertia of a section subject to tensile and compressive stresses simultaneously is equal to twice the second moment of the tension steel area about the neutral axis. Applying this to the beam in *Fig. 19(a)*,

$$I_B = 2 \times 15 \times 2.36 \times 8.5 = 5,100 \text{ concrete units,}$$

compared with 4,700 concrete units by the more precise calculation.

Applying this method to the tee-beam in *Fig. 19(d)*,

$$I_B = 2 \times 15 \times 1.57 \times 13.5^2 = 8,560 \text{ concrete units,}$$

compared with 8,600 concrete units.

CHAPTER VI

FOUNDATIONS

1.—Site Inspection.

THE complete design of a foundation involves a three-stage procedure. The first stage is to determine from an inspection of the site the nature of the ground, and, having chosen which of the available strata it is proposed to subject to loading, the maximum safe bearing pressure must be decided. The second stage is to select a suitable type of foundation, whether it shall be an isolated footing, a raft, or piles, and consideration may have to be given to alternative types. To design the selected type adequately to transfer and distribute the loads from the superstructure on to the ground constitutes the third stage.

Obviously the first step in a site investigation is to undertake a surface inspection and to determine from the appearance of the site itself or from the immediate vicinity whether the subsoil is rock, chalk, clay, gravel, sand, alluvial soil, or filling. Such a superficial inspection tells the engineer nothing as to the depth of the apparent strata, although in this first instance he can perhaps draw on his own or others' local experience, and with extreme caution can base his deduction on probabilities.

For more precise determination of the type of ground underlying the site he must resort to digging trial holes, sinking bores, or driving test piles. A trial hole can only be carried down to a moderate depth, but affords an excellent opportunity of studying the nature of the soil in its undisturbed form, of determining the difficulty or otherwise of excavation, and the need or otherwise of timbering and pumping. The chief objection to a trial hole is the localised nature of the inspection, but this can be overcome by sinking a number of holes at salient points.

A bore can be carried down very much deeper than a trial hole, but still has the objection that it is a localised investigation. It is a very good method of determining the geological formation below a site, especially when the successive strata are of greatly differing nature giving a sharply contrasted "throw-up." With alluvia, clays, and other recent sedimentary depositions, however, care must be taken with the deductions made from the material brought up by the bore, since some such materials may be quite compact in bulk and firm when buried, thus affording an excellent foundation material, but when brought up may be churned up into mud and become friable upon exposure to air and sun.

A test pile does not definitely indicate the kind of soil it has been driven through, but the driving data combined with local information may give a clue. A test pile is particularly valuable in determining the thickness of top crusts or the depth below poorer soil at which good bearing strata lie.

Having thus obtained some knowledge of the nature of the ground underlying the site, it is necessary to study the strata from the point of view of their value as a foundation-bearing material. There is little trouble with rocks except to determine whether they shall be classed as good, medium, or rotten. Inspection should be made for faults, splits, inclined strata, and the possible incidence of pockets of soft material. Some clue as to the thickness is also necessary as it is not uncommon for a thin layer of rock to overlie softer strata.

The thickness of the beds and the level of the strata are the most important data to obtain concerning clay, and for gravels and sands information concerning the thickness of the deposit and its water content is also necessary.

Alluvial soils and fillings present a more serious problem to the designer of foundations. It is sometimes difficult to distinguish between an old filling and virgin earth. Taking the extreme view, a "virgin earth" is merely a type of ancient and naturally deposited filling, but to minimise the risk it is always essential to decide whether the ground has been artificially filled or is a natural deposit. Whereas we can usually assume that a naturally deposited soil, such as old river mud, etc., will be of uniform compactness (and so may also be an artificially-placed hydraulic fill), we can by no means be so confident with ordinary tipped filling. Although the passage of time will generally render such a filling more compact, the time factor may also make it more insecure since it may contain material liable to decay, such as dumps of paper, timber baulks, or other material that may leave soft patches if not actual cavities in the sub-strata.

Inspection by bore or trial holes cannot ensure the exposure of isolated deposits of any treacherous material that may be lying below the general foundation level, and the occurrence of such a deposit or cavity below an isolated base is obviously dangerous. If, on account of time or expense, the idea of a bore hole at each foundation cannot be entertained, a little more confidence in the hidden strata can be gained if a $\frac{3}{4}$ -in. steel bar is driven a few feet into the soil and it is noticed that each successive hammer blow of equal force does not drive the rod further into the ground than the preceding blow. This is an inexpensive and satisfactory test and can be carried out at every base, or in several places under a large base.

Where there is any doubt as to whether a certain soil is or is not an artificial fill, it is always safe to assume it is artificial and to design the foundations with the necessary caution. If, in a soil that has all the attributes of "virgin earth," there is found a piece of glass, wood, iron or other foreign substance, it can at once be definitely decided that the soil is artificial fill. The finding of a Neolithic implement would not necessarily condemn a soil that has all the evidence of being naturally deposited, but any recent man-made product would convict the most natural-looking deposit.

Having decided that a particular soil is a filling, it is first necessary to decide how long it has been in place undisturbed, and, secondly, how deep it is and what underlies it. Its possible age may be judged by one or more of several criteria. Persons who actually witnessed the placing of the material should be fairly reliable, but such information should be independently confirmed if they attribute more than a few decades to the fill. The type of fill may give a clue as to its original source, and if this latter is investigated the date at which excavation took place there and hence the date of deposition may be determinable.

Relics found beneath the surface may assist in assessing the age, but in so estimating it must be borne in mind that whereas a piece of Roman pottery may not be taken as conclusive evidence that the filling was deposited fifteen hundred years ago, a fragment of modern English glass would be an almost certain clue that it was tipped or at least disturbed in recent times. The history of other structures on the same site may assist in arriving at the age of a deposit, and other dating data would most probably suggest themselves to the engineer investigating a particular site.

The question of age is of some importance, because one would look askance at the reliability of a deposit only five or ten years old, but with an established age of upwards of fifty years one might be inclined to deal more leniently with the problem. If the deposit were refuse from a town or industrial works it would immediately come under suspicion as not being a reliable bearing strata, although carefully designed raft foundations have been constructed on what was the common hillside rubbish tip of a sixteenth-century city.

The depth to which the filling extends is of equal importance and a consideration of this fact is inseparable from investigations of the strata's compactness and compressibility, and more obviously it is this depth that will be in a great measure the deciding factor in selecting the most suitable type of foundation.

2.—Safe Bearing Pressures.

The bearing pressures that can be imposed on soils of the ordinary types are well known and in certain districts are subject to regulation. On *Table No. 17* are given a list of soils with their generally accepted bearing values, together with the limiting values given in the current London County Council Regulations for Reinforced Concrete. The London Building Act (1931) allows 6 tons per square foot for chalk and pressures up to $\frac{1}{2}$ ton per square foot on alluvial soil, made ground, or very wet sand.

For soils of uncertain bearing resistance a study of existing buildings and their foundation loads on such soils may be instructive, or a bearing test can be carried out. It is an easy matter to arrange such a test by constructing a timber or concrete platform and loading it up with pig-iron or other available weights and taking periodical readings of any settlement. The platform should be monolithic and should cover 10 or 20 sq. ft.; it should be placed at the level at which it is proposed to build the foundation and in no case within 2 ft. of the ground surface. An initial load that gives 2 or 3 cwt. pressure per square foot should be imposed on the ground before observations are commenced, and these should be measured by a level sighted on an ordinary level staff. Readings should be taken at each corner of the platform in order to detect tilting, and readings should be taken before and after every increment of loading. The loading should advance in convenient stages and with due regard to symmetry until the ground is subjected to a bearing pressure at least double that for which it is proposed to design the foundations; it should be left thus loaded for 24 to 48 hours, when further observations should be taken to detect progressive settlement that might justify taking a less safe load than that anticipated.

Several types of machines for loading ground to be tested have been designed,

but they usually depend on a bearing test on a small area. The bearing resistance of successive strata can also be estimated by driving test piles, and apparatus has been designed for carrying out standardised tests.

3.—Types of Foundations.

The type of foundation most suitable for any given structure depends primarily upon the magnitude of the loading and the safe bearing pressure. The most simple type of column foundation is an independent base in one of the forms (a), (b), or (c) illustrated on *Table No. 18*. Such types are most suitable for ground having upwards of $1\frac{1}{2}$ tons per square foot safe bearing value, and for bases of small area the form (a) with a mass concrete sub-base is the most suitable. This type of base is also best adopted for foundations on rock, the mass concrete sub-base being formed in a pocket roughly hewn out of the rock into which bond bars should penetrate. In designing independent bases for the columns of a building founded on relatively compressible soils, to avoid in part the risk of local settlement the relative sizes of each base should be strictly in proportion to the dead load carried by the corresponding columns, and obviously the ground pressure under any base due to the combined dead and live load on the superimposed column should not exceed the safe bearing pressure of the ground.

When the columns in a structure are at fairly close centres in any one direction it is preferable to link up the bases to form a continuous strip footing, and a similar form of footing may be used for wall foundations.

Unless the foundation is on rock independent bases and strip footings should be founded at a level at least 2 ft. below the ground surface, since apart from considerations of spewing it is very seldom that virgin soil or a satisfactory consolidation is reached in less depth, and in clayey soils more than this depth is necessary to ensure protection of the bearing strata from effects of weathering. In the case of footings in which there is no mass concrete at the base forming an integral part of the base, the bottom of the excavation should be covered with a lean concrete (say Mix A, see *Table No. 23*) in order that a clean surface is provided upon which to place the reinforcement. The thickness of this layer depends upon the compactness and wetness of the bottom of the excavation, and would be from 1 in. to 3 in.

When the columns or other supports to a superstructure are at close centres in all directions, or when the column loads are so high and the safe ground pressure so low that the extent of independent bases almost or totally covers the space between the columns, the bases are linked up one with another in both directions and form a single raft foundation. The magnitude of the loading and the spacing of the columns determine the shear and bending moments that will in turn determine the thickness of the raft. If this thickness does not exceed 12 in. or 18 in. a solid slab is usually the most convenient and economical form; if this thickness must be exceeded a beam and slab construction designed as an inverted floor is more satisfactory. In cases where the total depth is of the order of five feet or more, a cellular construction consisting of a top and bottom slab with intermediate ribs gives the most economical design for rafts of large extent. These alternative designs are illustrated in *Fig. 20*.

In raft design consideration must be given to the upward pressure due to water, and especially in the case of rafts located below the natural water level with monolithic retaining walls constructed to above this level, as in *Fig. 20(d)*, the effect of water-pressure and buoyancy must be considered. Care should also be taken in the preparation of the ground upon which a raft is to be built, and everything reasonable done to ensure uniformity of bearing capacity. Weak

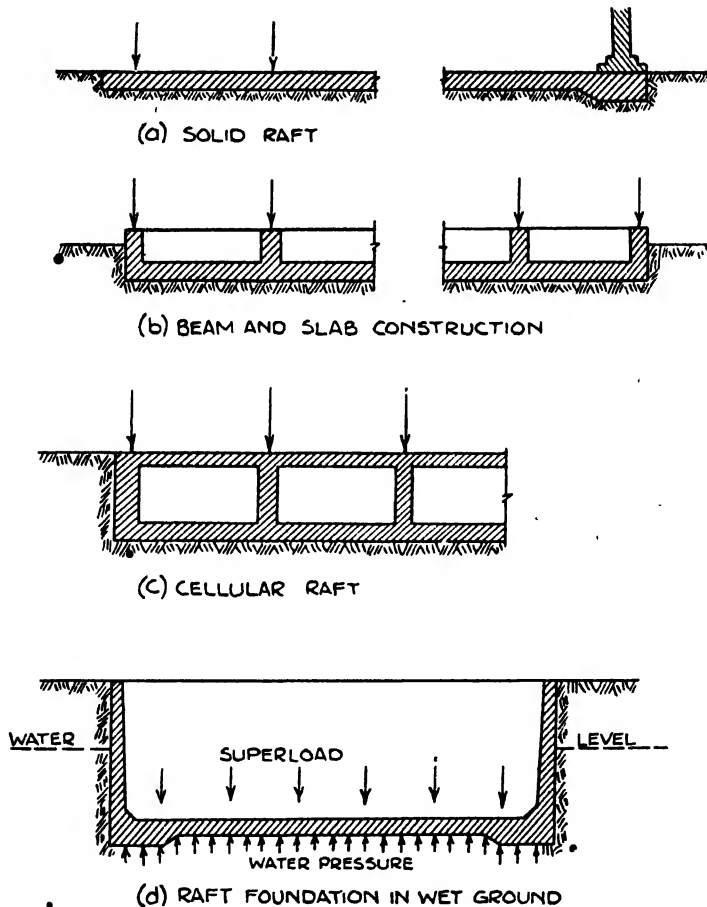


Fig. 20.—Raft Foundations.

places and hollows should be filled in with the same material as the ground itself, this filling being placed in thin layers well watered and rolled. Any patches of ground harder than the general nature of the site, such as rock outcrops, old walls, etc., should be cut away for a depth of 2 ft. or more below the foundation level and the hollows treated as specified above.

A layer of hardcore well rolled in all over the site thus prepared is also beneficial in producing a uniform quality of surface.

When a satisfactory foundation material is found at a depth of 5 ft. to 15 ft. below the natural ground level a suitable foundation can be made by building up piers from the low level to ground level, and constructing independent bases to the columns or other supports at ground level. The piers are generally in the form shown on *Table No. 18*, and can be constructed in brick, masonry, or mass concrete, whichever is most convenient. The maximum bearing pressure of the base on the top of the pier depends on the material of which the latter is constructed; safe values of this pressure for various materials are given on *Table No. 17* and are further discussed below. If the top strata are such that the excavation will not stand up without timbering, or if water is present in such quantities that continuous pumping is required, piers of this type are not usually economical. If neither of these disadvantages obtains, however, the minimum economic size of the pier is when the load it carries is great enough to require a pier equal in area to the smallest hole in which men can conveniently work. Otherwise unnecessary excavation has to be taken out and refilled. Thus, assuming a man can conveniently work in a hole one yard square at a depth of 12 to 15 ft., the minimum load would be in the neighbourhood of 90 tons for a Mix A mass concrete pier or a good brick pier. Generally it is better to provide as few piers as possible and to collect as much superload as practicable on each pier, thus making each pier of generous proportions. Usually it is not necessary to take out a bigger hole than is required for the stem of the pier, as at any reasonable depth the ground is firm enough to be undercut for the widening at the base. In the case of mass concrete piers this procedure eliminates shuttering.

Reinforced concrete columns can sometimes be economically carried down to moderate depths, but in order to avoid slender columns it is usually necessary to provide an adequate lateral support at ground level.

When piers are impracticable, either by reason of the depth at which a firm bearing strata occurs or otherwise, piles have to be adopted. Reinforced concrete piles are either pre-cast or cast *in situ*, and have been driven in general work in lengths up to 110 ft., although when exceeding 60 ft. it is necessary to give very special consideration to the design of the pile and of the handling and driving plant. Lengths less than 15 ft. are seldom economical. For ordinary work pre-cast piles have usually a square or octagonal section and are from 8 in. to 18 in. wide, although special piles have been designed in excess of these dimensions. For their support, piles either depend on direct bearing resistance on a firm strata or on side frictional resistance in soft strata.

In selecting a type of foundation suitable for any particular purpose the type of structure should be considered, and sometimes it has to be decided whether any injury to the superstructure may be risked as the result of a little local settlement in preference to incurring the expense of putting in a more elaborate foundation. In the case of silo foundations and the abutments of fixed-end arches, all risk of settlement must be eliminated, but for gantries and bases for large steel tanks a simple foundation can be provided and probable settlement allowed for in the superstructure design. In mining districts, where settlement can be reasonably anticipated, raft foundations should be provided for all major structures in order that the structure as a whole may settle equally in all parts.

The safe pressures on various materials tabulated on *Table No. 17* are con-

servative figures that can be used with the knowledge that they allow an ample margin of security. Those given in the column headed "L.C.C." are in accordance with the current London County Council Regulations for Reinforced Concrete. For work where the excellence of the materials can be assured the following safe pressures are acceptable :

Concrete Mix A	20 tons per square foot.			
Concrete Mix C	30	"	"	"
Blue brick	15 to 20	"	"	"
Hard brick	10 to 15	"	"	"
Ordinary brick	8 to 10	"	"	"

The pressure under the bases of steel stanchions bearing on a reinforced concrete raft or other type of foundation can be taken as 25 to 30 tons per square foot if the concrete is not leaner than Mix C.

4.—Pressure under Foundations.

When a foundation is subjected to a concentric load—when the centre of gravity of the superimposed load coincides with the centre of the base—the intensity of bearing pressure to which the ground is subjected equals the total applied load divided by the total area. When the loading is eccentrically placed on the base the pressure will not be uniformly distributed but will vary uniformly from a maximum at one edge to a minimum at the opposite edge, or to zero at some intermediate point. This variation of pressure depends on the magnitude of the eccentricity, and values of the maximum and minimum pressures are given by the formulæ on *Table No. 17*.

With a concentric load the pressure on the ground should not exceed the tabulated safe pressure, but these safe pressures can be exceeded by 10 per cent. when considering the maximum permissible pressure due to eccentric loads.

In order that the applied bearing pressure can be realised without risk of the material spewing out from under the base, it is essential that there shall be sufficient earth above the foundation level to counteract this effect by lateral pressure. The minimum depths of foundation levels in various kinds of soil are given on *Table No. 17*, and these depths are based upon consideration of spewing and upon the factors already mentioned in Paragraph 3 of this Chapter.

If bases are founded at depths greater than those tabulated, the safe pressures can be exceeded by an amount equal to the weight of earth between the foundation level and minimum depth. If advantage is taken of this concession, however, any earth immediately above and carried by the base must be included in computing the total load on the foundation.

5.—Design of Independent Bases.

Column footings of the form (a) shown on *Table No. 18* should be so proportioned that the bending stresses are negligible, and therefore the thickness of both the upper reinforced concrete footing and the lower mass concrete footing should be determined so that the load can be distributed on the mass concrete and ground by dispersion. When the angle of dispersion is assumed to be not less than 45 deg. to the horizontal the proportions are given by the formulæ

on *Table No. 18*. The thickness of each part of the footing must also be sufficient to keep the punching shear stresses within the permissible limits specified on *Table No. 23*, and expressions giving these thicknesses are also incorporated on *Table No. 18*.

The minimum thickness of reinforced concrete footings of the usual splayed type illustrated by (b) and (c) should also be determined from considerations of punching shear, and reinforcement provided to resist the bending moments specified on *Table No. 18*, where expressions for the moment at the edge of the column are given for square and rectangular bases. The resistance moment of splayed footings cannot be determined with precision, and the formulæ given on page 208 allow for the calculation to be made on a rational and conservative basis. When the size of the base relative to its thickness is such that the column load can be spread by dispersion over the whole area of the base, no bending moment need be considered and only nominal reinforcement need be provided.

When piers are provided to convey the column loads down to a suitable bearing strata, the proportions of the various parts of the pier should be in accordance with the data given on *Table No. 18*.

6.—Design of Combined Foundations.

When more than one column or load is carried on a single base the centre of gravity of the several loads should, if possible, coincide with the centre of the area of the base, in which case the pressure under the base will be uniformly distributed. The base should be symmetrically disposed about the line of the loads and should be either trapezium-shape as in *Fig. 21(a)*, or be made up of a series of rectangles as in *Fig. 21(b)*. In the latter case each rectangle should be so proportioned that the load upon it acts at its centre of area, and the area of each rectangle should be equal to the corresponding applied load divided by a safe bearing pressure, the value adopted for this pressure being the same for all the rectangles. If all the loads are equal and the columns placed at equal intervals the width of the base would be constant.

If it is not possible or practicable to proportion the bases as described the resultant load will be eccentric, and thus the centre of pressure of the upward ground pressure will have this same eccentricity relative to the centre of area of the base. If the base is thick enough to be considered as acting as a single rigid member, the ground pressure will vary according to the formulæ for eccentric loading and will give a pressure distribution diagram as in *Fig. 21(c)*. If the base is comparatively thin this theoretical distribution may not be realised and owing to the flexibility of the base the ground pressure may be greater immediately under the loads than at intermediate points, giving a pressure diagram somewhat as in *Fig. 21(d)*.

In the case of uniform distribution, or uniform variation of distribution, the longitudinal bending moment on the base at any section is calculable, being equal to the sum of the clockwise moments of each load to the left of the section minus the anti-clockwise moment of the upward pressure between the section and the left-hand end of the base. Since the moments due to the indefinite distribution of pressure indicated in *Fig. 21(d)* are indeterminable, it is suggested that for bases carrying any number up to five loads the theoretical distribution

be assumed and the longitudinal moments calculated accordingly. When a greater number of unequal loads are carried on a single strip base it should be sufficient if the longitudinal moment is calculated from the approximate formula given on *Table No. 17*.

When the load on a strip footing is uniformly distributed throughout its

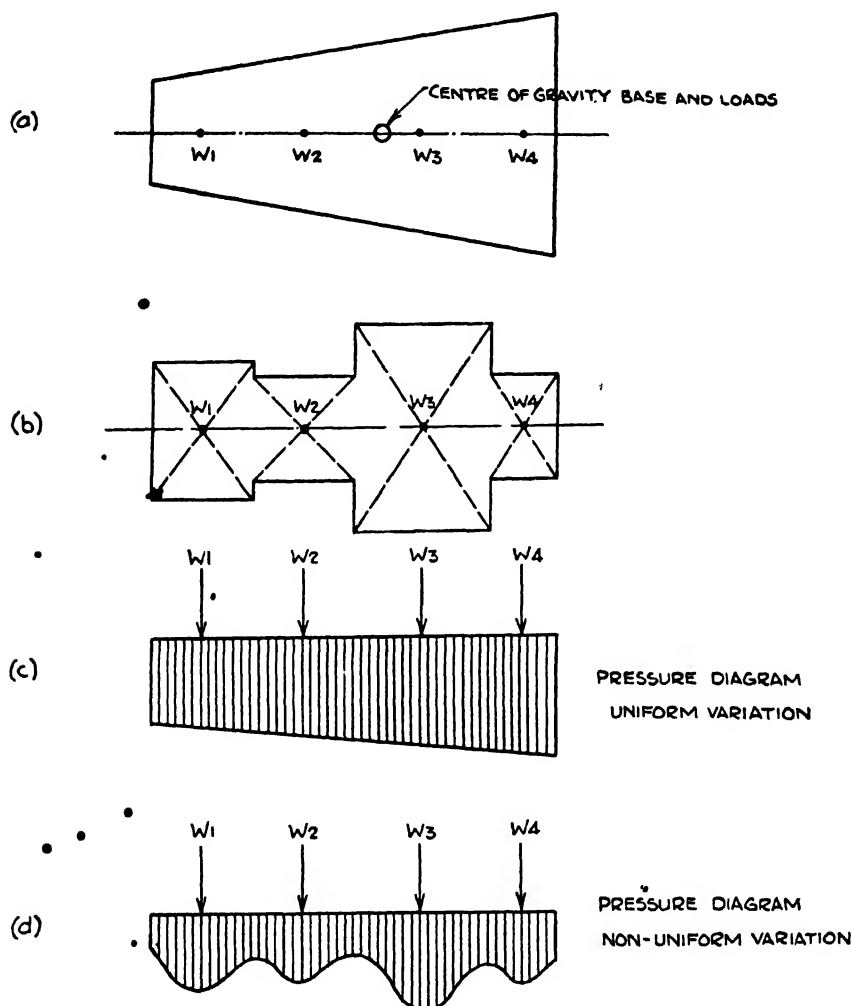


Fig. 21.—Combined Bases.

length, as in the general case of a wall footing, the principal bending moments will be due to the transverse cantilever action of the projecting portion of the base. If the wall is of concrete and is built monolithic with the base the moment is a maximum at the face of the wall, but if the wall is in brick or masonry the maximum moment occurs under the centre of the wall. Expressions for these moments are given on *Table No. 18*. When the overhang is less than the thick-

ness of the base the moments can be neglected, but in all cases the thickness of the base should be such that the punching shear stresses are not excessive.

Whether wall footings are designed for transverse bending or not, if the safe ground pressure is less than 2 tons per square foot a nominal amount of longitudinal reinforcement should be inserted to take possible longitudinal moments due to unequal settlement or non-uniformity of loading.

When it is not possible to place an adequate base centrally under a column or other load owing to site limitations, and when under such conditions the eccentricity would result in inadmissible ground pressures, a balanced foundation can be devised as illustrated diagrammatically in *Fig. 22 (a)*. The column

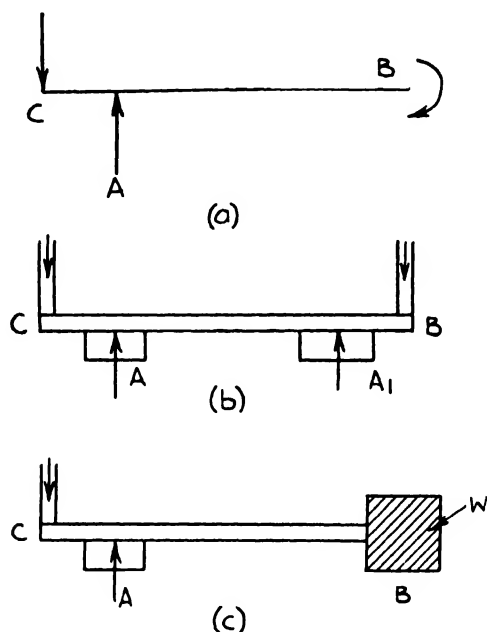


Fig. 22.—Balanced Foundations.

is supported on the overhanging end C of a beam BC, that is supported on a base at A and subjected to a counterbalancing effect at B. The reaction at A, which depends on the relative values of BC and BA, can be carried by an ordinary reinforced concrete or mass concrete base designed for a concentric load. The counterbalancing can usually be provided by the load from another column as in *Fig. 22(b)*, in which case the dead load on this column at B should be sufficient to counterbalance the dead and live load on the column at C, and vice versa. Formulae giving the resulting values of the reactions at A and A₁ are given on *Table No. 18*; from these reactions the shears and moments in the beam can be readily calculated.

If no column loads can be conveniently brought into service to counterbalance the column at C an anchorage must be provided at B by other means, such as the construction of a mass concrete counterweight or the provision of tension piles.

7.—Safe Load on Piles.

The load that can be safely carried on a pile depends on the maximum load that the pile can carry when considered as a column and on the minimum load that would produce settlement or further penetration of the pile into the ground. So many are the factors that enter into a consideration of the settlement load for any particular pile that the results of calculations made to determine this load do not have quite the conclusive value of the results of test loads on driven piles. The value of calculations, however, is greatly enhanced when they can be used comparatively in conjunction with loading tests. Such tests are often inconvenient and expensive, and for most normal cases the engineer has to fall back on safe loads computed by one of the many pile formulæ that have been put forward from time to time. When a number of these formulæ are applied to any one problem, the range of the results will be so wide that at first sight it would appear that the use of formulæ is most unreliable. This apparent inconsistency is largely due to the fact that each formula has its own limitations of application, and no particular formula should be used unless one is satisfied that the problem in hand falls within the conditions appertaining to the formula.

Pile driving formulæ fall into two main groups: (i) Impact formulæ which are applicable to bearing piles, and which, if carefully selected, can be very reliable. (ii) Friction formulæ which are applicable to piles that are supported by the frictional resistance of the ground in which they are embedded; these formulæ are not very reliable and should be associated with test loadings.

For the sake of comparison of impact formulæ, those which follow have been set out in similar form using the notation

W_1 = safe load on pile (tons).

w = weight of moving parts of hammer (tons).

H = fall of hammer (inches).

n = number of blows per final inch of penetration.

k = a coefficient to allow for characteristics of pile, type of hammer and method of operation, nature of soil, condition of dolly, factor of safety, and other factors not directly represented in the formula. (This coefficient is often fallaciously referred to as simply the "factor of safety.")

P = weight of pile (tons), including helmet, dolly, cushioning, and any stationary parts of the hammer resting on the pilehead.

$$R = \frac{P}{w}$$

A = effective cross-sectional area of pile (square inches).

L = length of pile as cast (feet).

E = elastic modulus of material of which pile is made (tons per square inch).

$$K = \text{elastic constant} = \frac{AE}{12L}$$

Impact formulæ can be conveniently divided into four classes:

CLASS I.—These formulæ involve only the weight of the hammer and the

drop, and are sometimes accompanied by a statement regarding the set. McAlpine's formula and Haswell's formula are of this type and are useless for reinforced concrete piles.

CLASS II.—This class takes the simple form

$$W_1 = \frac{wHn}{k}$$

The formulae proposed by Sanders ($k = 8$), Merriman ($k = 6$), and Berg ($k = 10$) are in this class, but this type of formula is unreliable unless the value of k can be fixed with regard to all the variables encountered in pile driving. The Wellington series of formulae for timber piles represents the factor k by an expression involving n , as does also the Nicholson formula for concrete piles. For well-driven concrete piles a value of k between 20 and 30 seems to be suitable.

CLASS III.—This class would include those formulae that, in addition to the primary factors w , H and n , also involve the ratio of pile weight to hammer weight. The best known is possibly the Dutch formula in its simplified form

$$W_1 = \frac{wHn}{k(1+R)}$$

For concrete piles the value of k would lie between 3 and 9. For piles subjected to dead loads only and driven by a winch-operated drop hammer the value of k should be taken as 6 if a helmet and dolly are used, but if no dolly is provided $k = 4$, and these values can be decreased by 10 per cent. if a single-acting steam-hammer is used. The safe loads calculated by this formula with these values of k have been tabulated on Table No. 19 assuming the set to be $\frac{1}{16}$ -inch per blow (i.e. $n = 10$) and the hammer drop to be 3 ft. For other values of n and H an expression for adjusting the tabulated loads is given, but this formula gives rather too high a value of W_1 for sets smaller than those represented by $n = 10$. High values of k should be used if the soil is resilient and up to 50 per cent. higher if the working load will be vibratory.

The Brix formula is also commonly used for reinforced concrete piles driven with drop hammers. This formula,

$$W_1 = \frac{wHnR}{k(1+R)^2}$$

can be used for higher values of n than the Dutch formula, using the same range of values for k as advocated for that formula. An American range of formulae of this class is of interest as indicating the effect of the type of the hammer:

$$\begin{aligned} \text{Drop hammer,} \quad W_1 &= \frac{wHn}{6(1+R)} \\ \text{Single-acting hammer,} \quad W_1 &= \frac{wHn}{6(1+0.1Rn)} \\ \text{Double-acting hammer,} \quad W_1 &= \frac{(ap+w)Hn}{6(1-cRn)} \end{aligned}$$

in which a = piston area (square inches).

p = steam pressure (tons per square inch).

c = constant = 0.1 to 0.3.

CLASS IV.—This class includes the more complex formulæ, and embraces a group of formulæ that incorporate the elasticity of the pile. Representative of this group is Rankine's formula

$$W_1 = \frac{1}{k} \left[\sqrt{wHK + \left(\frac{K}{n} \right)^2} - \frac{K}{n} \right]$$

where $k = 3$ for dead loads, pile driven without dolly.

$= 4$ for general work (dead loads and dolly).

$= 5$ for vibrating working loads.

The formulæ proposed by Redtenbacher, Weisbach, and Bennett are somewhat similar, and such formulæ can be satisfactorily used for most concrete pile problems, or as an alternative the use of the more modern and comprehensive formula proposed and extensively used by Mr. Hiley is advocated.

This formula embraces all the major variants occurring in pile driving problems, such as weight and type of hammer, drop, penetration per blow, length of pile, shape of point, type of helmet, nature of ground, and material of which the pile is made. In the form given on *Table No. 19* the constant c takes into account the energy absorbed in temporarily compressing the pile, the helmet, and the ground, and values of c for various degrees of severity of driving reinforced concrete piles of various lengths are given. These values assume that the pile is fitted with a helmet with packing and dolly of the type usually employed in connection with concrete piles. When no helmet is used the value of c should be reduced by 0.1 in. Since the quake of the earth below the pile shoe is included in this value it follows that the nature of the ground affects c , and the tabulated values are applicable to a firm gravel sub-stratum. If the pile bears on clay an additional 0.05 in. should be allowed, and if the material is of a peaty character with soft ground below it as much as 0.20 in. can be added to the tabulated value of c for normal driving conditions. The factor $2c$ is a quantity that is measurable on any pile that is being driven if proper arrangements are made, since it represents the difference between the permanent penetration for one blow and the maximum instantaneous depression of the pile-head as measured at the top of the helmet.

The factor e represents the efficiency of the blow and depends on the $\frac{P}{w}$ ratio; values of e are given on *Table No. 19*, together with values of $\frac{H_1}{H}$ which allow for the freedom or otherwise of the drop. Settlement loads as calculated by the Hiley formula are subject to a factor of safety allowance of $1\frac{1}{2}$ to 3 for normal structures.

When piles are driven into soft material and depend solely upon friction for their support, the safe load can be calculated by either of the formulæ given below :

$$(i) W = L[k_1 L(B + D) + k_2]$$

$$(ii) W = k_3(B + D)L + k_4 BD$$

where L = embedded length of pile (ft.).

B = breadth of pile (in.).

D = width of pile (in.).

The factors k_1 , k_2 , k_3 , and k_4 depend on the character of the ground driven through and into, and appropriate values are given on *Table No. 19*.

Formula (i) is more suitable when the material under the shoe of the pile is not of much better quality than that in which the greater length of the pile is embedded; formula (ii) is more applicable to cases where the lower few feet of the pile penetrate a firmer strata than that surrounding the major portion of the pile.

When a pile is driven through a fairly soft material to refusal on hard strata (rock) the safe load is determined by the strength of the pile considered as a column.

8.—Design of Piles.

Reinforced concrete piles should be designed to withstand the stresses due to handling, driving, and load carrying, with appropriate factors of safety. A pile made from normal-hardening cement should not be driven or lifted until it is six weeks old unless special precautions have been taken early in the curing stage to assist maturing. If the pile is cast in warm weather and is kept protected from the direct rays of the sun for several days and constantly wetted throughout this period, the interval between casting and driving may be reduced to four weeks. If rapid-hardening cement is employed this interval can be reduced to ten or seven days. Overstressing young concrete during the handling and slinging operations can be guarded against by so arranging the position and number of the points of suspension that the bending moments produced by the weight of the pile itself are within the safe resistance moments of the section. For these resistance moments a factor of safety of about three on the ultimate concrete strength at six weeks can be assumed; therefore for square piles having a normal percentage of longitudinal steel the maximum moment due to bending about an axis parallel to one side of the section should not exceed

$$M = 0.16cD^3 \text{ inch lb. approximately}$$

where c is the standard compressive stress tabulated for the appropriate concrete mix given on *Table No. 23*, and D = length of side in inches.

If toggle holes are provided some degree of assurance that the pile will not be lifted so as to bend about a diagonal is attained. The toggle holes should be so arranged that the minimum bending moments are experienced during lifting, and the appropriate positions of the holes for this condition for single and double-point lifting are given on *Table No. 19*. The size and length of the pile will determine whether single or two-point lifting should be adopted.

Until piles are three to four weeks old (or four to seven days with rapid-hardening cements) they should not be lifted or carted, and any moving should be by rolling the pile. In no case should the pile be disturbed until two weeks old, although the side forms may be struck in two or three days.

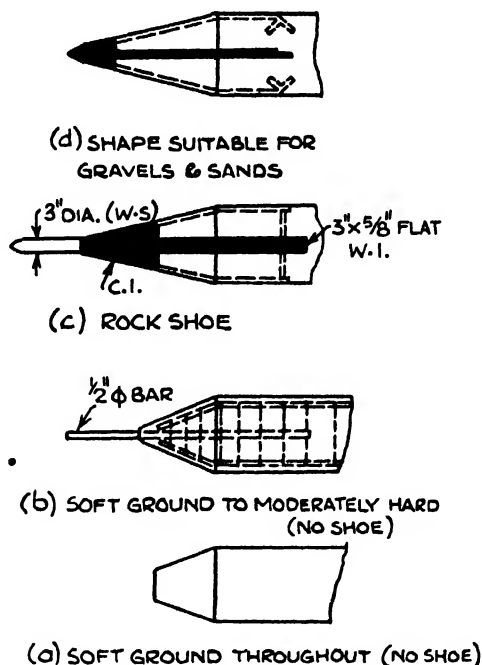
The maximum compression stresses experienced in a pile are usually those due to driving and occur at the head of the pile, generally as final set is approached. In the simplest case this stress would be approximately the driving force divided by the total effective cross-sectional area, but even with a well-centred pile it is probable that the maximum stress is at least double the mean stress. The driving force can be approximately calculated from the factors involved in the Hiley formula, and thus the probable maximum stress is given by

$$c_m = \frac{2wH_1en}{A(1 + cn)}$$

where A = total effective cross-sectional area of the pile.

If the value of c_m exceeds one-and-a-half times the standard allowable stress c_1 , there is danger that the cover concrete may spall off at the head of the pile, and therefore if driving conditions are anticipated in which by calculation $c_m > 1.5c_1$, either the richness of the mix or the effective cross-section of the pile should be increased. If c_m exceeds c_1 , the stresses in the pile will be unduly high unless special attention is given to the provision of sufficient helical binders in the topmost few feet of the pile. Octagonal piles usually have helical binders throughout their whole length.

In Fig. 24 is illustrated a typical detail of the reinforcement in a square reinforced concrete pile, in which helical binders at the head of the pile are



(a) SOFT GROUND THROUGHOUT (NO SHOE)

Fig. 23.—Pile Shoes.

provided. The arrangement of the toggle holes and spacers is also indicated. For driving into clays, gravels, or sands a pile shoe having a two to one overall taper, as shown in Fig. 24, is usually satisfactory, but for other types of soil other shoe shapes are necessary. If the pile has to be driven through soft material to take a bearing on a thin bed of gravel overlying softer ground it is necessary to have a blunter shoe to prevent punching through the thin strata. For friction piles driven into soft material throughout a shoe is not absolutely necessary, and a blunter end should be formed as in Fig. 23(a). When driving through soft material to a bearing on soft rock or stiff clay, the form of pile end shown in Fig. 23(b) is satisfactory so long as driving ceases as soon as the firm strata is reached or is only just penetrated. When driving down to hard rock, or where heavy boulders are anticipated, a rock shoe as shown in Fig. 23(c) should be fitted.

Irrespective of the load the pile can carry before risk of settlement is incurred, the stresses produced by the working load on the pile acting as a column should be considered. For calculation of the stress reduction due to slenderness (see Chapter XIII) the effective length of the pile can be considered as two-thirds of the length embedded in soft strata (or one-third of the length embedded in fairly firm strata) plus the length projecting above the ground, and usually the end conditions of a pile are equivalent to the general case of one end fixed and one end hinged. In no case should the effective column length be assumed at less than 5 ft. plus the projecting length.

In preparing pile lay-outs attention must be given to practicability of driving as well as to effectiveness for load carrying. In order that each pile in a group

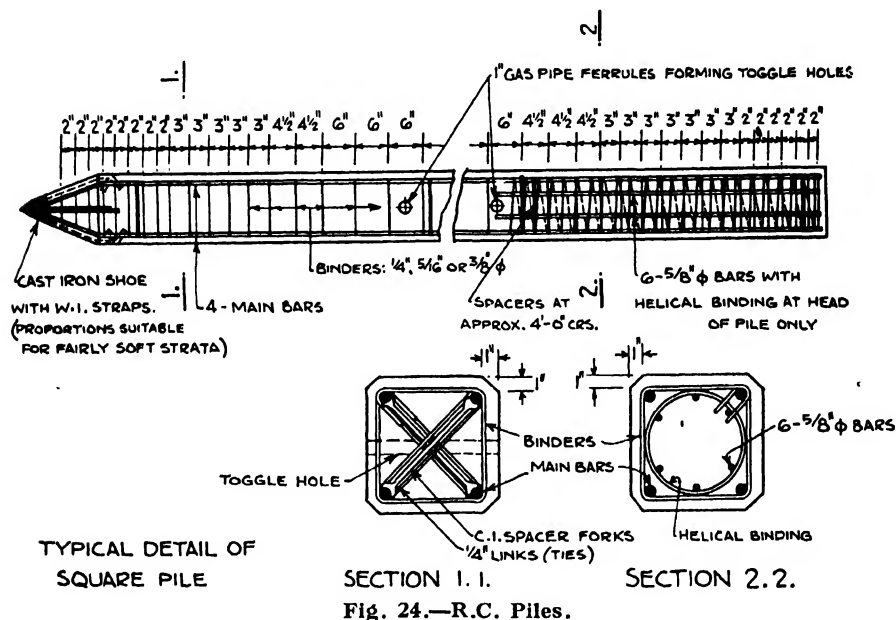


Fig. 24.—R.C. Piles.

shall carry an equal share of the load the centre of gravity of the pile group should coincide with the centre of the superimposed load. The clear distance between any two piles should generally be not less than 2 ft. 6 in., except in clays where experiments indicate that a clear distance equal to the size of the pile is most effective. So far as possible piles should be arranged to lie along a series of straight lines in both directions throughout any section of a particular job, as this form of lay-out minimises the amount of movement of the driver. The lay-out should also allow for driving to proceed in such a way that any displacement of earth due to the consolidation in the piled area shall be free to take place in a direction away from the piles already driven.

Pile caps should be designed primarily for punching shear around the heads of the piles and around the column base. The thickness of the cap should also be sufficient to provide adequate bond length for the pile bars and column dowel bars. If the thickness is such that the column load can all be transmitted to

the piles by dispersion no bending moments need be considered, but usually when four or more piles are placed under one column it is necessary to reinforce the pile cap for the bending moments produced. *Figs. 25(a) and 25(b)*, illustrate

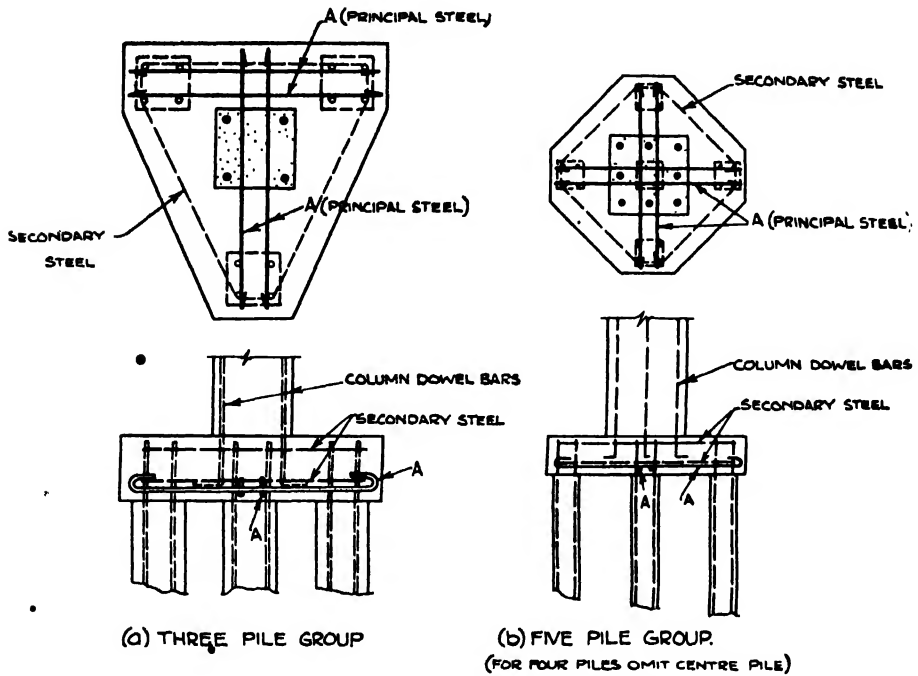


Fig. 25.—Typical Details of Pile Caps.

typical details of pile caps suitable for three, four, and five piles in a group, and in each case each of the bands of steel marked *A* is provided to give a resistance equal to a single pile load multiplied by the distance from the centre of the pile to the face of the column. The shear and compressive stresses on the concrete must be investigated in these cases.

CHAPTER VII

RETAINING WALLS AND CONTAINERS

1.—Types of Containers.

FOR convenience of reference to the various types of structures that are designed primarily to contain or retain materials deposited within or against them, the following meanings will be attached to the various terms employed :

RETAINING WALL.—A vertical or inclined wall holding up a bank of earth or other filling placed behind it, the wall to be of such form in plan that it does not return upon itself to form a continuous unbroken line.

CYLINDRICAL TANK.—A container that is circular in plan and is designed to contain liquids, or if it is used for storing granular materials is of such dimensions that the horizontal pressure exerted by the material at any depth is proportional to the depth.

BUNKER.—A container that is polygonal in plan, that contains either dry material or liquids, and is of such dimensions that the horizontal pressure at any depth is proportional to the depth.

SIL.—A container that is circular or polygonal on plan, is designed to hold granular materials, and is of such a depth that the horizontal pressure does not increase in direct proportion to the depth.

HOPPER BOTTOM.—The part of a polygonal container embracing the outlet, and in the form of an inverted pyramid, usually truncated.

CONICAL BOTTOM.—A hopper bottom in the form of an inverted truncated cone.

SUSPENDED BOTTOM.—The bottom of a container that is proportioned so that under maximum loading only tensile forces are produced ; usually in the form of an inverted parabola.

The method of calculating the horizontal pressures due to retained earth, and to liquids and dry materials contained in tanks, bunkers, and silos, is given in Chapter III, and the data ordinarily necessary for this calculation are given on *Tables Nos. 5 and 6*. The succeeding paragraphs deal with the general design of retaining walls and container structures, and with the calculation of the forces and moments produced by the pressure of the retained or contained materials.

2.—Types of Retaining Walls.

Retaining walls are essentially vertical cantilevers, and when constructed in reinforced concrete can be one of the following types :

- 1.—Simple cantilever walls.
- 2.—Counterforted walls.
- 3.—Sheet-pile walls.

The simple cantilever wall is suitable for small and moderate heights and can be constructed either with a base projecting backwards under the filling, as in *Fig. 26 (a)*, or with a base projecting forward as in *Fig. 26 (b)*. The former type is generally more economical. Designing methods for both types are discussed in Paragraph 4 of this Chapter.

Counterforted design is suitable for high walls or great pressures. The slab panels span horizontally between vertical counterforts or buttresses as in *Fig. 27(a)*, although in extreme cases the slab spans vertically on to horizontal beams (as in *Fig. 27(b)*) which react on the counterforts. By graduating the spacing of the beams the maximum bending moments in each span can be kept

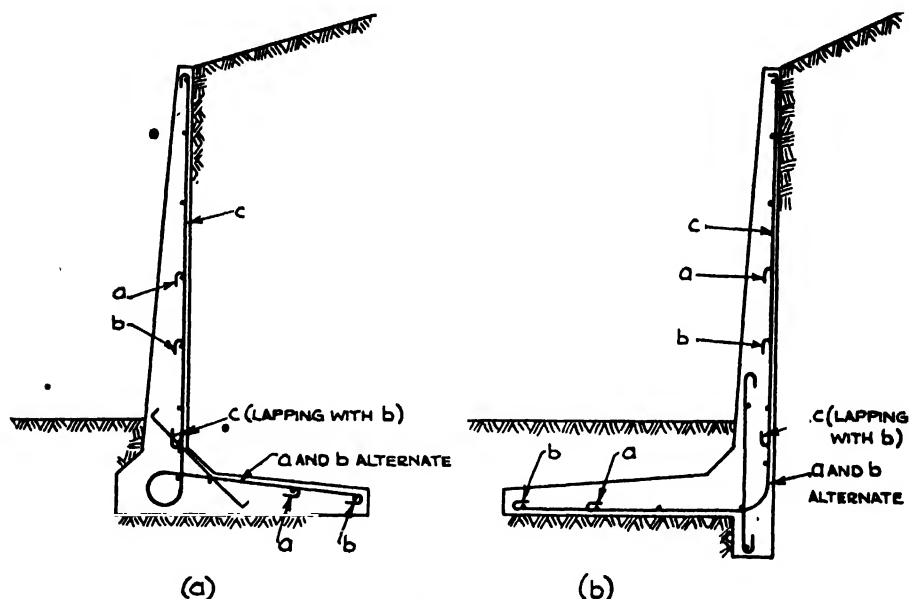


Fig. 26.—Typical Details of Simple Cantilever Retaining Walls.

equal and the slab maintained the same thickness throughout. When the horizontal shearing stresses will allow, the web of the counterfort can be cut out as indicated by the dotted lines in *Fig. 27(a)*. Counterfort type walls are designed in accordance with the same principles with regard to stability, sliding, and ground pressures as are simple cantilever walls, except that these factors are investigated for a length of wall equal to the distance apart of the counterforts instead of a length of 1 ft.

When satisfactory bearing strata are not encountered at a reasonable depth below the surface of the ground in front of a retaining wall, sheet pile walls are usually adopted. Such walls are constructed by driving pre-cast reinforced concrete sheeting into the ground a sufficient distance to obtain an anchorage for the vertical cantilever effect and security against sliding forward and spewing. This type of wall is particularly suitable for waterside works, and in the simplest

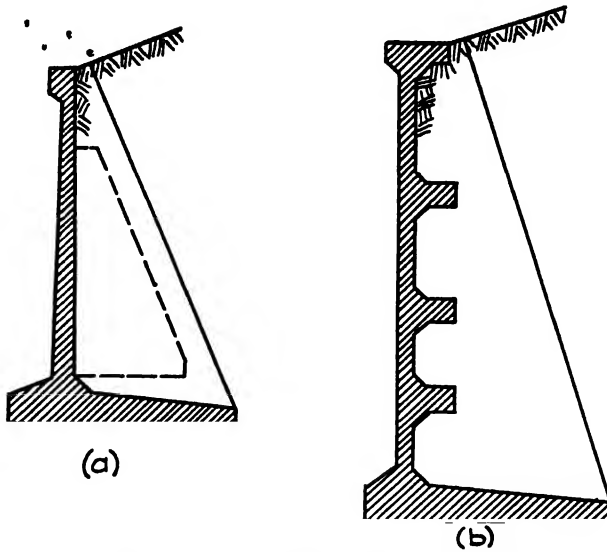


Fig. 27.—Counterfort Retaining Walls.

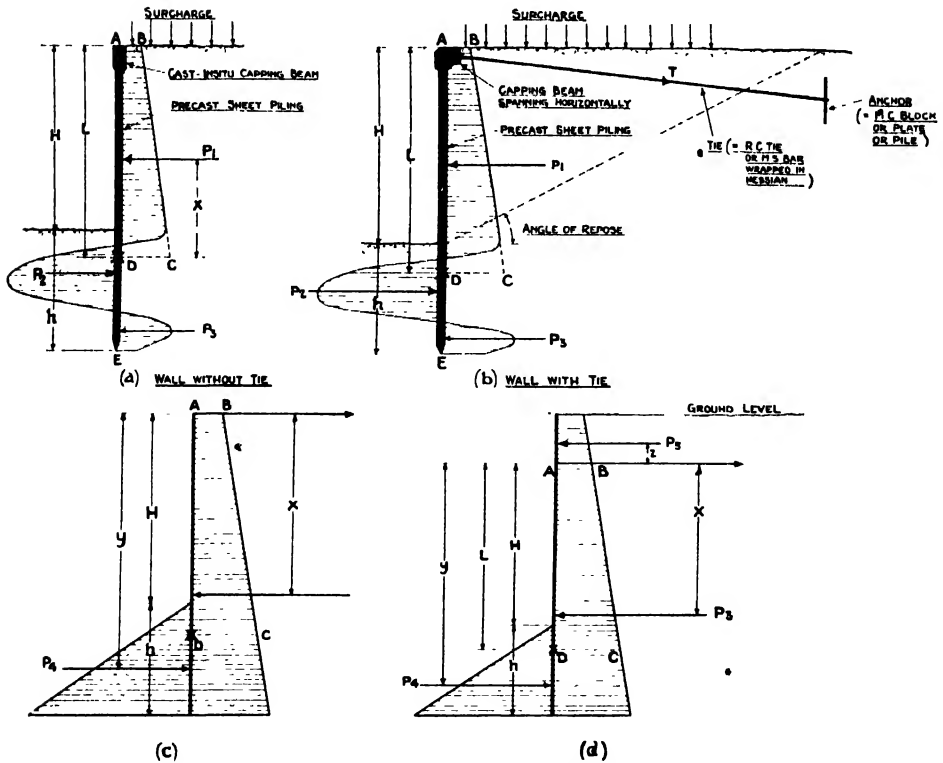


Fig. 28.—Sheet Pile Walls.

form the sheeting simply cantilevers out of the ground, the individual pile heads being generally stripped and bonded into a cast-in-situ capping beam.

Typical sections of sheet piles and common designs for the tongue-and-groove jointing that is necessary to maintain alignment during driving are given

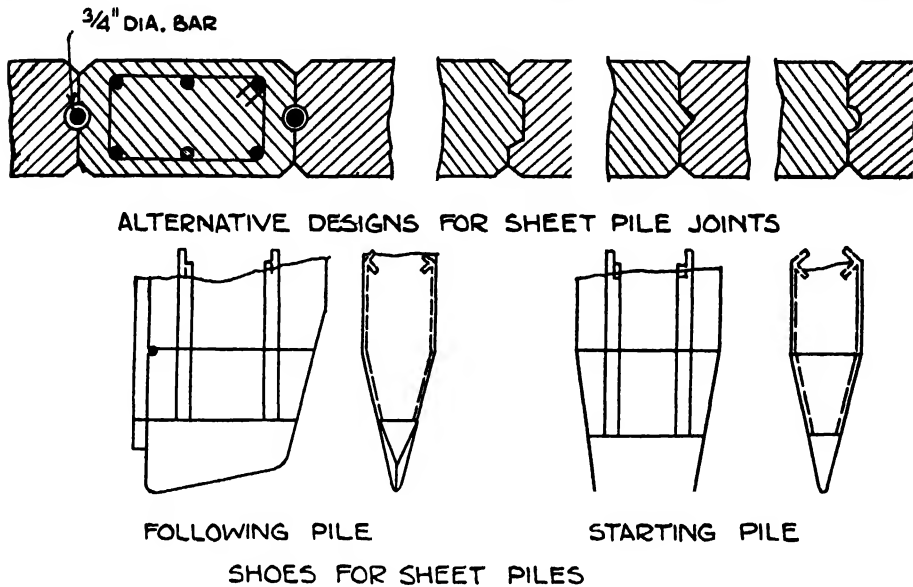


Fig. 29.—Sheet Piles.

in Fig. 29, which also illustrates typical shoe shapes for "starting piles" and "following piles."

If the height of the wall and the pressure on the sheeting are such that an excessive pile section is required the introduction of a tie at capping beam level will reduce the maximum moment. This tie can be either constructed in reinforced concrete or can be a mild steel bar, properly anchored into the capping beam and wrapped with hessian to protect it from corrosion. The capping beam must be designed to span between the ties and to take the horizontal reactions from the top of the sheeting. The remote end of the tie should be anchored behind the natural slope of the ground, and the anchorage should be provided by a block of mass concrete, a vertical concrete plate, or preferably by an anchor pile. Although the force in the tie is increased, moments can be reduced by placing the tie at a point below the top of the wall; a suitable beam must then be provided at this level. The forces and moments on sheet pile walls are considered in the next paragraph.

3.—Sheet Pile Walls.

The forces on a simple cantilever sheet pile wall are as indicated in Fig. 28(a), in which P_1 is the active pressure due to the filling and surcharge behind the wall and P_2 and P_3 are passive pressures producing the necessary restraint

moment to resist the overturning effect of P_1 . The maximum moment in the sheeting will occur at some point D , and the distance L is calculated by the factors k_1 given in the column headed "free" on *Table No. 20* for different angles of repose for the ground in which the pile is embedded; these factors, together with k_2 in the same column, are based on the more comprehensive range deduced by A. Freund. It is sufficient if the bending moment on the sheeting be calculated as P_1x , the value of P_1 being conveniently represented by the area of the trapezium $ABCD$ in *Fig. 28(a)*, which value can be determined from *Table No. 5*. The distance x locates the centroid of the area. The embedded length of the sheeting must be great enough to enable sufficient passive pressures to be produced, and the factors k_2 referred to enable this length to be calculated.

When a tie is introduced at the top of the sheeting the forces acting on the sheeting are as shown in *Fig. 28(b)*, which are similar to those in *Fig. 28(a)* except for the introduction of the horizontal force T in the tie. It is not simple to determine the variations of the pressure by mathematics with any precision, and therefore it is recommended that design problems be tackled in the following manner. The factors k_2 in the column headed "hinged" on *Table No. 20* will give the minimum value for h for the production of sufficient restraint moment. The embedded length h must not, however, be less than either of the minimum values required to resist forward movement of the toe or to prevent spewing. The sheeting will be stable if

$$P_3x > \frac{2}{3}P_4y$$

where P_3 = total active pressure on the whole depth of the sheeting as shown in *Fig. 28(c)*.

P_4 = total maximum passive pressure that can be brought into play in front of the sheeting.

The value of P_3 can be computed from the data on *Table No. 5*, and P_4 will be calculated from the formula given in Paragraph 6 of Chapter III. The factor $\frac{2}{3}$ is introduced in order to allow a margin between the theoretically calculated passive resistance and that actually required.

To prevent spewing in front of the sheeting the embedded length should not be less than $\frac{k_2 W}{w}$

where W = weight per square foot at point E of earth and surcharge above this point.

w = weight per cubic foot of earth in front of sheeting.

k_2 = pressure factor taken from *Table No. 5*.

The bending moment on the sheeting can be calculated by first determining L from the factors given in the column headed "hinged" on *Table No. 20*. The sheeting can be considered as a propped cantilever of span L built in at D and propped at A and subject to a trapezoidal load represented by the area $ABCD$. This load can be divided into a uniformly distributed load and a triangularly distributed load, for both of which the moment coefficients can be read off *Table No. 7*. This Table also gives the reaction on the prop, and in this case this reaction represents the force in the tie. Since the security of this design of wall depends on the efficiency of the anchorage, no risk of underrating the force in the tie should be incurred; it is better to increase the force to be provided for

from the value given by the theoretical reactions to half the value of P_1 . The value of this force should certainly not be less than $P_3 - \frac{2}{3}P_4$, which is the unbalanced part of the outward active pressure.

In Chapter III it was observed that the pressures behind flexible walls adjust themselves in such a way that the moments on the walls are reduced. Mr. Stroyer has put forward a formula by which the amount of this reduction can be assessed for reinforced concrete sheet pile walls. Without restricting the thickness-height ratio, this formula gives the following reduction factors, which are more conservative than those put forward by the Danish Society of Engineers.

MOMENT REDUCTION FACTORS FOR FLEXIBLE WALLS

Angle of Repose.	Ratio of Wall Thickness to Height:	0.02.	0.10.	0.20.	0.40.	0.60.
		0.02.	0.10.	0.20.	0.40.	0.60.
5 Deg. or less	.	1.00	1.00	1.00	1.00	1.00
14 "	.	0.82	0.89	0.92	0.96	0.98
20 "	.	0.67	0.80	0.86	0.92	0.96
30 "	.	0.52	0.67	0.76	0.85	0.92
35 "	.	0.44	0.61	0.70	0.82	0.89
45 " or more	.	0.31	0.49	0.61	0.75	0.85

The case of the anchored wall illustrated in *Fig. 28(b)* and *28(c)* assumes that the connection between the tie and the head of the sheeting is equivalent to a hinge; that is, the moment at *A* is zero. If the wall is extended above *A*, either by continuing the sheeting or by constructing a cast-in-situ wall, as shown in *Fig. 28(d)*, a moment is introduced at *A* equal to P_5z where P_5 is the total active pressure on the extended portion of the wall *AF*. This moment will introduce a negative moment in the sheeting at *A*, but will reduce the positive moment in the sheeting between *D* and *A* and will also reduce the negative bending moment at *D*. If this moment at *A* is large enough to produce conditions amounting to complete fixity at *A*, then the span *L* can be calculated by the factors k_1 given in the column headed "fixed" on *Table No. 20*. In this case also the factor k_2 in the same column will give the minimum embedded length *h*, but at the same time *h* must be sufficient to prevent spewing and forward movement as already described. The equation for stability will be

$$P_3x - \frac{2}{3}P_4y = P_5z.$$

The force allowed for in the tie will be given by

$$T = P_3 - \frac{2}{3}P_4 + P_5$$

$$\text{or } T = 0.5P_3 + P_5$$

whichever is greater. The moment on the sheeting will be calculated from the pressure represented by the area of the trapezium *ABCD* considering the beam fixed at both ends and using the appropriate coefficients given on *Table No. 7* and the reduction factors already tabulated.

When the moment at *A* is insufficient for complete fixity, the moments, forces, and values of *L* and *h* will be intermediate between those for hinged and fixed conditions at *A*.

A horizontal slab supported on a system of king piles is sometimes provided at *A*, and this has a sheltering effect on the sheeting inasmuch that if it is carried far enough back it can completely relieve the sheeting below *A* from any active pressure due to earth or surcharge above the level of *A*.

4.—Design of Cantilever Retaining Walls.

The factors affecting the design of simple cantilever walls are usually considered per foot length of wall when the wall is of uniform height, but when the wall is of varying height a section of convenient length, say 10 ft., should be treated as a complete unit. The principal factors to be considered are:

1.—*Stability*.—A minimum factor of safety of $1\frac{1}{2}$ against overturning should be allowed.

2.—*Ground Pressure*.—The maximum pressure should not exceed the allowable pressure for the type of ground upon which the wall is founded.

3.—*Sliding*.—The total vertical load multiplied by an adequate coefficient of friction should exceed the total horizontal pressure by at least 50 per cent. For sands, gravels, rock, and other fairly dry soils, a coefficient of 0.4 is suitable, but for less reliable clayey soils a rib should be provided on the underside of the slab to ensure a more definite resistance to sliding. In walls of the form shown in *Fig. 26(b)*, where the vertical load is always small compared with the horizontal pressure, such a rib should always be provided.

4.—*Resistance Moments*.—The safe resistance moment of the stem of the wall should be equal to the bending moment, which can be calculated from the pressure behind the wall, and will be a maximum at the top of the haunches at the base of the stem. The base slab should be made the same thickness as the bottom of the wall stem and the same steel should be provided. The base slab can be tapered as indicated in *Fig. 26*.

Formulae giving the limiting values of the moments, forces, and dimensions, etc., for simple cantilever retaining walls are given on *Table No. 20*.

The tapering of the base slab in front of and behind the wall serves not only to economise in concrete but also to assist drainage, the consideration of which is of some importance in retaining wall construction, especially for walls that have been designed for low pressures. Where the fill behind the wall is gravel or sand, a French drain of loosely-packed rubble should be constructed along the base of the back of the wall, and weep-holes, 3 in. to 6 in. diameter, provided at intervals of about 10 ft. A weep-hole should be provided in every "pocket" formed by counterforts, and the top surface of any intermediate horizontal beams should be given a slight slope away from the back of the wall. With backings of poor porosity, hand-packed rubble placed behind the wall for almost the whole length, in addition to weep-holes, materially assists effective drainage of the filling, and since the rubble is partially self-supporting the pressure on the stem is to a certain extent relieved. The filling behind the wall should not be tipped from a height into position, but should be carefully deposited in shallow horizontal layers.

Walls of greater length than, say, 100 ft. should be provided with expansion joints at 60-ft. to 100-ft. centres depending on the aspect and exposure of the wall. To provide against contraction and temperature cracks occurring on exposed faces of walls reinforced on the earth face only, a mesh of reinforcement, say, $\frac{3}{8}$ -in. diameter at 12-in. centres horizontally and vertically, should be provided on the exposed face if the wall thickness exceeds 8 in.

5.—Cylindrical Tanks.

The walls of a cylindrical tank are primarily designed to take the direct tension due to the horizontal pressures of the contained materials, and if p lb. per square foot is the unit pressure at any depth considered the value of the direct tension on a ring 1 ft. in depth is given by

$$T = 0.5pD \text{ lb.}$$

where D = internal diameter of the tank (ft.). Sufficient horizontal steel must be provided to take this tension, and if t lb. per square inch is the safe tensile stress and A_T is the area of the cross section of the reinforcement (sq. in.)

$$A_T = \frac{T}{t} = \frac{pD}{2t}.$$

For tanks containing liquids the value of t should be from 12,000 lb. to 14,000 lb. per square inch, but for dry materials a stress of 16,000 lb. to 18,000 lb. per square inch can be allowed. These stresses and the length of overlap allowed on the bars should be subject to the remarks in Chapter X. For cylindrical tanks containing liquids the thickness of the walls should be determined with relation to the total direct tension. In order that tension cracks should not localise, the tensile stress in the concrete should be kept within reasonable limits; suitable maximum values of the concrete tensile stress c_t on the total effective section are given on *Table No. 23* for various mixes. The minimum thickness of the wall d , at any depth h , for any value of t adopted in the design of a tank containing a liquid weighing w lb. per cubic foot, is given by

$$0.042whD \left[\frac{1}{c_t} - \frac{(m-1)}{t} \right].$$

Underground or submerged tanks will be subjected to external pressures due to the surrounding earth or water, which will produce direct compression in the walls. The stress produced by this compression will be a maximum when the tank is empty, and will be

$$c = \frac{p_e D}{2[12d + (m-1)A_T]}$$

where p_e = intensity of external pressure at the depth considered.

This stress is usually well within safe limits. Unless conditions are such that the permanence of the external pressure is assured, the tensional relief provided by the compression should be disregarded altogether in the design of the tank when full. When empty, the structure should be investigated for flotation if it is submerged in a liquid or is in waterlogged ground.

In addition to the horizontal tension in the walls of a cylindrical container the bending moments produced by fixity at the base of the walls must be considered. Unless a definite joint is made at the foot of the wall there will be a certain amount of continuity between the walls and the base slab that will cause vertical cantilever action for a certain height up the wall. In consideration of this problem there are three principal factors, namely, (a) the bending moment at the base of the wall, (b) the point at which the maximum ring tension occurs, and (c) the magnitude of the maximum ring tension.

The coefficients and formulæ for determining these factors given on *Table*

No. 20 are derived from Mr. H. Carpenter's simplification of Dr. Reissner's exhaustive mathematical consideration of the subject. In the complete study the shape of the wall enters into consideration, but the difference between the moments at the bases of triangular or rectangular walls (other conditions being equal) is so small that the usual intermediate case of a trapezoidal section can be considered the same as a rectangular wall. The error involved would partially offset the error of assuming perfect fixity at the junction of the wall and base slab.

The design procedure is first to determine the maximum vertical moment and provide an equal resistance moment at the base of the wall. This is followed by the determination of the maximum ring tension and the point on the wall at which this occurs; sufficient steel (and concrete) must be provided at this point to take this maximum tension. Above this point the steel can be uniformly graded down to a nominal amount, and below this point the steel can be maintained equal to that required for the maximum ring tension.

6.—Bottoms of Elevated Tanks.

The design adopted for the bottoms of elevated tanks depends on the diameter of the tank and the head of water sustained. For very small tanks a flat beamless slab bottom is satisfactory, but beams are necessary for tanks from 10 ft. up to, say, 25 ft. diameter. Alternative arrangements are indicated in *Fig. 30(a)* and *30(b)*. In *Fig. 30(a)* each beam is designed to span between opposite columns and takes one-quarter of the load on the tank bottom. The remainder of the load not taken by the two beams, together with the weight of the walls and the roof load, is taken directly on to the columns through the walls. In the arrangement shown on *Fig. 30(b)*, each length of beam between columns takes the loading on the shaded area, and the remainder of the loading on the floor of the tank together with the weight of the walls and the roof load is equally divided between the eight cantilevered portions of the beams.

For large diameter tanks domical bottoms (and roofs) of either of the types shown in *Figs. 30(c)* and *30(d)* are most economical, and although the shuttering is much more costly the saving in material over beam and slab construction is considerable. The ring beams marked *R* take the horizontal component of the thrust from the domes, and the thicknesses of the domes are determined by the magnitude of this thrust. The working compressive stress in the roof domes should be kept low, say 20 per cent. of the safe standard stress, in order to guard against localised increases due to incidental point loading or to non-uniform distribution of the superload. For the bottom domes, where the uniformity of the loading is more assured, a higher stress can be worked to, and about one-half to 1 per cent. of reinforcement should be provided in each direction. The shear around the periphery of the dome should also be investigated and sufficient concrete thickness provided to resist the shearing forces. Expressions for the maximum axial thrust and vertical shear around the edge of the dome, together with the resultant ring tension in the ring beams, are given on *Table No. 22*.

The *Intze* form of tank bottom illustrated diagrammatically in *Fig. 30(d)* produces an economical design for large diameter tanks. The outward thrust from the top of the conical section is taken by the ring beam *S*, and the difference

between the thrust from the bottom of the conical section and the thrust from the domical section is taken by the ring beam *T*. Referring again to *Table No. 22*, expressions are given for the forces, etc., in the various parts of such a tank bottom; the proportions of the rises and diameters of the conical and domical sections can be so arranged that the resultant thrust on *T* is zero. The walls of the cylindrical portion of the tank should be designed in accordance with

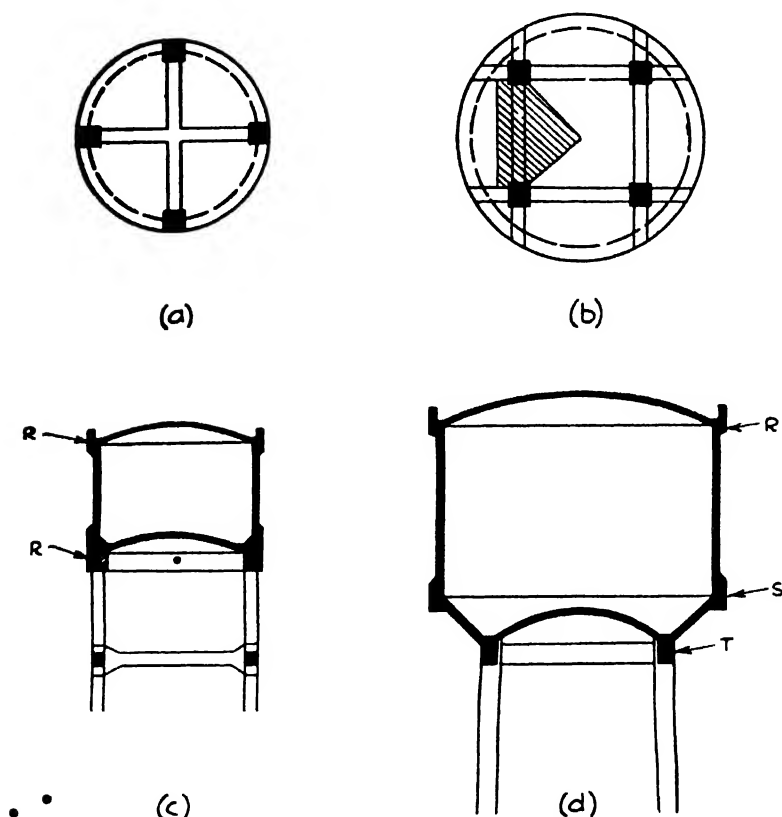


Fig. 30.—Water Tower Bottoms.

the principles described in the previous paragraph, due account being taken of the cantilever moment at the base of the wall and its effect as a transverse moment in the conical section.

7.—Bunkers and Silos.

The walls of bunkers and silos have to be designed to take bending moments and direct tensions induced by the contained material. If the wall slabs span horizontally they have to be designed for the combined effect of the moments and direct forces in accordance with the methods described in Chapter XIV.

If the walls span vertically horizontal steel should be provided to take the direct tensions and vertical steel to take the bending moments. In this case also the horizontal bending moments due to continuity at corners should be considered, and it is usually sufficient if as much horizontal steel is provided at any level at the corners as is normally required for vertical bending at this level; the maximum amount of steel provided for this purpose, however, need not exceed the amount of vertical steel required at one-third the height of the wall.

For walls spanning horizontally the bending moments and forces will depend upon the number and arrangement of the compartments. For multi-compartment structures the intermediate walls act as ties between the outer walls, and on *Table No. 21* expressions are given for the negative moments in the outer walls of rectangular bunkers with various arrangements of intermediate walls or ties. The corresponding expressions for the reactions, which are a measure of the direct tensions in the walls, are also tabulated.

Any particular span of an exterior wall is subject to its maximum stresses when the adjacent compartment is loaded, since in this condition it is subjected to both the maximum moment and maximum direct force due to the filling. An intermediate wall is subject to maximum bending moment when the compartment on one side of it is filled, and to maximum direct force (but minimum moment) when both adjacent compartments are loaded.

In the case of an elevated bunker the whole load is usually transferred to the columns by the walls, and when the span exceeds twice the depth of the wall the latter can be designed as a beam. If the magnitude of the moment warrants, a compression head can conveniently be provided at the top of the wall, but there is usually ample space to accommodate the tension steel in the base of the wall. When the span between columns is less than twice the wall height the true beam action becomes less apparent; the wall is then more in the nature of an arch and requires reinforcement to take the tie force along the base of the wall. Sufficient reinforcement should be provided to take a direct force equal to one-quarter of the total load carried by the wall.

In addition to the stresses produced by vertical bending moment (if any) the stresses due to loads, other than the filling, carried by walls must be investigated. These loads may be due to the roof or superstructure or to cranes mounted above the bunker and to the weight of the wall itself. In the case of big bunkers the moments and forces due to wind pressure should be calculated, and in silos the direct compression force induced in the leeward walls by wind pressure is one of the principal forces to be investigated. The equally important stress due to the weight of filling sustained by friction on silo walls (see Chapter III) must be added to the stresses produced by wind pressure, and at the base and top of the walls there may be additional bending stresses due to continuity with the base slabs or covers.

Where walls differ in thickness between the top and base there are a number of factors to consider before deciding whether the wall should be tapered uniformly from top to bottom or whether the reduction in thickness should be made in a number of definite steps. The shuttering is somewhat more costly for tapered than for stepped walls, especially in the case of circular containers. Stepped walls, however, may induce high secondary stresses at the change of section, and since the daywork joints are usually arranged at the change of

section liability to cracking is greater. Stepping on the outside is often objectionable as providing ledges for the collection of coal, cement, or other dust

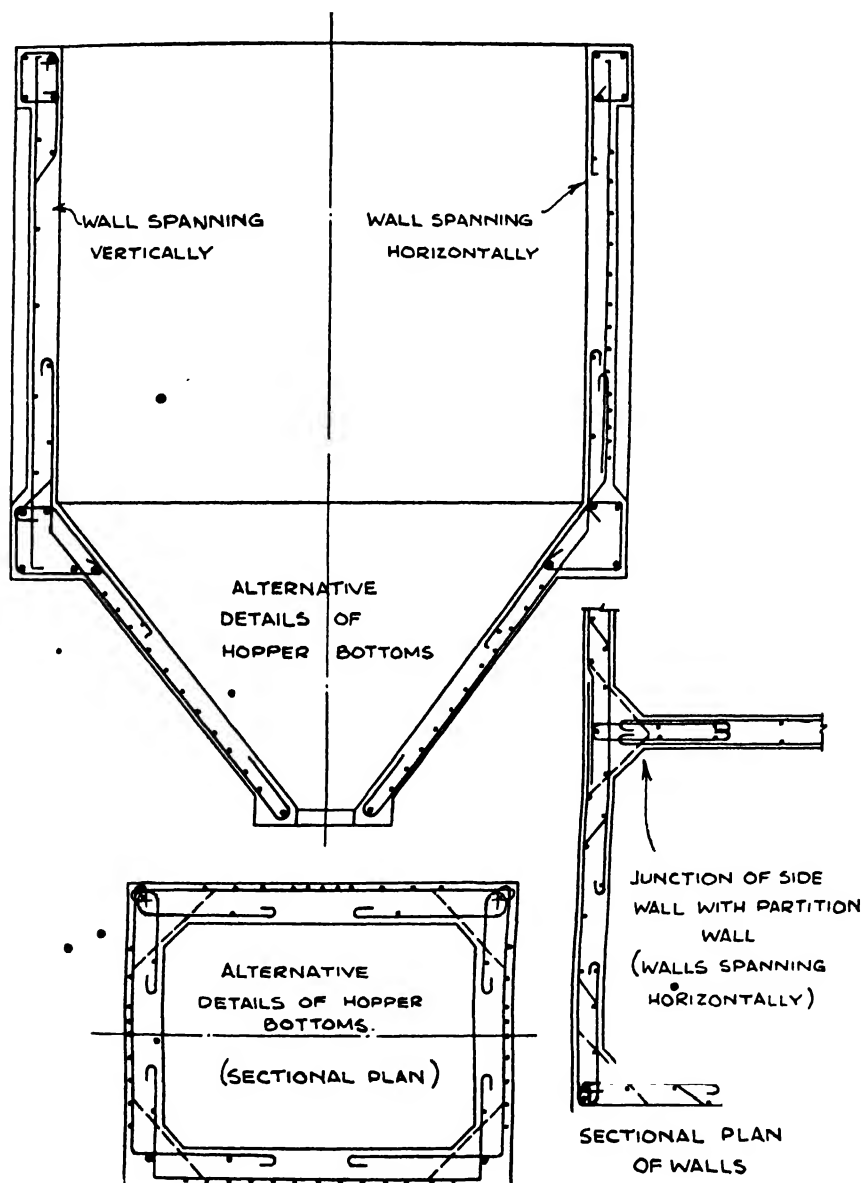


Fig. 31.—Typical Details of Bunkers.

that is usually in the air in an industrial works, and stepping on the inside may interfere with the free flow of the filling when emptying the bunker or silo.

In Fig. 31 typical arrangements of reinforcement in the walls and bottom

of a bunker are illustrated, both for a structure where the walls span vertically and for one where they span horizontally. In the latter case the reinforcement will vary from a maximum at the bottom to a merely nominal amount at the top of the wall, and the vertical steel need only be sufficient to keep the horizontal bars in place; for general cases $\frac{3}{8}$ -in. bars at 12-in. to 15-in. centres are satisfactory. In the case of tall bunkers each lift of vertical steel should not exceed about 10 ft., to prevent distortion due to "whippiness."

The useful life of a reinforced concrete bunker is considerably in excess of that of a steel or timber structure, but this life can be further prolonged by careful detailing. By thus reducing the liability to cracking, the concrete does not deteriorate so rapidly under service conditions due to abrasion from falling and moving coal or stone or other hard filling. If attention is given to the secondary stresses and care is exercised to obtain a good dense concrete, lining the wearing faces of the structure with tiles or plates is unnecessary except when there is the liability of spontaneous combustion. The top surfaces of ties and intermediate walls exposed to falling material should be made in the form of an inverted V and should be protected by a replaceable metal shield.

8.—Hopper Bottoms.

The design of hopper bottoms in the form of inverted truncated pyramids consists of finding a "centre of pressure" and the normal pressure at this point for each sloping side. With a determined mean span the bending moments at the centre and edge of the sloping side considered can be calculated. The horizontal direct tension is then computed and the horizontal steel determined. The direct tension acting along the slope at the mean centre and the moment at this point are combined to find the reinforcement necessary in the underside of the slab at this point. Likewise at the top of the slope the moment and the component of the hanging-up force are combined in order to calculate the steel required in the upper face of the sloping side at this point.

The "centre of pressure" and the mean span can be most economically found by inscribing on a normal plan of the sloping side considered a circle touching three of the sides. The diameter of this circle is "the mean span" and the centre is the "centre of pressure." The total load normal to the slab at this point is the sum of the normal components of the vertical and horizontal pressures at the "centre of pressure" and the dead weight of the slab; the values of the pressures, etc., and the resulting moments, together with the direct tensions both along the slope and horizontally, are given on *Table No. 22* for the critical parts of the structure. An example of a hopper bottom design is given in the "Additional Examples" that follow *Table No. 40*.

In adopting this method of design it should be remembered that, although the horizontal span of any side is considerably reduced towards the outlet, the steel should not be reduced below that determined for the "centre of pressure," since in determining the moment based on the mean span adequate transverse support from the steel towards the base is assumed. In *Fig. 31* a commonly adopted detail of reinforcement for a hopper bottom is illustrated.

CHAPTER VIII

BRIDGES, BUILDINGS AND OTHER STRUCTURES

1.—Structural Design.

ANY given structure is an assemblage of members each of which is subjected either to pure bending or to direct forces or to combined bending and direct force. The designer's art embraces not only the computation of and the provision for the moments and forces in a given assembly of members, but also the arrangement of these members in such a way that the moments and forces produced are reduced to minimum values consistent with the requirements of the structure and the limitations of the site. In previous Chapters, in addition to outlining the methods of computing the moments, shears, and forces produced by the external loading, some indication has been given of the application of these methods to such structures as foundations, retaining walls, tanks, bunkers, and silos. In this present Chapter it is proposed to deal briefly with other types of structures that are met with in designing practice. The necessary sections required to resist the calculated moments and forces on the various parts of the structure should be determined in accordance with the methods set out in later Chapters.

2.—Arch Design.

Within the scope of this volume it is impracticable to give to arch design the detailed consideration that the subject merits and necessitates. All that will be attempted is to give (Paragraph 3) the data for the design of fixed-end arches for those cases when the designer has a free hand in choosing the profile of the arch, whether the ratio of span to rise is specified or not. The reader is referred to the appended Bibliography for a list of suitable authorities that specially treat the simpler problem of hinged arches and the more difficult problem of designing arches to specified profiles that do not approximate to parabolic.

Arches are of two principal types, namely, "hinged" or "fixed." A hinged arch can be either hinged at both supports, or at both supports and the crown. A fixed arch can either be rigidly fixed at the springings, or can be partially fixed if it is one of a series of continuous arches. A fixed arch is more economical in material than a hinged arch, but the calculations usually presuppose absolute rigidity at the supports and freedom from settlement at these points. Unless these conditions can be absolutely assured it is safer to adopt a hinged design; further, the design procedure for a hinged arch is much simpler than that for a fixed arch, since at the hinges there is no bending moment.

Any particular section of a fixed arch, whether it be an arch rib or an arched slab, is subjected to a bending moment and a thrust; the determination of the magnitude of these at the critical sections is the objective of the calcula-

tions. Arch design is a matter of trial and error since the section and shape of the arch rib or slab enter into the essential calculations, but it is possible to select a preliminary section that will give results that reduce repetitive arithmetical work to a minimum. The following is a suggested method of determining the possible section at crown and springing, and is based on treating the fixed arch as a hinged arch. Referring to *Fig. 32*, draw a horizontal line through the crown *C*, and find *G*, the point of intersection with the vertical through the centre of gravity of the dead load on half the arch span. Set off *GT* equal to the dead load on the half span, drawn to a convenient scale; draw a horizontal through *T* to intersect *GS* produced in *R*. Draw *RK* perpendicular to *GR*, and *GK* parallel to the tangent to the arch axis at *S*. In the same weight units as *GT*, scale off *TR* = H_c and *GK* = H_s . If *c* is the maximum

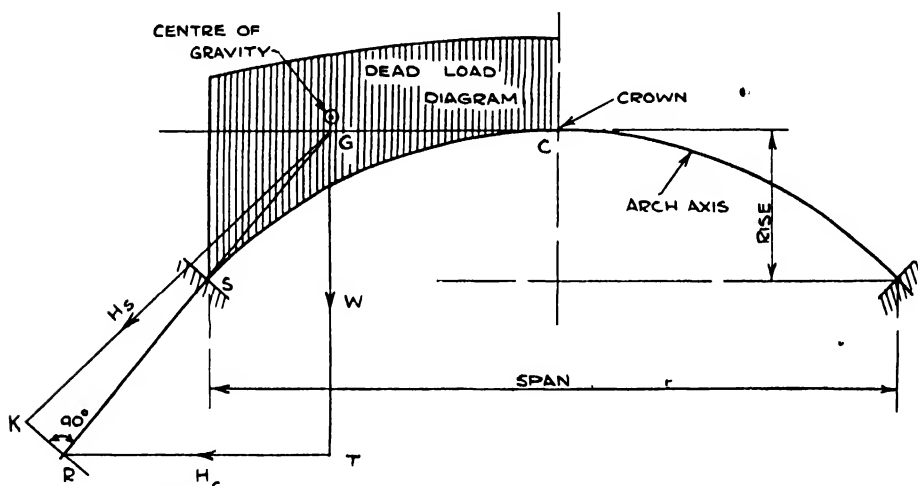


Fig. 32.—Approximate Method of Determining Thrusts.

allowable compressive stress in the concrete, *d* is the arch thickness at the crown, *d_s* is the arch thickness at the springing, and *b* is the assumed breadth of arch rib (12 in. for an arch slab), then the following expressions approximately apply :

$$d = \frac{1.7H_c}{cb}$$

$$d = \frac{2H_s}{cb}$$

Having fixed approximate sizes thus, or otherwise, a calculation is made to determine the thrusts and moments, and the stresses produced by these are computed. These stresses will determine the suitability or otherwise of the assumed sections.

3.—Fixed Arches.

Consideration in this paragraph will be limited to symmetrical, approximately parabolic, fixed-end arches that can be either open or closed spandrel

arch rib or arch slab design as shown in *Fig. 33*, and the treatment is based on Mr. H. Carpenter's development of Dr. Strassner's methods. In all the cases considered it is assumed that the axis of the arch is made to coincide with the line of pressure due to the dead load, which gives the most economical design and simplest methods of calculation. If the increase in thickness of the arch ring from crown to springing is a parabolic variation, only the moments and thrusts at the crown and springing sections need be investigated.

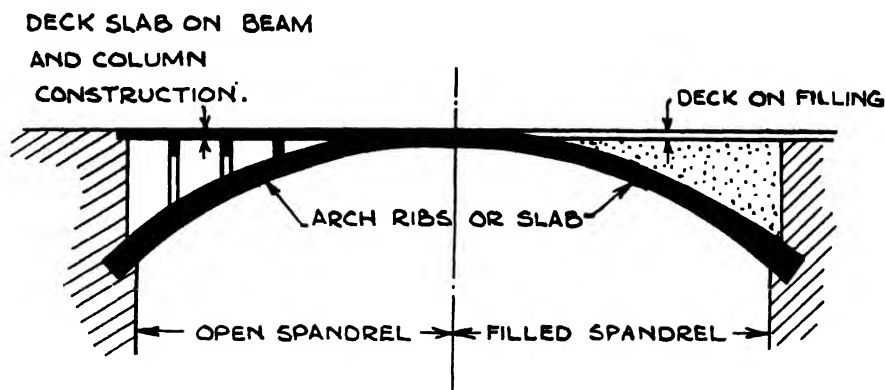


Fig. 33.—Arch Types.

Due to the dead loading alone the horizontal thrust is given by

$$H = \frac{k_1 w_d L^2}{R}$$

where w_d = dead load per unit length at the crown

L = span and R = rise of arch axis

and k_1 = a coefficient depending on the dead load at the springing ; values are given on *Table I*.

Due to the elastic deformation produced by the thrust along the arch axis the assumption of rigid abutments produces an anti-thrust $= H_D$ which, while slightly reducing the thrust due to the dead load, renders this thrust eccentric, producing a positive bending moment at the crown and a negative bending moment at the springings. If d = thickness of arch at crown

$$H_D = k_2 \left(\frac{d}{R} \right)^2 H$$

where k_2 is a coefficient depending on the relative thicknesses at the crown and springing, and values of k_2 are given on *Table II*.

Due to dead load and arch shortening, the resultant thrusts H_C and H_S at the crown and springing respectively and parallel to the arch axis at these sections, are given by

$$H_C = H - H_D$$

$$H_S = \frac{H}{\cos \phi} - H_D \cos \phi$$

where ϕ is the angle between the horizontal and the tangent to the arch axis at the springing. Values of $\cos \phi$ for various types of arches are given on Table III.

The moments due to the eccentricity of H_C and H_S are given by

$$\begin{aligned} M_C &= k_3 RH_D \\ M_S &= (k_3 - 1) RH_D \end{aligned}$$

For values of k_3 see Table IV.

For arches of small rise-span ratios, or with thick rings compared with the span, the stresses due to arch shortening may be excessive and the difficulty is overcome by introducing temporary hinges at the crown and springings. These hinges are filled with concrete after the arch shortening deformations have taken place; this method eliminates all bending stresses due to dead loading.

The additional horizontal thrust due to increase of temperature and the anti-thrust due to fall in temperature are given by the expression

$$H_T = \pm k_4 \left(\frac{d}{R} \right)^2 dt$$

where t = rise or fall in temperature in deg. F. and where k_4 has the values given on Table V. If d and R are in feet, H_T will be in lb. per foot width of arch. The values of k_4 are based on an elastic modulus for concrete $E_c = 2,000,000$ and a linear expansion coefficient $e = 0.000066$ per deg. F. If other values, say E_1 and e_1 are adopted, then k_4 should be multiplied by $0.076 E_1 e_1$. A value of t equal to 15 deg. F. is ample for structures in the British Isles, but careful consideration should be given to those factors that may necessitate an increase or may justify a decrease in the temperature range.

At the crown the increment or decrease in normal thrust due to temperature change equals H_T and the moment produced is given by $-k_5 RH_T$; due account must be taken of the sign of H_T when substituting. The normal thrust at the springing due to temperature change is given by $H_T \cos \phi$, and whether the thrusts due to dead load, etc., are augmented or decreased thereby depends upon the sign of H_T . At the springing the moment is given by $(1 - k_5) RH_T$ and the sign will be the same as that of H_T .

The shrinkage that takes place when concrete hardens produces anti-thrusts equivalent to a drop in temperature, and with the usual sectional method of constructing arch rings shrinkage may be allowed for by assuming it equal to a 15-deg. F. drop in temperature and using the formula for H_T given above.

The disposition of live loading on an arch to produce the maximum stresses on the critical sections follows from a study of influence lines, and the following approximate conclusions have been deduced.

(a) The maximum positive bending moment at the crown occurs when the mid-third section of the arch is loaded.

(b) The maximum negative moment at the springing occurs when four-tenths of the span adjacent to the springing considered is loaded.

(c) The maximum positive moment at the springing occurs when the whole span is loaded except the length of four-tenths of the span adjacent to the springing considered.

The maximum bending moments and thrusts are given by the following expressions, in which w equals the intensity of uniformly distributed loading equivalent to the specified live load

Maximum positive B.M. at crown, $k_5 w L^2$

Corresponding horizontal thrust, $k_6 w \frac{L^2}{R}$

Maximum negative B.M. at springing, $k_7 w L^2$

Corresponding horizontal thrust, $k_8 w \frac{L^2}{R}$

„ vertical reaction, $k_9 w L$
Maximum positive B.M. at springing, $k_{10} w L^2$

Corresponding horizontal thrust, $k_{11} w \frac{L^2}{R}$

„ vertical reaction, $k_{12} w L$

The value of the coefficients k_5 to k_{12} are given in *Tables VI, VII, and VIII*. If H and V are the appropriate horizontal thrust and vertical reaction, the corresponding normal thrust at the springing is given by

$$T = H \cos \phi + V \sqrt{1 - \cos^2 \phi}$$

In the foregoing the live loading is expressed in terms of an equivalent uniformly distributed load ; for the Ministry of Transport loading, Mr. Carpenter has deduced the equivalent distributed loads given in *Table IX* for inclusion in various parts of the arch analysis calculations. It is of interest to compare this table with *Fig. 1*, which gives the Ministry of Transport values for the distributed load equivalent to the standard train ; it should be remembered that the official figures have to be read in conjunction with a knife-edge load.

TABLE I.—HORIZONTAL THRUST DUE TO DEAD LOAD
Values of k_1

Rise \div span . . .	0.10	0.15	0.20	0.25
Uniform dead load	0.125	0.125	0.125	0.125
Open spandrel .	0.135	0.145	0.144	0.148
Filled spandrel .	0.160	0.176	0.190	0.204

TABLE II.—HORIZONTAL THRUST DUE TO ARCH SHORTENING
Values of k_2

Rise \div span	0.10	0.15	0.20	0.25
Uniform dead load $\frac{d_1}{d} = 1.25$	1.10	1.07	1.03	0.99
1.50	1.42	1.37	1.32	1.25
1.75	1.68	1.63	1.58	1.53
Open spandrel $\frac{d_1}{d} = 1.25$	1.13	1.08	1.03	1.00
1.50	1.44	1.39	1.33	1.27
1.75	1.73	1.68	1.63	1.58
Closed spandrel $\frac{d_1}{d} = 1.25$	1.19	1.13	1.08	1.00
1.50	1.53	1.48	1.42	1.33
1.75	1.86	1.82	1.76	1.69

TABLE III.—INCLINATION OF ARCH AXIS AT SPRINGING

Values of $\cos \phi$

Rise \div span	0.10	0.15	0.20	0.25
Uniform dead load	0.930	0.848	0.781	0.709
Open spandrel	0.918	0.820	0.740	0.650
Closed spandrel	0.893	0.764	0.665	0.565

TABLE IV.—MOMENTS DUE TO ARCH SHORTENING AND TEMPERATURE CHANGE AND ECCENTRICITY OF THRUST

Values of k_s

Rise \div span	0.10	0.15	0.20	0.25
Uniform dead load $\frac{d_s}{d} = 1.25$	0.284	0.293	0.300	0.307
1.50	0.248	0.253	0.258	0.263
1.75	0.223	0.227	0.231	0.235
Open spandrel $\frac{d_s}{d} = 1.25$	0.279	0.280	0.281	0.282
1.50	0.240	0.243	0.247	0.251
1.75	0.218	0.220	0.222	0.224
Closed spandrel $\frac{d_s}{d} = 1.25$	0.255	0.261	0.265	0.270
1.50	0.224	0.226	0.228	0.230
1.75	0.200	0.200	0.200	0.200

TABLE V.—HORIZONTAL FORCE DUE TO TEMPERATURE CHANGE

Values of $k_h \div 10^3$

Rise \div span	0.10	0.15	0.20	0.25
Uniform dead load $\frac{d_s}{d} = 1.25$	2.54	2.42	2.32	2.22
1.50	3.42	3.27	3.14	3.01
1.75	4.26	4.10	3.94	3.78
Open spandrel $\frac{d_s}{d} = 1.25$	2.58	2.46	2.33	2.19
1.50	3.49	3.34	3.18	3.02
1.75	4.37	4.21	4.04	3.83
Closed spandrel $\frac{d_s}{d} = 1.25$	2.72	2.53	2.38	2.17
1.50	3.74	3.55	3.34	3.12
1.75	4.69	4.48	4.29	4.12

TABLE VI.—HORIZONTAL THRUSTS DUE TO LIVE LOAD

Values of k_6 , k_8 , and k_{11}

Rise \div span		0.10	0.15	0.20	0.25
Uniform dead load	k_6	0.059	0.059	0.059	0.059
	k_8	0.030	0.039	0.039	0.039
	k_{11}	0.080	0.086	0.086	0.086
Open spandrel	k_6	0.062	0.064	0.065	0.066
	k_8	0.038	0.038	0.037	0.037
	k_{11}	0.088	0.089	0.090	0.092
Closed spandrel	k_6	0.070	0.074	0.077	0.080
	k_8	0.037	0.035	0.033	0.032
	k_{11}				
	$d_s \begin{cases} 1.25 \\ 1.50 \\ 1.75 \end{cases}$	0.093 0.095 0.097	0.097 0.098 0.100	0.098 0.101 0.104	0.100 0.103 0.106

TABLE VII.—VERTICAL REACTIONS DUE TO LIVE LOAD

Values of k_9 and k_{12}

Rise \div span		0.10	0.15	0.20	0.25
Uniform dead load	k_9	0.358	0.358	0.358	0.358
	k_{12}	0.149	0.149	0.149	0.149
Open spandrel	k_9	0.354	0.352	0.350	0.349
	k_{12}	0.150	0.151	0.153	0.155
Closed spandrel	k_9	0.342	0.337	0.330	0.321
	k_{12}	0.160	0.164	0.170	0.177

TABLE VIII.—BENDING MOMENTS DUE TO LIVE LOAD

Values of k_5

Rise \div span		0.10	0.15	0.20	0.25
Uniform dead load	$\frac{d_s}{d} = 1.25$	0.0048	0.0049	0.0051	0.0052
	1.50	0.0045	0.0046	0.0046	0.0047
	1.75	0.0042	0.0043	0.0043	0.0044
Open spandrel	$\frac{d_s}{d} = 1.25$	0.0052	0.0054	0.0057	0.0060
	1.50	0.0048	0.0050	0.0052	0.0054
	1.75	0.0044	0.0046	0.0048	0.0050
Closed spandrel	$\frac{d_s}{d} = 1.25$	0.0060	0.0069	0.0077	0.0084
	1.50	0.0056	0.0062	0.0068	0.0075
	1.75	0.0052	0.0058	0.0063	0.0068

Values of k_7

Rise ÷ span		0.10	0.15	0.20	0.25
Uniform dead load $\frac{d_s}{d} = 1.25$		0.019	0.019	0.018	0.018
	1.50	0.021	0.021	0.020	0.020
	1.75	0.022	0.022	0.022	0.022
Open spandrel $\frac{d_s}{d} = 1.25$		0.018	0.018	0.017	0.017
	1.50	0.020	0.020	0.019	0.018
	1.75	0.022	0.021	0.020	0.020
Closed spandrel $\frac{d_s}{d} = 1.25$		0.017	0.015	0.014	0.014
	1.50	0.018	0.017	0.016	0.015
	1.75	0.020	0.018	0.017	0.016

Values of k_{10}

Rise ÷ span		0.10	0.15	0.20	0.25
Uniform dead load $\frac{d_s}{d} = 1.25$		0.019	0.019	0.018	0.018
	1.50	0.021	0.020	0.020	0.020
	1.75	0.022	0.022	0.022	0.022
Open spandrel $\frac{d_s}{d} = 1.25$		0.020	0.021	0.021	0.021
	1.50	0.022	0.023	0.023	0.023
	1.75	0.024	0.025	0.025	0.025
Closed spandrel $\frac{d_s}{d} = 1.25$		0.024	0.025	0.025	0.026
	1.50	0.026	0.027	0.028	0.028
	1.75	0.029	0.030	0.031	0.031

TABLE IX.—MINISTRY OF TRANSPORT LOADING EXPRESSED AS EQUIVALENT UNIFORMLY DISTRIBUTED LOAD (lb. per sq. ft.)

Span of arch (ft.)	50	100	150	200
Horizontal Thrusts:				
Max. Pos. B.M. at crown	450	300	270	260
Max. Neg. B.M. at springing	300	260	250	240
Max. Pos. B.M. at springing	370	290	260	250
Vertical Reactions:				
Max. Neg. B.M. at springing	400	290	260	250
Max. Pos. B.M. at springing	340	280	270	250
Bending Moments: $D = 2$ ft.:				
Max. Pos. B.M. at crown	600	390	350	320
Max. Neg. B.M. at springing	400	310	290	275
Max. Pos. B.M. at springing	320	280	270	260
Bending Moments: $D = 4$ ft.:				
Max. Pos. B.M. at crown	480	360	325	200
Max. Neg. B.M. at springing	380	300	280	270
Max. Pos. B.M. at springing	290	270	260	260

 D = thickness from road surface to soffit of arch at crown.

An example of the application of the foregoing analytical method of determining the moments and forces in an arch ring indicates the procedure.

DATA.—A fixed ended arched slab ; open spandrels.

Span : 150 ft. (measured horizontally between the intersection of arch axis with springing section).

Rise : 22 ft. 6 in. (being the rise of the arch axis within the 150-ft. span).

Slab thickness : 3 ft. 4½ in. at springing, 2 ft. 3 in. at crown.

Dead Load : 250 lb. per square foot, excluding the weight of the arch slab.

Live Load : Ministry of Transport loading, which for the example in hand can be considered as equivalent to the following uniform loadings (see Table IX) :

Horizontal thrusts and vertical reaction : 260 lb. per square foot.

Maximum positive crown moment : 350 lb. " "

Maximum negative springing moment : 290 lb. " "

Maximum positive springing moment : 270 lb. " "

Temperature range : ± 15 deg. F. ; $E_c = 2,000,000$; $e = 0.0000066$.

Shrinkage : equivalent to temperature drop of 15 deg. F.

FACTORS.—Rise : span ratio $= \frac{22.5}{150} = 0.15$

$$\frac{\text{Thickness at springing}}{\text{Thickness at crown}} = \frac{d_s}{d} = \frac{3.375}{2.25} = 1.5$$

Dead load at crown : as above = 250 lb. per square foot.

27-in. slab = 338 lb. " "

Total : $w_d = 588$ lb. " "

Angle of inclination of arch axis at springing (from Table III),
 $\cos \phi = 0.820$.

A strip of slab 12 in. wide will be considered.

Horizontal thrusts due to dead load, etc. :

$$\text{Dead load (Table I) } H = 0.140 \times 588 \times \frac{150^2}{22.5} = 82,200 \text{ lb.}$$

$$\text{Arch shortening (Table II) } H_p = -1.39 \left(\frac{2.25}{22.5} \right)^2 82,200 = -1,145 \text{ lb.}$$

$$\text{Temperature change (Table V) } H_T = \pm 3.34 \times 10^3 \left(\frac{2.25}{22.5} \right)^2 \times 2.25 \times 15 = \pm 1,130 \text{ lb.}$$

$$\text{Shrinkage (as for temperature) } = -1,130 \text{ lb.}$$

Crown : Maximum positive B.M.

Moment Thrust
in. lb. lb.

Dead load and arch shortening

$$H_c = 82,200 - 1,145 = 80,855$$

(Table IV)

$$M_c = 0.243 \times 22.5 \times 1,145 \times 12 = 75,100$$

Temperature drop : thrust H_T as above

$$-1,130$$

(Table IV)

$$\text{B.M.} = 0.243 \times 22.5 \times 1,130 \times 12 = 74,300$$

Shrinkage (as for temperature)	74,300	— 1,130
Live load (<i>Tables VI and VIII</i>)		
Thrust : $0.064 \times 260 \times \frac{150^2}{22.5}$	=	16,620
B.M. : $0.00050 \times 350 \times 150^2 \times 12$	= 473,000	
<i>Total Moment and thrust :</i>	696,700	95,415
<i>Springing : Maximum negative Moment :</i>	Moment	Thrust
	in. lb.	lb.
Dead load and arch shortening :		
$H_s = \frac{82,200}{0.820} \quad 1,145 \times 0.820$		99,260
M_s (<i>Table IV</i>) : $0.757 \times 22.5 \times 1,145 \times 12 =$	— 234,200	
Temperature drop :		
Thrust = — 1,130 $\times 0.820$		— 925
B.M. (<i>Table IV</i>) : — $0.757 \times 22.5 \times 1,130 \times 12 =$	— 231,000	
Shrinkage (as for temperature)	— 231,000	— 925
Live load (<i>Tables VI, VII, and VIII</i>)		
$H = 0.038 \times 260 \times \frac{150^2}{22.5} =$	9,880	
$V = 0.352 \times 260 \times 150 =$	13,700	
Normal thrust :		
$(9,880 \times 0.820) + (13,700 \sqrt{1 - 0.820^2}) =$		12,580
Moment : $0.0195 \times 290 \times 150^2 \times 12 =$	— 1,528,000	
<i>Total Moment and thrust :</i>	— 2,224,200	109,990
<i>Springing : Maximum positive Moment :</i>	Moment	Thrust
	in. lb.	lb.
Dead load and arch shortening as before	— 234,200	99,260
Temperature rise and shrinkage neutralise.		
Live load (<i>Tables VI, VII, VIII</i>)		
$H = 0.089 \times 260 \times \frac{150^2}{22.5} =$	23,150	
$V = 0.151 \times 260 \times 150 =$	5,900	
Normal thrust :		
$(23,150 \times 0.820) + 5,900 \sqrt{1 - 0.820^2} =$		20,928
Moment = $0.0226 \times 270 \times 150^2 \times 12 =$	1,650,000	
<i>Total Moments and thrusts :</i>	1,415,800	120,188

With the corresponding thrusts and moments thus determined the area of reinforcement and the stresses at crown and springing are found in accordance

with the appropriate methods described in Chapter XIV. There only then remains to determine the intermediate sections and the profile of the arch axis. If the dead loading is uniform throughout (or practically so) the axis will be a parabola, but if it is non-uniform the axis must be shaped to coincide with the line of pressure for dead load. The latter can be plotted by force and link polygons (after the manner of ordinary graphic statics), the necessary data being the magnitude of the dead loading and the value of the horizontal thrust due to dead load and vertical reaction (which equals the dead load on half the span) at the springing. The line of pressure, and therefore the arch axis, being established, and given the requisite thicknesses of the arch at the crown and springing, the lines of the extrados and intrados can be plotted to give a parabolic variation between the two extremes. Thus, the thickness (normal to the arch axis) at any point is given by $[(d_s - d)r + d]$ where r has the following values:

Ratio of distance of point from springing measured along arch axis to half length of arch axis	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$
Value of r	0.563	0.250	0.063

4.—Girder Bridges.

Certain aspects of the design of girder bridges have already been dealt with, viz. loading for road bridges, wind loading, pavement loads, dispersal of wheel loads and width of slab considered as carrying a point load (all in Chapter II), influence lines for point loads passing over a system of continuous spans, etc. (Chapter IV). Bridges of less than 15 ft. width are most economical if the deck slab is spanned transversely between two outer longitudinal girders. These girders may be the parapets of the bridge, but for major structures the use of the parapets as principal structural members is not considered good practice. If the width of the bridge exceeds 15 ft. the most efficient design is produced by providing several longitudinal girders, usually spaced at about 7-ft. centres for bridges designed for the maximum loadings. The roadway is generally a multiple of 10 ft. in width, and the deck slab is designed for one or more maximum wheel loads placed in such positions as to give maximum moments. The longitudinal girders are subjected to maximum bending moments when a train of wheels is directly over the beam and a parallel train as near the former train as possible.

Footpaths are sometimes cantilevered off the principal part of the structure, and water, gas, and electric services are generally installed in a duct provided under the footpaths.

Girder bridges do not usually exceed 50 or 60 ft. in span, as beyond this limit an arch design is generally more economical. Even when a bridge has an arch-shape soffit, if the span-rise ratio exceeds ten, or if it is impracticable to provide adequate abutments to take the thrusts from an arch design, it is usual to design the bridge as a girder construction; in assessing the bending moments in such a case it is essential to take into account the variation in moment of inertia (Chapter IV), since the inertia is usually considerably higher at the supports than at the crown. The moment of resistance at midspan should be about 25 per cent. in excess of the calculated bending moment at this point

to allow for increases due to slight settlement of supports and for other variations from the design assumptions.

5.—Bridge Piers and Abutments.

The piers for girder bridges are usually only subjected to vertical loading due to the reactions from the girders; the abutments of girder bridges have to be designed for the vertical reaction from the ends of the girders and the horizontal thrusts due to earth back-filling. It is usual to provide the girders with a sliding seating on the abutments unless continuity with the abutments has been assumed.

The piers and abutments of arch bridges have to be designed to take both vertical reactions and the horizontal thrusts from the arches. Stability is obtained by constructing massive piers in plain concrete or masonry, or by providing tension and compression piles, or by a cellular reinforced concrete box construction filled with earth. Part of the horizontal thrust on the abutments can be resisted by the active (or passive) earth pressure behind the abutment, but in the case of fixed arches passive pressure should only be assumed when the structure reacts upon solid rock abutments, and care should be exercised before considering the counter-thrust from active earth pressure to ensure that this pressure will always be effective. Adequate resistance to sliding should be assured, and the possibilities of uplift from water-pressure should be investigated.

Mid-river piers, if not protected by independent fenders, should be designed to withstand blows from passing vessels or floating debris, and should be provided with cutwaters.

6.—Building Details.

The floors of buildings are either beam and slab construction or "mushroom" (flat slab) construction (Chapter IV). If the spans of the slabs between the beams exceed 10 ft. it is more economical to provide a hollow-tile floor, which is light in weight and uses less concrete. Such a floor consists of a thin top slab ($1\frac{1}{2}$ in. to $3\frac{1}{2}$ in. thick) overlying a series of concrete ribs extending the full thickness of the floor construction. These ribs may be provided at 6-in. to 24-in. centres and may be from 2 in. to 5 in. wide. The spaces between the ribs can be kept open, but in order to simplify the shuttering they are usually filled with hollow clay tiles (comparable with permanent shuttering). The combined depth of the rib and slab is determined in the same way as the depth of a solid slab, and the thickness of the top slab is made sufficient to provide adequate compression area. The tensile steel is located in the bottom of the rib, usually a single bar of sufficient diameter in each rib. The space or tile between the ribs merely replaces the useless concrete below the neutral axis. There are several proprietary types of hollow-tile floor construction, but the simplest construction is not covered by patents.

Concrete roofs can also be constructed economically on this principle. Although expensive in shuttering, domed roofs also lead to light-weight construction; the forces produced and method of design of such a type have already been dealt with when considering tanks (see Chapter VII). When asphalt is not specified for roofs, weather-tightness can be assured by treating the top

surface of the concrete slab with one coat of cement grout, two coats of tar, and a generous sprinkling of sand. The grout should be dry before the tar is applied, and the first coat of tar should be dry before the second coat is applied; the sand should be laid while the second coat is wet and a $\frac{1}{4}$ -in. layer of sand is not unduly thick. Roof slabs should not be less than 4 in. thick whether treated for watertightness or not, and should be laid to a fall of 1 in. in 10 ft. to facilitate drainage.

Holes formed in roof or floor slabs should be trimmed on all sides with reinforcing bars, unless the hole is large compared with the span of the slab (for example, stair wells or lift openings), in which case trimmer beams should be provided. When the holes are small, as in manholes in tank roofs, ventilation ducts, etc., the cross-sectional area of the trimmer bars placed parallel to the principal reinforcement should be at least equal to the area of principal reinforcement cut out by the hole. Other trimmer bars need only be nominal in size.

Stairs can be designed to span transversely or longitudinally. When spanning transversely (parallel to the nosings) supports must be provided at both sides of the flight, either by providing walls or stringer beams. In this case the "waist" or thinnest part of the stair construction need only be, say, 3 in. thick, the effective lever-arm for resisting the bending moment being about one-half the maximum thickness from the nosing to the soffit measured normal to the soffit. When spanning longitudinally the thickness of slab required to resist bending determines the thickness of the "waist." The loadings for which stairs should be designed have been discussed in Chapter II, and the bending moments should be calculated from the total weight of the stairs and the total superimposed load combined with span as measured on plan. The stresses produced by the longitudinal thrust are only small and are usually neglected. Unless circumstances dictate otherwise a reasonable profile for a step is 7 in. rise with 10 in. tread.

As an example of the design of a simple flight of stairs by the alternative methods, consider the problem illustrated in *Fig. 34*.

(a) The flight supported by longitudinal stringers along both edges; assume a minimum "waist" of 3 in., as in *Fig. 34(a)*.

Clear span = 4 ft. Effective span = 4 ft. 6 in.

Dead load: Step = $11 \times 7 \times \frac{1}{2}$ = $38\frac{1}{2}$ lb. per ft. run of step.

Waist = $13\frac{1}{2} \times 3$ = $40\frac{1}{2}$ " " "

Granolithic = $\frac{1}{2} \times 11$ = $5\frac{1}{2}$ " " "

 $84\frac{1}{2}$ " " "

Live load: See *Table No. 1*. = $\frac{1}{2} \times 120$ = 110 " " "

Total load = $194\frac{1}{2}$ " " "

B.M. = $\frac{1}{8} \times 195 \times 4.0 \times 4.5 \times 12$ = 5,260 in. lb.

Alternative B.M. with 300 lb. central point load:

= $(\frac{1}{8} \times 85 \times 4.0 \times 4.5 \times 12) + (\frac{1}{4} \times 300 \times 4.5 \times 12)$ = 6,350 in. lb.

Effective depth = $3\frac{1}{2}$ in. approx.; assuming maximum stresses of 17,000 and 700 lb. per square inch, from *Table No. 27* $Q = 116.7$.

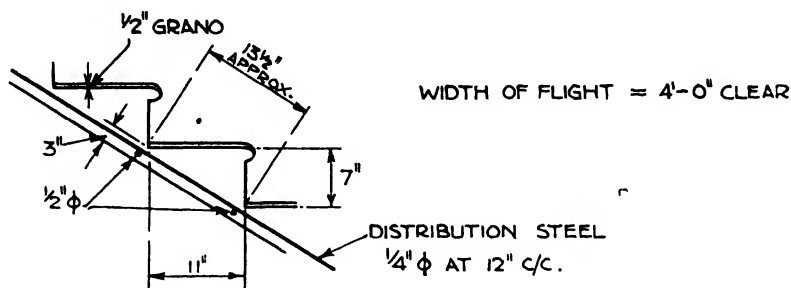
R.M. of compression = $116.7 \times 13.5 \times 3.5^2 = 21,200$ in. lb.

If this resistance moment did not equal or exceed the applied bending moment, it would be necessary to provide top steel or to increase the effective depth by increasing the "waist" dimension.

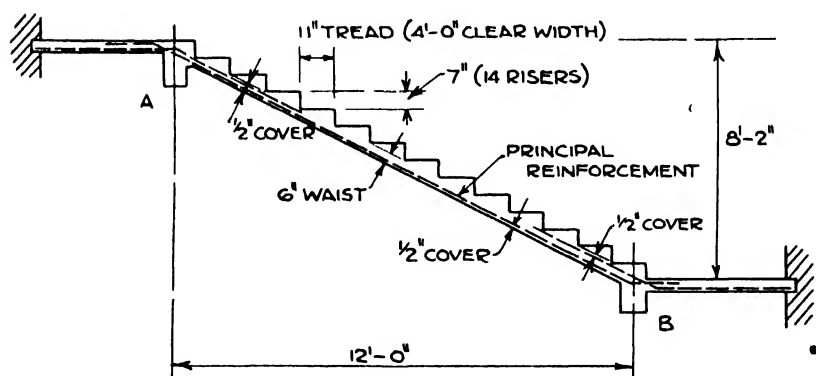
Area of reinforcement

$$= \frac{6,350}{0.87 \times 3.5 \times 17,000} = 0.123 \text{ sq. in. say, one } \frac{1}{2}\text{-in. bar per step.}$$

(b) The flight supported at the top and bottom on trimmer beams (or walls), beyond which the flight is continuous with landing slabs as shown in *Fig. 34(b)*.



(a) SUPPORTED ON STRINGER BEAMS.



(b) SPANNING BETWEEN BEAMS A AND B.

Fig. 34.—Simple Stair Designs.

Assume 6-in. waist; hence minimum effective depth = 5.25 in. Effective span = 12 ft.

Dead load : Step	$= 11 \times 7 \times \frac{1}{2}$	$= 38\frac{1}{2}$	lb. per ft. run of step.
Waist	$= 13\frac{1}{2} \times 6$	$= 81$	" " "
Granolithic	$= 11 \times \frac{1}{2}$	$= 5\frac{1}{2}$	" " "
Live load	$= 120 \times \frac{1}{12}$	$= 110$	" " "
Total load		$= 235$	" " "

Total load on ft. width of flight (12 steps net) = $12 \times 235 = 2,820$ lb.
 B.M. = $\frac{1}{12} \times 2,820 \times 12.0 = 2,820$ ft. lb.

From *Table* No. 30 : Minimum effective depth = $0.0926\sqrt{2820} = 4.93$ in.
 Principal longitudinal reinforcement

$$= \frac{2,820 \times 12}{0.87 \times 5.25 \times 17,000} = 0.436 \text{ sq. in. say, } \frac{1}{2}\text{-in. bars at 5-in. centres.}$$

Curtain, panel, or filling walls in building structures, that is, walls not designed to take vertical loading or horizontal pressures, should be made not less than 4 in. thick for constructional reasons, and should be reinforced so as to span, if necessary, across their shortest dimension. Usually nominal reinforcement, say, $\frac{3}{4}$ -in. bars at 12-in. centres placed horizontally and vertically on both faces, is sufficient. Heavier bars should be provided above and at the sides of all door and window openings, and $\frac{1}{2}$ -in. bars 3 ft. long should be placed diagonally across the corners of all such openings.

Plain walls of any great height should be avoided as, apart from the provision of an architectural feature, the formation of a cornice, band, ridge, or other break forms a convenient position for a construction joint and also a support for the shuttering above, as well as providing an opportunity for correcting any errors in vertical alignment.

7.—Industrial Structures.

Utility rather than appearance is the predominant factor in the design of structures in connection with colliery works, steel works, by-product and gas works, quarries, and other essentially industrial plants, although some attention is given to the architecture of such prominent structures as water towers, cooling towers, and tall bunkers. In addition to the capability of the various members to sustain the principal moments and forces to which they are subjected, there are other considerations peculiar to each type of industrial structure. Vibration must be allowed for in the design of the substructures for coal screening and stone crushing and screening plants. Impact and extra security are obtained in pit-head gear design by adopting a factor of safety of ten. Watertightness is an essential quality required in slurry basins, coal draining bunkers, settling tanks, and similar hydraulic structures, while airtightness is essential in gas purifiers and in airlock structures in connection with colliery work. The freedom of concrete from corrosion by the fumes that are inseparable from certain industrial processes is one of the qualities that recommend the material for industrial construction, but protection is needed to prevent contact with certain liquids (see next Chapter). Provision should be made for expansion in connection with steelworks, coke ovens, gas retorts, etc. Boiler foundations, especially on clay, should be made sufficiently thick to prevent undue heating and drying out of the subsoil. Firing floors, coke benches, and rolling mill floors should be protected from extreme temperatures and abrasion by being covered with steel plates.

Structures in mining districts should be designed to resist the moments and forces produced by a not unreasonable settlement of part of the ground upon which they are founded. Thus raft foundations that have at any section equal resistance to negative and positive moments are commonly adopted. If isolated

foundations are provided, as for gantries, the beams should be designed for freely-supported moments at midspan, although some negative moment should be allowed for over the supports.

8.—Chimneys.

The two principal loadings on a chimney are the horizontal wind pressure and the dead weight of the stack. At any section the cantilever bending moment due to the former is combined with the direct force due to the weight of the chimney above the section considered to find the maximum stresses. Suitable values for wind pressures on square, octagonal, and circular stacks are given in Chapter II, and an example showing the method of determining the stresses is described in Chapter XIV, Paragraph 8. Generally the preliminary determination of the section required is a matter of trial and error, but Messrs. Taylor, Glenday, and Faber have established the following expressions for circular stacks :

$$\text{Total area of steel required} = \frac{M}{ar} - \frac{W}{b}$$

$$\text{Total thickness of concrete} = \frac{M}{cr^2} + \frac{W}{dr}$$

where M and W are the moment and direct force at any section and r is the mean radius at the section ; the factors, a , b , c and d have the following values :

Stress in concrete, lb. per square inch	600	600	600	400
Stress in steel, lb. per square inch	12,000	14,000	16,000	16,000
a	6,408	7,551	8,685	8,600
b	7,776	8,990	10,230	9,698
c	4,582	3,409	2,867	1,376
d	1,372	1,410	1,419	911

Since the heat of flue gases in the stack reduces the strength of the concrete (principally by causing slow deterioration in the cement) it is essential to keep the stresses in the steel and concrete low in those parts of the chimney that are not lined with firebrick. Throughout the construction, and especially where the fumes can come in contact with the concrete, the concrete should be well compacted and of low water content to ensure a dense concrete that will limit reaction between sulphurous fumes and free lime to surface effect only.

It has been suggested that firebrick linings should be provided wherever the anticipated temperature of the flue gases exceeds 750 deg. F. ; below this temperature ordinary crushed stone or ballast aggregates are satisfactory. Linings will not prevent considerable temperature rise in the concrete walls, and therefore in all parts of the structure liberal and reasonably closely spaced vertical and horizontal bars should be provided to resist the tendency to cracking due to the difference of temperature between the inner and outer faces of the stack.

9.—Culverts.

The bending moments produced in rectangular culverts of normal design are determined by considering the four sides as a continuous beam of four spans with the moments at the end supports equal; thus by applying the Theorem of Three Moments three equations containing three unknowns are obtained for unsymmetrical loading. The loading can be conveniently divided into the following components:

- (a) Uniformly distributed loading on top slab and equal reaction from earth below bottom slab.
- (b) Superimposed point loads on top slab and equal reaction from earth below bottom slab.
- (c) Upward pressure on bottom slab due to the weight of the walls.
- (d) Triangularly distributed horizontal pressure on walls due to earth.
- (e) Uniformly distributed horizontal pressure on walls due to earth and surcharge.
- (f) Internal horizontal pressure from contents of culvert.

These loads are indicated in *Fig. 35* and the bending moments at the corners due to the various types of loading can be found from the following formulæ which are applicable when the thickness of the top slab, walls, and bottom slab are practically equal:

$$\text{For all formulæ: } k = \frac{H}{L}$$

$$\text{For all loadings: } M_A = M_B; M_C = M_D$$

All moments expressed in ft. lb. per foot run of culvert.

$$\text{Loading as in Fig. 35(a), } M_A = -\frac{w_1 L^2}{12(k+1)} = M_C$$

Loading as in *Fig. 35(b)*, d = width of slab assumed to be supporting point load.

$$M_A = -\frac{WL}{12d} \left[\frac{2k+4.5}{(k+3)(k+1)} \right]$$

$$M_D = -\frac{WL}{24d} \left[\frac{k+6}{(k+3)(k+1)} \right]$$

$$\text{Loading as in Fig. 35(c), } M_A = +\frac{w_2 L^2}{12} \left[\frac{k}{(k+3)(k+1)} \right]$$

$$M_D = -\frac{w_2 L^2}{12} \left[\frac{3+k}{(k+3)(k+1)} \right]$$

$$\text{Loading as in Fig. 35(d), } M_A = -\frac{p_1 H^2}{60} \left[\frac{(2k+7)k}{(k+3)(k+1)} \right]$$

$$M_D = -\frac{p_1 H^2}{60} \left[\frac{(3k+8)k}{(k+3)(k+1)} \right]$$

$$\text{Loading as in Fig. 35(e), } M_A = -\frac{p^2 H^2 k}{12(k+1)} = M_D$$

Loading as in *Fig. 35(f)*, Moments as for loading in *Fig. 35(d)*, but reverse sign.

Loading as in *Fig. 35(g)*, Moments as for loading in *Fig. 35(e)*, but reverse sign.

The loading w_1 on the top slab would include the weight of the filling, any uniformly distributed superload, and the weight of the top slab. When a trench has been excavated in consolidated ground for the construction of the culvert

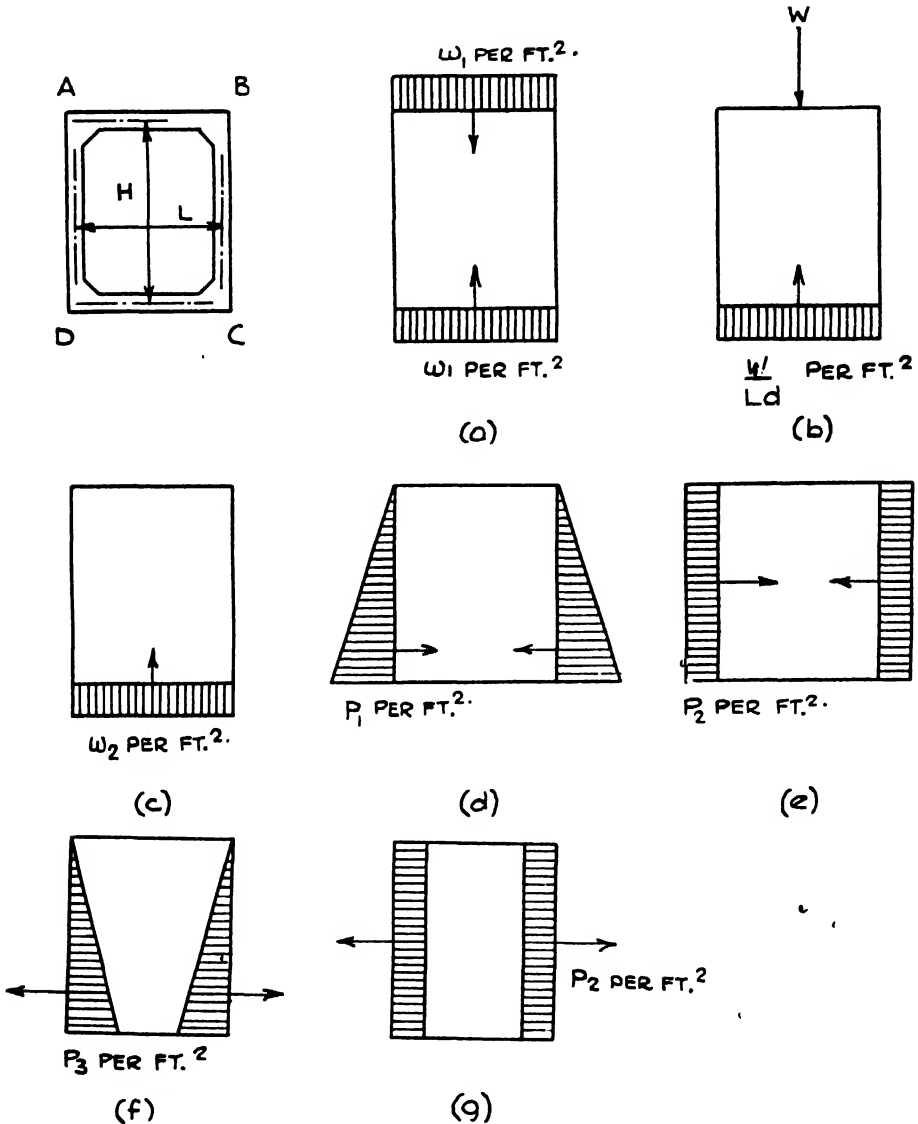


Fig. 35. Loading on Culverts.

and the depth from the surface of the ground to the roof of the culvert exceeds $3L$, it is permissible to take the maximum load on the culvert roof as due to a head of earth equal to $3L$. Although the roof of a culvert passing through even a newly-filled embankment is probably not subjected to the weight of the

full head of earth above the roof, there is little reliable data of the actual loading carried, and therefore any load reductions due to "earth arching" should be made with discretion. If there is no filling and the wheel loads or other point loads can bear directly on the roof of the culvert, the total point load should be considered as carried on a certain width of slab ($= d$) as determined by the methods described in Chapter II. The point load effect is somewhat modified if there is any filling above the culvert roof, and if the depth of filling is D the point load W can be considered as spread over an area of $4D^2$.

When D equals or exceeds $\frac{L}{2}$ the point loading is equivalent to a load of $\frac{W}{4D^2}$ lb. per square foot uniformly distributed over a length of culvert equal to $2D$. For values of D intermediate between $\frac{1}{2}$ and zero the moments will be something between those due to a uniformly distributed load and a central point load.

The weights of the walls of the culvert can be assumed to be resisted by a uniform upward ground pressure as in *Fig. 35(c)*. The weights of the bottom slab and the contents of the culvert are resisted directly by equal reactions from the ground below the slab and thus do not produce bending moments, although these factors must be taken into account when assessing the maximum ground pressures. The horizontal pressure due to the fluid contents of a culvert would produce an internal triangular loading as shown in *Fig. 35(f)*, or even a trapezium load distribution if the surface of the water outside the culvert is higher than the roof.

The magnitude and distribution of the horizontal pressure due to the earth against the sides of the culvert would be calculated in accordance with the formulæ given in Chapter III.

The maximum bending moments at the various critical sections of the culvert can be calculated by considering the possible incidence of loading, and generally there are only two conditions to consider:

(a) Culvert empty; full loading and surcharge on top slab (*Fig. 35(a)* and/or *35(b)*), dead weight of walls (*Fig. 35(c)*) and maximum earth pressure on walls (*Fig. 35(d)* and *Fig. 35(e)*).

(b) Culvert full; minimum loading on top slab (*Fig. 35(a)*) and minimum (or zero) pressure from earth on walls (*Fig. 35(d)* and *35(e)*), dead weight of walls (*Fig. 35(c)*) and maximum horizontal pressure from contents of culvert (*Fig. 35(f)* and *Fig. 35(g)*).

In special circumstances these conditions may not produce the maximum positive or negative moments at any particular section, and the effect of every probable combination should be considered. With the moments should be combined the direct thrusts and tensions due to various loadings.

10.—Roads.

Reinforced concrete roads can be divided into two classes:

(a) Reinforced concrete slabs underlying macadam, granite sets, asphalt, wood blocks, or other surfacing.

(b) Reinforced concrete slabs that form the complete road.

With either type the ground upon which the slab is laid should be carefully

prepared. All vegetation or patches of existing roads should be stripped off the site and hollow places filled with earth laid in superimposed layers 8 in. to 12 in. thick. Each layer should be well watered and rolled, a suitable weight for the roller being about 5 tons. Where the whole site has to be filled to the formation level of the road the filling should be left for about two months to settle before the concrete is laid. In awkward places where a roller cannot operate a rammer weighing up to $\frac{1}{4}$ ton and operated by several men should be used. Immediately before the concrete is placed the ground should be tamped with a $\frac{1}{4}$ -cwt. hand rammer and well watered. When the surface lacks homogeneity it is advisable to roll in a layer of hardcore, and for very dry, dusty, or sandy surfaces the application of a thin layer of lean concrete will prevent the water in the slab concrete from soaking into the ground. If the top of the ground is shaped in conformity with the camber of the finished road surface, the advantages of improved formation, drainage and economy of concrete accrue.

For all-concrete roads the slab can be from 4 in. to 8 in. in thickness, depending on the weight of traffic and the type of subsoil; the reinforcement should be between 6 lb. and 10 lb. per square yard, provided in a single layer in the bottom of the slab. With exceptionally heavy traffic on poor subsoils a layer of reinforcement in the top and bottom of the slab giving a total weight of 10 lb. to 15 lb. per square yard may be required. The provision of fabricated mesh reinforcement merits consideration in preparing road designs, and if cold-drawn hard steel is used a weight equal to only two-thirds of the required weight of rolled mild steel is necessary.

The arrangement of the reinforcement depends on the width of the road and spacing of the transverse joints. If these joints are at distances apart equal to the width of the road the steel should be arranged to give equal strength in both directions (square mesh), but if transverse joints are not provided, or are provided only at long intervals, at least nine-tenths of the total reinforcement should be parallel to the length of the road. For intermediate size panels an intermediate ratio between 0.5 and 0.9 of the total steel should be placed longitudinally.

The transverse bars should be bent up into and be attached to the longitudinal bars in the kerb if this is cast monolithic with the main slab. Often a granite kerb set on an independent mass concrete base is provided. If the ground under the edges of the road slab is likely to be affected by ground water, frost, etc., the outer 2 or 3 ft. of the slab should be tapered from the normal thickness to 2 in. or 3 in. more at the extreme edge.

The concrete for road foundation slabs should be not less rich than Mix C (see Table No. 23), and for all-concrete road construction not leaner than Mix E. For the wearing surface, gravel or other rounded aggregates are definitely not advocated and a hard crushed stone should be used. In districts where such crushed aggregates are costly an economical and durable road slab can be obtained by making the bulk of the slab of Mix C concrete with a selected gravel aggregate, and the top $1\frac{1}{2}$ in. of Mix E concrete with a crushed granite aggregate graded from $\frac{1}{2}$ in. to $\frac{1}{4}$ in. and clean coarse sand. The grading of sand and aggregates suitable for road work are given in Chapter IX.

Exposure to weather and abrasion from traffic impose a severe test on concrete used for roads, and thus every reasonable precaution to attain a high

standard of excellence should be taken. The water content should be subject to strict control and slump tests should be periodically made, the maximum slump being never more than $1\frac{1}{2}$ in. and the nearer zero as practicable the better (see Chapter IX). Some specifications require the water content to be from 8 per cent. to 10 per cent. of the weight of the dry materials. For small water contents it is essential adequately to mix the materials, and the best practice calls for dry mixing for half a minute, and, after admitting the water, wet mixing for 1 to $1\frac{1}{2}$ minutes with a drum speed of 15 to 20 revolutions per minute.

When the concrete has been placed it should be well and continuously tamped by hand or mechanical tampers in order to bring to the surface the small amount of excess water previous to the immediate commencement of the finishing operations, the first of which is further tamping of the concrete surface with a timber beam shaped to the required camber of the finished profile. This beam is usually about 2 in. wide, extending the full width of the road, and worked off the side forms. This operation should be followed by further consolidation by a wide roller weighing about 1 cwt. dragged transversely across the slab by workmen standing at the edges. There should then follow the minor operations involving surfacing by hand at joints, accidental hollow places, etc., previous to the final operation of drawing a canvas belt to and fro across the surface of the concrete. After the initial set has taken place the slab should be flooded with water, and later completely covered with a few inches of sand or earth from the excavation. This covering should be maintained saturated for about three weeks, and the road should not be opened to traffic until one month after pouring; in frosty weather this period should be lengthened. If rapid-hardening Portland cement or aluminous cement has been used the interval can be reduced.

Although many concrete roads are constructed without transverse joints the provision of such joints, and in wide roads a central longitudinal joint, assists in eliminating temperature cracks, shrinkage cracks, and the opening of construction joints. A convenient interval between joints is 30 ft., and the end of each day's concreting operations should coincide with a joint. A clear gap of about $\frac{1}{2}$ in. should be left between the faces of adjacent panels and the space filled with a bitumastic material. The filling material is often in the form of sheets or strips shaped to the profile of the road and kerbs and projecting from the joint so that it can be flattened down on the top surface of the slab, as shown in *Fig. 36(e)*, to protect the top edges of the panel faces. In the centre of the slab, and at intervals of about 2 ft. 6 in. across the width of the road, $\frac{1}{2}$ -in. bars 2 ft. or 3 ft. long should project horizontally from one panel to connect with the succeeding panel; these bars are usually greased in order to allow free expansion while preventing any one panel rising relatively to its neighbour.

11.—Joints in Concrete Construction.

Joints in monolithic construction are required to allow free expansion due to temperature changes, as in roads, retaining walls, and long buildings, or to provide for unrestrained deformation under loading as in the case of vertical walls of cylindrical containers. In *Fig. 36* are illustrated a number of typical designs for joints for various purposes; the diagrams should be self-explanatory. If the joint indicated at (b) is adopted there is no need to provide for vertical

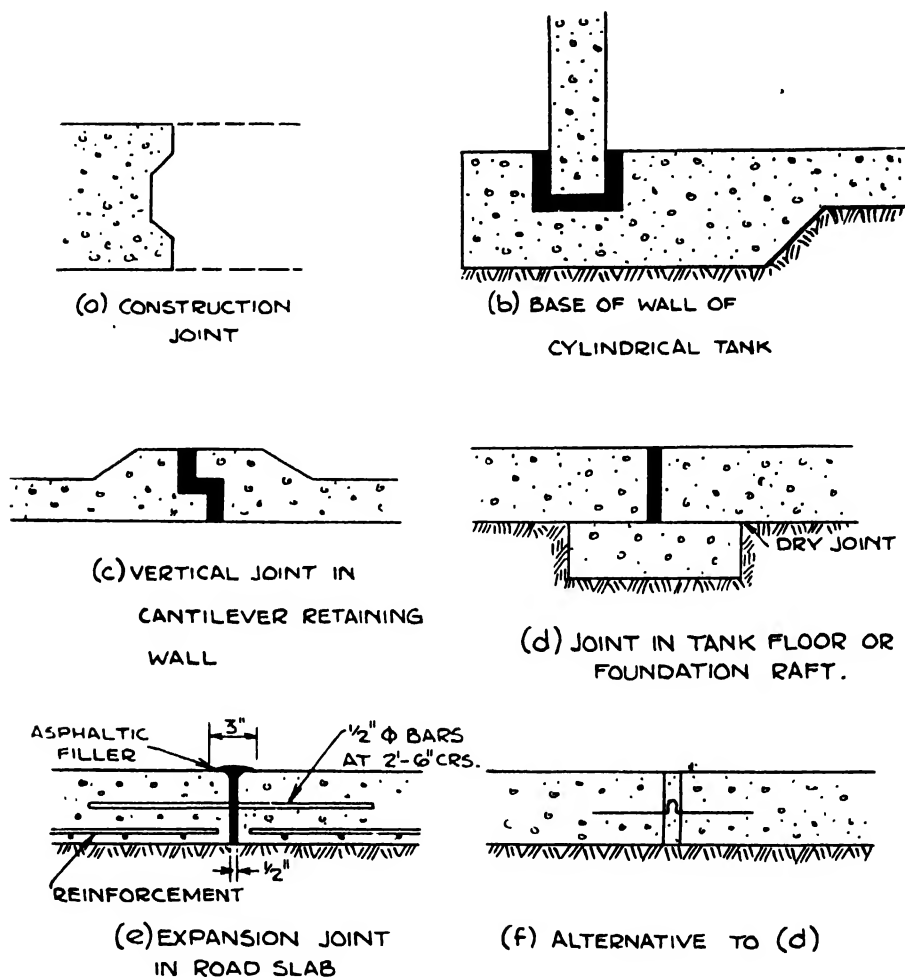


Fig. 36. Typical Joints.

bending in the tank wall. At (a) the profile of a slab construction joint is indicated; such joints when at right angles to the principal reinforcement should be formed at mid-span of the slab; the reinforcement should cross the joint without interruption. This joint is applicable to suspended slabs, but for slabs laid directly on the ground (not forming part of a raft), construction joints should be made permanent joints in predetermined positions. Figs. 36 (d) and (f) show designs for such joints making definite breaks in the continuity of the slab. For very firm subsoils (f) can be adopted, the copper strip being inserted if the joint is to be watertight (as in swimming bath floors). The pad in (d) prevents one panel settling relative to the other when laid on soft ground. The joint can be filled with bitumen sheeting, unless it should be watertight, when hot asphalt should be used.

CHAPTER IX

CONCRETE MATERIALS, MIXES, AND STRESSES

1.—Quality of Cement.

A KNOWLEDGE of the properties and variation of quality of the materials he intends using is essential to the concrete engineer, and the importance of employing the most suitable materials in correct relative quantities in the manufacture of the concrete calls for no emphasis. The essential qualities of the sand, stone, and cement are laid down in all regulations and specifications.

The physical and chemical properties of the cement are adequately set out in the British Standard Specification for this material, the 1931 edition of which requires a 3 to 1 mortar to have a minimum strength of 300 and 375 lb. per square inch at 3 days and 7 days respectively. The requirements for fineness stipulate that not more than 10 per cent. shall be retained on a sieve having 170 meshes to the square inch and not more than 1 per cent. on a 72-mesh sieve. Of equal importance to the contractor as to the designer is the stipulation that for slow-setting (or normal) cements the initial set shall take place not less than 30 minutes after mixing, and final set not more than 10 hours after mixing. The corresponding times for quick-setting cements are 5 minutes and 30 minutes. The specification also gives the limiting percentages of the constituents in the composition of the material, and details and limiting results of the Le Chatelier test. An optional test on neat cement briquettes that should attain a tensile breaking strength of 600 lb. per square inch at 7 days is also prescribed.

In addition to these tests there are certain other requirements that are not given in the B.S.S. and which, if not absolutely necessary, are desirable for important work. Pats of neat cement should be made and placed on a piece of glass. The thickness of the pat should not exceed $\frac{3}{8}$ in. and when set the pat should be immersed in water for seven days. At the end of this time the specimens should not show any signs of cracking, buckling, or "flying." When broken the sections should be of uniform colour and hardness, without any indication of surface flaking.

Faija's test is another useful indication of the quality of the cement. Pats of neat cement are placed in a moist air bath for four hours and thereafter immersed for 20 hours in water at a temperature of about 115 deg. F. At the end of this treatment the specimens should show no signs of buckling or cracking.

The value of these additional tests is that they can be made on the site immediately before using the material and thus are a safeguard against deterioration, since the quality of cement is affected greatly by the storage conditions. If the material has to be kept for any length of time it should be stored in a weatherproof shed having a floor raised clear of the ground. For temporary

storage not exceeding two or three days the cement can be packed on a floor raised clear of the ground if the containers are effectively covered by tarpaulins. Any cement found to be defective, for example, partially set due to dampness from the store or elsewhere, should be condemned and removed from the site.

2.—Quality of Aggregate.

Cleanness and hardness are the essential properties to be looked for in both the fine aggregate (sand) and the coarse aggregate (stone). "Clean, sharp and free from foreign and vegetable matter and from adhering soil, clay, etc.," should be the wording of the specification for both materials. For the coarse aggregate, whether it be crushed stone or river ballast, uniformity of quality should be stipulated.

Given a suitable material as regards strength, cleanness, roughness, impermeability, and freedom from crusher dust, the shape of the particles should be considered. Laminated and wedge-shape pieces of stone and splinters should be avoided. Thus crushed granite, beach shingle, and pit ballast, if properly treated to remove dust, traces of salt, and loam respectively, are the best aggregates. Broken brick and flint have certain desirable qualities but should be given careful consideration before being used in a normal reinforced concrete structure. Broken brick is a cheap and useful aggregate for mass concrete work as in foundations, and is ideal for fireproofing purposes, whereas flint, and to a certain extent granite, are not the most suitable materials for the latter purpose. If good hard broken brick is used in concrete for structural members it should be free from lime mortar dust, and should be thoroughly saturated with water immediately before mixing. Coke breeze, cinders, etc., should not be used for load-carrying members, but they make an economical and light concrete for partition wall and similar construction. For thin walls and long columns and other cases where the concrete has to be poured among intricate reinforcement it is found that smooth rounded ballast aggregate is preferable to angular broken stone.

Stone and sand are usually bought by the ton, and the unit weights vary from 90 lb. to 110 lb. per cubic foot depending on the type of material and moisture content. For average figures one can take 22 cubic feet of aggregate to the ton of either fine or coarse aggregate. Mixed aggregate usually measures only 20 cubic feet to the ton. The quantities of dry materials required for a cubic yard of any particular mix are given on *Table No. 39*.

3.—Concrete Mixes.

The proportions in which the three constituents—cement, fine aggregate, and coarse aggregate—should be mixed are usually stated by volume to a unit volume of cement. Experience has shown that a mix of 1:2:4 produces a suitable concrete both as regards cost and strength. Weaker mixes are not recommended for reinforced work, but 1:3:6 is a suitable mix for mass concrete, for concrete temporarily placed that is to be cut away at a later date, and for a "blinding" layer under reinforced concrete foundations. Mixes richer in cement than 1:2:4, for example, 1:1½:3 and 1:1:2, are stronger and

more expensive owing to the higher cement content. Such richer mixes are not economically justified in members subject to flexure (since so much of the concrete is in tension and is therefore ineffective in adding to the strength of the member according to generally accepted methods of calculation), but often produce economical designs for heavily loaded columns. Richer mixes than 1 : 1 : 2 contain such a large percentage of cement that the contraction upon hardening is a serious matter apart from the high cost. On *Table No. 23* is given a self-explanatory series of recommendations for the most suitable mix for various parts and types of structure, and these suggestions should be of some guidance in selecting a suitable mix for any particular work. Later in this Chapter are given some observations on mixes for special purposes.

The practice of proportioning as generally specified by relative volumes is open to a number of objections, and concretes indiscriminately made to such a specification can vary considerably in compactness and strength. Apart from the question of economically using the available materials, such concrete may be good enough as regards strength if the conservative working stress of 600 lb. per square inch is used for a 1 : 2 : 4 concrete, but the designer wishing to work to the higher stresses justified by modern cements must give some consideration to the proportioning of his materials.

4.—Proportion of Cement.

Since the strength of concrete depends on its cement content, and since weight is an absolute measure of quantity, the cement should be proportioned by weight. Thus by eliminating the risk of more or less loosely bulking the cement in a batch measurer, the collateral risk of getting less (or more) than the specified amount of cement in the mix is also avoided.

Having regard to this, therefore, the practice of referring to concrete mixes as being Mix A, Mix B, etc., is strongly advocated, and the make-up of a series of such mixes is given on *Table No. 23*. Taking Mix C as an example, this would be comparable to a 1 : 2 : 4 mix in so far that it contains approximately one part by volume of cement to six parts by volume of aggregate. Thus, considering 1 cb. ft. of normal-hardening Portland cement as weighing 90 lb., an engineer specifying a 1 : 2 : 4 mix expects to get 90 lb. of cement to each 6 cb. ft. of unmixed aggregate, and such a quantity of cement can only be assured by definitely specifying it. Since cement is supplied in 1-cwt. and 2-cwt. bags it is more convenient to express the mix in terms of a unit of 1 cwt. of cement ; thus Mix C is 112 lb. of cement to 2½ cb. ft. of fine aggregate and 5 cb. ft. of coarse aggregate.

5.—Proportions of Fine and Coarse Aggregate.

The ratio between the amounts of fine and coarse aggregate should necessarily depend on the grading of these materials in order that the volume of the "sand" shall be sufficient to fill the voids in the coarse aggregate. Until the material for any particular job has been purchased it is not possible to say what will be the exact grading of either the sand or the stone, therefore this information will not usually be available when the designs and specifications are being

prepared. To the engineer determined to have good dense concrete there are one or two courses open when specifying the mix.

He can specify the proportions of a particular sand and a particular stone, each taken from a known source and both being materials with the properties of which he is well acquainted. The gradings of these materials should be fairly constant throughout all commercially supplied consignments. It is preferable if one or two independent sources of supply are available, since if the product of a single pit is specified the price quoted may be raised above the competitive level thus offsetting in part the economical advantage of the better concrete. It is obvious that any material so specified should be obtained in the locality of the site of the work in order to reduce freight charges. Thus one would not specify whinstone for a London job or Thames ballast for a structure in Glasgow. Even when the required material is thus particularised it is wise to specify the limits of variation of the essential properties.

As an alternative the engineer can specify the proportions of a coarse and a fine aggregate having definite gradings (within limits) and leave it to the contractor to find his own supply of material falling within these limits. A clause should be inserted to the effect that the quality and shape of the aggregate should be subject to approval. For the sake of economy the engineer must assure himself that materials of his specified gradings are commercially obtainable within reasonable distance of the site.

A third method which is open to the engineer is to state the provisional ratio of coarse to fine aggregate, the maximum and minimum sizes, and the general properties of each, but he should insert a clause that will allow him to alter the proportions upon examination of the samples of materials submitted for his approval. The engineer, however, must use caution in exercising his power to put this clause into effect, since his alteration of the proportions may require the provision of more of the expensive material and less of the cheaper, and thus lead to trouble over unit prices. On the other hand the adjustments may be such that less of the expensive material is required than allowed for in the estimate, in which case it should be possible to adjust the prices so that the client secures the benefit. Taking all aspects into consideration this method is perhaps the one that would be recommended in general cases.

6.—Grading of Aggregate.

The chief point to observe is that the proportion of fine to coarse material depends on the materials chosen, and that the volume of fine aggregate should be at least 5 per cent. in excess of the volume of voids in the coarse aggregate. The grading of the constituent materials varies considerably with the nature and source of the material, and the grading required depends upon the class of work. For ordinary building work the fine aggregate should be graded from dust to $\frac{1}{4}$ -in. particles and the coarse aggregate from $\frac{1}{4}$ -in. to $\frac{3}{4}$ -in. particles, although for mass concrete work $1\frac{1}{2}$ or 2 in. could be the permissible maximum sizes for the coarse material. For floor finishes, watertight work, etc., the maximum size of the particles should be less than $\frac{3}{4}$ in.

The ideal grading of the material between the limits mentioned is obtained when the voids in a bulk of any particular size of particle are completely filled

by the bulk of all the particles smaller than the one considered. For general work the relative quantities of fine and coarse particles in the sand should be approximately as follows.

Retained on a	$\frac{5}{16}$ -in. mesh,	nil
" "	$\frac{1}{4}$ -in. mesh,	not more than 10 per cent.
" "	0.0125-in. mesh,	{ not more than 90 " "
" "		{ not less than 70 " "
" "	0.005-in. mesh,	not less than 95 " "

For work subjected to attrition, such as road slabs, floors of garages, workshops, etc., where these are not specially finished, a coarse and more fully-graded sand is recommended in which the relative proportions are :

Retained on	$\frac{1}{4}$ -in. mesh,	nil
" "	0.05-in. mesh,	{ not more than 65 per cent.
" "		{ not less than 50 " "
" "	0.02-in. mesh,	not less than 80 " "
" "	0.01-in. mesh,	not less than 95 " "
Dust,		nil

The grading of the coarse aggregate for general work should conform to the following specification :

Retained on	$\frac{7}{8}$ -in. mesh,	nil
" "	$\frac{3}{4}$ -in. mesh,	not more than 5 per cent.
" "	$\frac{1}{4}$ -in. mesh,	not less than 90 " "
" "	$\frac{1}{8}$ -in. mesh,	not less than 95 " "

Since the grading of the coarse aggregate should bear some relation to that of the fine, it is recommended that for road slabs the grading of a suitable large granite aggregate for use with the sand specified above would be as follows :

Retained on	$1\frac{1}{2}$ -in. mesh,	nil
" "	$1\frac{1}{4}$ -in. mesh,	not more than 50 per cent.
" "	$\frac{1}{2}$ -in. mesh,	not less than 70 " "
" "	$\frac{1}{8}$ -in. mesh,	not less than 95 " "

7.—Water Content.

The strength of concrete depends to a very great extent on the amount of water used in mixing. There is a certain amount of water in any given mix with any given materials that will produce a concrete of maximum strength for this particular mix and these materials. Less than this amount of water gives a decrease in strength, and about 10 per cent. less gives insufficient water to ensure complete setting of the cement and produces an unworkable concrete. More than this amount increases ease of workability, but also decreases strength. An increase of 10 per cent. of water reduces the strength by approximately 15 per cent. and with 50 per cent. increase the strength is reduced by half. With more than 50 per cent. increase the concrete becomes too wet and owing to segregation is not workable.

A rule recommended by some authorities to give the quantity of water for almost maximum strength combined with workability is

Weight of water = 28 per cent. weight of cement
+ 4 per cent. weight of total aggregate.

This rule assumes non-absorbent and dry materials, and sufficient extra water should be added to provide for full absorption. If the materials are wet the quantity of water required should be reduced by the amount of water therein. Other authorities base the water content on the weight of all the materials when dry, and fix a figure of from 8 per cent. to 10 per cent. This gives quantities somewhat in excess of the rule already stated.

A more practicable method of proportioning the amount of water is by the well-known slump test, wherein a sample of the concrete is placed in a standard, open-ended, truncated metal cone, 12 in. high, 8 in. diameter at the bottom and 4 in. diameter at the top, stood with the large end downwards on a board. When the cone is lifted vertically free from the wet concrete the material will spread out over the board, and the difference between 12 in. and the maximum depth of the concrete spread on the board is a measure of the slump. The amount of slump generally varies from practically nil to 9 in. or more, depending on the consistency of the concrete. Suitable maximum values for various classes of work are given on *Table No. 23*, on which is also tabulated the nominal quantities of water required in accordance with the formula aforementioned. The slump test has the practical advantage of being a measure of the wetness of the concrete immediately before it is placed, and in a simple way allows for the porosity and dampness of the aggregates and for the quantity of cement.

8.- Special Mixes and Mortars.

Certain classes of concrete construction call for special treatment. Floors subject to abrasion can be hardened by treating with sodium silicate, or they can be surfaced with granolithic.

The granolithic finish applied to floors subject to hard wear is made by mixing one part of Portland cement to three parts of washed granite chippings graded from $\frac{3}{8}$ in. downwards. It is preferable to lay the surfacing at the same time as the main slab is placed in order to create a good bond between the two materials. Such a course, however, is open to the objection that unless properly protected the surfacing may be damaged by workmen, etc., during subsequent construction. The usual thickness of the surfacing is $\frac{3}{4}$ in. to 1 in. Thicknesses of less than 1 in. must be considered as wearing surfaces only and cannot be taken into account as increasing the resistance moment of the slab. If a layer of 1 in. or upwards is applied at the same time as the slab itself it is only fair to consider the upper $\frac{1}{2}$ in. as wearing surface and the remainder as additional effective slab thickness, but to justify this consideration the granolithic should be placed within twenty minutes of placing the main slab. If several hours elapse between the two operations the bond is of doubtful value, and if the interval is a matter of days a plane of cleavage is so definitely formed as to render the surfacing useless as part of the constructional thickness.

Steps are often finished with granolithic treads and a non-slip surface formed

by the addition of carborundum. The carborundum dust should be lightly trowelled into the surface of the concrete while still green, the amount of material required being about 6 oz. per square yard.

Falls on roof and floor slabs are often most conveniently formed by adding a screeding over and above the slab thickness. Such a screeding can be a mortar of cement and sand mixed in the proportions of 1 to 3.

It is sometimes necessary to "blind" the bottom of an excavation with concrete before a clean smooth surface is obtained on which to place the reinforcement. Such a layer can be from 1 in. to 3 in. of lean concrete in the proportions of, say, 1 part of cement to 10 parts of aggregate.

An important factor in deciding the effectiveness of watertight construction, such as the construction of the walls and bottoms of tanks and reservoirs, is the quality of the concrete. A watertight concrete should contain a little more cement (such as Mix D or E), it should be exceedingly well and carefully rammed (this point cannot be over-emphasised), and special care should be taken in the formation of the construction joints. Attention given to these factors, together with the use of a moderate tensile stress and adequate wall thickness, ensures that tanks of moderate depths are proof against seepage from water and other liquids that do not attack the concrete. If for any reason careful and expert construction and supervision are not available, the tank can be made watertight by either internal rendering or the addition of a waterproofing compound to the concrete, by using a waterproofed cement, or by asphaltting.

If rendering is adopted the wall could be built with Mix C concrete of normal consistency, and the rendering could be a $\frac{1}{2}$ -in. to $\frac{3}{4}$ -in. thickness of mortar composed of 3 parts of sand to 1 part of cement. The inside faces of the walls should be hacked while the concrete is green to form a key for the cement rendering and thoroughly wetted, and with advantage can be given a coat of cement grout immediately before applying the rendering. The latter should be applied in two layers.

This method of rendering should be satisfactory for cases where the head of water is anything up to 30 ft. When only a few feet deep an application of cement grout, well brushed in, gives good results. Oil tanks up to 20 ft. deep can be proofed by careful construction of the walls in Mix D concrete; immediately the shuttering is removed the interior surface should be given a skimming coat of cement (preferably rapid-hardening cement) well worked into the pores of the concrete with a trowel. Before drying out this coat should be given two successive coatings of cement grout applied with a brush. There are on the market a number of proprietary waterproofing compounds, applied either in the form of a rendering or mixed with the concrete, while hydrated lime is also useful in this direction.

The problems of waterproofing and of protecting the concrete from deterioration by contact with the contained liquid are more or less bound together. In practice the following list gives some satisfactory treatments for various liquids:

Water-heads up to 6 ft.—Cement-grout wash on ordinary concrete.

Water-heads up to 20 ft.—Well designed tanks, proper construction, rich mix; or asphalt lining to ordinary construction.

Water-heads up to 30 ft.—3:1 mortar rendering or asphalt.

Water-heads above 30 ft.—Rendering with waterproofing compound or asphalt.

Weak acids.—As for water.

Heavy fuel oils, benzole, creosote, etc.—Successive applications of cement grout washes as described on page 115.

Tanning liquor.—As for water.

Hot vegetable oils.—Should not be stored in concrete tanks.

Strong sulphuric and other acids.—Lead or acid-resistant brick linings.

Fermenting beer.—Stainless steel or enamelled linings.

Chlorine solution.—As for water.

Caustic soda solution (4 per cent.).—As for water.

Sulphate liquor.—Dense concrete with coating of paraffin wax.

Cider vinegar.—As for sulphate liquor.

Petroleum.—Spar varnish.

Tar.—3 : 1 mortar rendering.

Some remarks on the most suitable types of joints for tank work have already been given in the preceding Chapter.

For non-reinforced work involving large masses of concrete a mix as weak as Mix A, or even weaker, can be allowed. If the dimensions of the work warrant, such as massive bridge piers, heavy foundations, concrete filling, etc., the use of hard stone "plums" (for example, boulders, old broken concrete) should be permitted. Neither the distance between any two "plums" nor the distance between any single "plum" and the face of the work should be less than the width of the "plum." Without sacrificing strength such practice leads to economy both in material and labour.

9.—Properties of Concrete.

Foremost among the properties of concrete that the designing engineer requires to know is the weight of hardened concrete. Plain concrete may weigh about 130 lb. to 140 lb. per cubic foot. When reinforced the composite material may weigh up to 150 lb. per cubic foot, but 144 lb. per cubic foot is a very convenient weight to take for dead load calculations since each square inch of cross-section is equivalent to 1 lb. per lineal foot of the member considered.

In the design of vertical shuttering the horizontal pressures exerted by wet concrete are the primary factor, and a maximum figure of 140 lb. per square foot of vertical surface per foot depth should be adequate for moderate depths of filling. For greater heights this unit pressure can be decreased in accordance with the values given on Table No. 23, and the pressure will be even more reduced for narrow wall-work, for drier concretes, and where the reinforcement is especially plentiful and intricate.

The coefficient of linear expansion and contraction due to temperature changes increases with the cement content, as is shown by the values given on Table No. 23. This coefficient averages 0.000066 per deg. F., which is practically the same as that for mild steel. This factor enters into the design of exposed or long lengths of construction, such as roads, arches, warehouses, tanks containing hot liquids, etc., and adequate provision must be made in the design to take up the stresses due to temperature changes or to limit these stresses by the provision of joints. Suggested designs for joints have been given in the

previous Chapter, together with joints safeguarding against contraction cracks. In addition to the normal hardening contraction, concrete appears to be subject to a progressive shrinkage over an unlimited period of time. This shrinkage, together with the correlated phenomena of plastic flow, is the object of serious modern investigations. Although neglected in everyday practice, the established values of the shrinkage coefficients are of interest. The coefficient is about 0.00025 at 28 days and 0.00035 at three months, after which period the increase is less pronounced, until after one year it seems to reach a maximum figure approaching 0.0005.

10.—Concrete Strengths and Standard Stresses.

The ultimate compressive strength of concrete depends upon age and cement content, increase in either of these factors giving an increase in strength. Values of maximum strength, attained at approximately one year, vary from 1,500 lb. per square inch for lean concrete to upwards of 8,000 lb. per square inch for rich mixes. The rate of increase of strength with age is almost independent of the cement content, and in 28 days about 60 per cent. of the strength attained at 12 months is obtained, three-quarters in 2 months, and about 95 per cent. in 6 months. For design purposes the working stresses are usually based on the strength at three to four months, which is about 85 per cent. of that at one year. The allowable stress, based on a factor of safety of 4 on the presumed ultimate at four months, is in the most conservative practice determined by the expression

$$c = 900 - 50V$$

where V is the volume of sand plus coarse aggregate per unit volume of cement.

With modern cements used in conjunction with well-graded and selected materials, and with proper attention paid to the water content and supervision of mixing and placing, these working stresses can be increased with safety by at least one-sixth. Thus for Mix C ($V = 6$) the safe compressive stress in accordance with conservative design would be 600 lb. per square inch, whereas for general practice the value advocated as a standard stress is 700 lb. per square inch. The values of the standard stresses suggested for other mixes are given in Table No. 23, and variations from these stresses are discussed in the next paragraph.

The Ministry of Transport's 1931 Memorandum demonstrates official recognition of the strength of modern concretes. If at 28 days (with ordinary Portland cement) or 7 days (with rapid-hardening Portland cement) the compressive strength of 6-in. concrete cubes equals or exceeds $(15A + 900)$ lb. per square inch, the working stress can be taken as $(5A + 300)$ lb. per square inch, where A = weight (lb.) of cement per 2 cb. ft. of fine aggregate and 4 cb. ft. of coarse aggregate. Applying these expressions to some of the mixes tabulated on Table No. 23, the following values are obtained:

Mix	28 (or 7) day strength (lb. per square inch)	Working stress (lb. per square inch)
C	2,250	750
E	2,700	900
F	3,600	1,200

Although based on a factor of safety that may be as low as three on the 28-day strength of the concrete, these stresses do not lack conservatism since they are associated with a substantial impact factor applied to a generous live load (see Chapter II, *Fig. 1*, and *Table No. 2*).

11.—Modifications to Standard Stresses.

The standard compressive stress is that which should be adopted when the loads and moments are computed in the usual way (that is, without particular refinement of calculation), but under certain conditions these stresses may be exceeded, and in others lower stresses should be adopted. Among the factors permitting an increase over the standard stress for a given mix are the following :

(1) When the ultimate test strengths of an adequate number of samples exceed four times the standard strength, the working stress can be one-quarter of the average of the ultimate unit stress. This condition applies when specially selected and graded materials are used with the object of obtaining a high-strength concrete with a minimum quantity of cement.

(2) When slump tests give values less than 6 in. for beams and slabs, and so long as there is adequate water to ensure a workable mixture, the standard stresses could be increased by 10 per cent. (if advantage is taken of this increase, means must be taken to ensure that the small slump is maintained throughout all batches of concrete poured by making an adequate number of tests).

(3) When rapid-hardening cements are used any increase in working stress allowed should be governed by the observations in Paragraph 14.

(4) If the moments and forces are precisely determined in accordance with the elastic theory, if the loading is accurately known, and if the reinforcement is disposed in accordance with the calculations, an increase of 10 per cent. to 15 per cent. over the standard stress should be permitted.

(5) If the approximate support moment coefficients are employed for the moments in tee beams supported on main beams, the support sections can be designed for a compressive stress 10 per cent. in excess of the standard stress.

(6) If a section subjected to bending has an area of compressive reinforcement equal to the tensile reinforcement, and the resistance moment is not calculated on the "steel beam theory," the standard stress may be increased as described in Paragraph 4, Chapter XI.

(7) Permissible extreme fibre stress on members calculated for combined stresses on "straight-line stress variation" theories can be 10 per cent. in excess of the standard stress.

(8) Certain conditions of column design allow increased core stresses (see Chapter XIII).

(9) If extreme or infrequent live loading is designed for, stresses for these loads can be increased at the discretion of the designer, but the dead load stress should be the standard stress. No increase should be taken under this heading if allowances have already been made under headings (4) and (6).

(10) If the extreme live loading can be guaranteed not to come on to the structure until after the expiry of twelve months after construction the standard stress can be increased by 10 per cent.

The working stress should be less than the standard stress when any of the following conditions prevail:

(a) If tests on samples give lower results than four times the standard stress, the working stress should be one-quarter of the average ultimate strength.

(b) When excessive water is employed, or when high-grade materials are not obtainable, or when the work is to be constructed by inexperienced men or organisations.

(c) When the calculations are only approximately made or when the maximum loading is uncertain.

(d) Certain conditions of slender-column design require stresses less than the standard value (see Chapter XIII).

(e) When the structure is subject to vibration or impact the standard stress should be reduced. Impact is usually allowed for in the assumed magnitude of the live loading, and incessant vibration can be allowed for by decreasing the working stress to at least 15 per cent. below the standard stress.

(f) If the maximum live loads can operate within three months of the completion of construction of any particular member the working stress should be reduced in the ratio of the ultimate strength at the time of loading to the ultimate strength at three months. Refer to *Table No. 23* for strength-with-age variations.

(g) If all the load is dead load the stresses should be reduced unless means can be taken (for example, by having temporary supports) to ensure the full load not coming on to the structure until three months after completion.

(h) When the work is difficult to construct and concrete is to be placed under difficulties, or where the concrete is to be deposited under water.

In adopting these modifications much has to be left to the individual judgment of the designer and consideration of the circumstances peculiar to any given structure, but they are put forward as suggestions for the refinement of designing principles. The German Regulations for reinforced concrete construction give various working stresses for different parts of structures and for various methods of calculation.

12.—Tensile and Shear Stresses.

In the design of members subjected to bending it is usual to neglect the value of the concrete in direct tension, but in certain cases, such as walls of cylindrical watertight tanks, etc., and more commonly in the consideration of shear resistance, the tensile strength of the concrete becomes an important factor in the design. The ultimate tensile strength depends on the cement content and increases with an increase in cement ratio. For a Mix C concrete a value upwards of 200 lb. per square inch may be obtained and the ultimate is generally taken as one-tenth of the ultimate compressive strength. Modern research seems to show that the tensile strength is not directly proportional to the compressive strength, but varies more nearly as the compressive strength to the two-thirds power; i.e. if we consider the safe diagonal tensile stress as 60 lb. per square inch for Mix C, the maximum safe shear stress for richer mixes would be as given on *Table No. 23*. The safe unit shear stresses on a section are applicable when the stresses are calculated as explained in Chapter XII.

The safe punching shear stress, as occurs in column footings, pile caps, etc., can be taken as twice the allowable figures mentioned. Three times the safe diagonal tensile stress should be considered the absolute maximum for the direct tensile stress in the walls of watertight cylindrical tanks when this stress is calculated on the total effective section as described in Chapter VII. *Table No. 23* gives the limiting values for punching shear and effective direct tension. For mixes weaker than Mix C the direct tension value of the concrete should be neglected.

13.—Modulus of Elasticity.

Young's Modulus for concrete (E_c) increases with the richness of the mix, and actual values by test seem to be between the limits of 1,000 to 1,500 times the ultimate compressive strength. Thus E_c for a Mix C concrete might average 3,000,000 lb. per square inch; with a modulus for steel $E_s = 30,000,000$ lb. per square inch a modular ratio $m = \frac{E_s}{E_c}$ 10 is obtained. For the generally accepted formulae for calculations an arbitrary modular ratio of 15 (corresponding to $E_c = 2,000,000$ lb. per square inch) is found to give resistance moments consistent with test results on beams made with concretes of the strength of Mix C. This higher ratio seems to compensate for the errors involved in the consideration of reinforced concrete as a theoretically elastic substance and for the neglect of the tensional value of concrete in bending. For calculations of the deflection of members a value of E_c more equal to the true value should be taken. Suggested values of E_c for calculations for ordinary bending and for direct stress, and values for deformation calculations, together with the appropriate modular ratios are given on *Table No. 23* for the mixes tabulated.

14. Rapid-Hardening Cements.

The special cements that attain high strengths in shorter time than normal Portland cement are of two kinds, (a) aluminous cement and (b) rapid-hardening Portland cement.

As the name implies one of the main constituents in aluminous cements is alumina (Al_2O_3), which is the active agent in the setting properties of all cements. In normal-hardening Portland cements the amount of this substance is somewhere between 5 per cent. and $6\frac{1}{2}$ per cent., but it may be between 35 per cent. and 40 per cent. in aluminous cements. The initial setting time of commercially supplied aluminous cement varies from one to three hours under test, and the final set follows very rapidly being between three and four hours from the time of mixing. Tests on 3 : 1 mortar specimens show strengths up to 600 lb. per square inch in 24 hours and ultimate strengths of 700 lb. per square inch are obtained in three to five days. These values are double that required by the specification for normal cements, but tests of Portland cement mortar specimens may often give values exceeding 500 lb. per square inch at 7 days. From a close study of test results it would appear that aluminous cement concretes could be safely stressed to 50 per cent. in excess of the figure taken for concretes employing

normal-hardening cements, given a hard stone aggregate. The necessity of saving time is the chief factor which offsets the high cost of this cement.

In the case of rapid-hardening Portland cements, however, the extra cost of such cement is, in all but very small jobs, more than offset by the saving accruing by the repeated uses of the shuttering and general shortening of the contract time. There is little difference chemically between rapid-hardening and normal Portland cements, but the former are much more finely ground. Whereas on a 172 or 180 sieve 10 per cent. is the maximum residue allowed, ordinary Portland cements may give as low as $6\frac{1}{2}$ per cent. whereas rapid-hardening Portland cements give usually less than $2\frac{1}{2}$ per cent. residue. The initial set of rapid-hardening Portland cement takes place within three-quarters to two hours of mixing, and final set from two to three hours. Tensile strength tests on 1 : 3 mortar briquettes show strengths at 24 hours at least equal to the three days specification; at seven days 75 per cent. of the ultimate strength is reached, and 95 per cent. of the ultimate at 28 days.

From a comparison of the ultimate values it would seem that rapid-hardening Portland cement concrete can be stressed to values 20 per cent. in excess of the working stresses for ordinary cement concrete. This advantage can only be taken, however, if the cement is proportioned by weight as specified on *Table No. 23*. This proviso is important, since one cubic foot of rapid-hardening Portland cement weighs less than an equal volume of normal cement, and if proportioned by volume the quantity of cement in the mix is not only insufficient but uncertain. Since the atmospheric temperature during the initial setting stage and the fineness of the sand seem to have an appreciable effect on the ultimate strengths attained by rapid-hardening cements, some engineers are averse to allowing higher stresses for such cements. Given suitably coarse sands and moderate temperatures, there seems to be some justification for using a higher working stress, one point in favour of this being that there is less likelihood of damage due to over-stressing from incidental loading during the first few weeks after placing the concrete. The use of higher stresses with rapid-hardening cement should always be associated with tests on the concrete.

CHAPTER X

REINFORCEMENT

1.—Quality of Material.

FOR all normal work the steel used for reinforcement should in all respects conform to the British Standard Specification for mild steel. The main provisions of this specification are an ultimate stress of 28 to 33 tons per square inch and an elongation not greater than 20 per cent. on a test piece 8 diameters in length. The bars should be capable of being bent double when cold without fracture to a radius not greater than $1\frac{1}{2}$ diameters if the bar is over 1 in. diameter; or to a radius equal to the diameter if 1 in. or less.

The reinforcement is usually in the form of rolled round bars. Notwithstanding certain advantages inherent in deformed bars, the use of the latter is restricted to specialised construction owing largely to the extra cost.

Many steels of foreign origin, although having sufficient tensile strength, will not stand up to the bending tests required by the British Standard Specification, and for bars larger than $\frac{3}{8}$ in. diameter it is necessary to heat the bars before bending. This practice should be discouraged, and when such steel is employed care should be taken to ensure that the steel benders do not cool the bars by immersion in water, as this increases the brittleness of the material and lowers the tensile resistance in the neighbourhood of the heated portion. Whenever possible it is well to specify round bars of mild steel complying with the B.S.S., and to insert the precautionary clause that all bending is to be done cold. Moreover, the bars should be thoroughly clean and free from scale, loose rust, grease, dirt, paint, etc.

The use of cold-drawn (as opposed to rolled) mild steel, usually supplied in the form of an electrically-welded mesh, has a wide scope as an economical and reliable reinforcement for slab construction. The essential difference between cold-drawn and rolled mild steel is a higher elastic limit accompanied by a higher ultimate tensile strength. The limits of the tensile strength are given in the British Standard Specification dealing with hard-drawn steel wire for reinforcement as 37 tons and 42 tons per square inch. The stipulated bending test is to bend cold around a radius equal to the diameter of the wire through an angle of 90 deg., then to be bent back in the opposite direction through an angle of 180 deg.; upon bending back to its original position the test piece should show no signs of fracture.

2.—Working Stresses

For rolled mild steel having an ultimate strength of about 64,000 lb. per square inch and working to a safety factor of 4 on the ultimate strength, it would seem that the safe working stress in tension would be 16,000 lb. per square inch. This is the value allowed in conservative practice, but general experience shows that with normal reliable British steel and reasonably expert design a stress of 17,000 lb. per square inch is safe for ordinary work such as industrial structures, bridges, etc. For building work a stress of 18,000 lb. per square inch is justified, but for liquid storage structures where the walls are in direct tension a much lower stress of 14,000 lb. per square inch or even 12,000 lb. per square inch is usual. The Ministry of Transport adopts a stress of 16,000 lb. per square inch for bridge-work. For shear reinforcement the limiting stresses are discussed in Chapter XII.

Since the yield point of rolled mild steel is about 32,000 lb. per square inch, a working stress of 16,000 lb. per square inch represents a factor of safety of two on the stress at the yield point, and this factor is the true measure of the margin of security. With cold-drawn mild steel having a yield point stress of 75,000 lb. per square inch and a factor of safety of three on this figure, a working tensile stress of 25,000 lb. per square inch is obtained. This is the value of the working stress usually adopted for floor and roof slab design when using cold-drawn mild steel reinforcement.

When deciding on a suitable tensile working stress for the reinforcement in any particular part of a structure, the factors given in the last Chapter permitting an increase or enforcing a decrease in the value to be adopted should be considered. Those factors cited that merely represent an increase or decrease in the ultimate strength of the concrete should not be considered operative as regards the steel stress, and corresponding conditions that may affect the ultimate steel strength should be substituted.

The compressive stress to which reinforcement may be subjected is controlled by the compressive stress in the surrounding concrete. Since the strain in the two materials is equal so long as the bond is not destroyed, the stresses will be proportional to the elastic moduli. Mild steel has an elastic modulus of 30,000,000 lb. per square inch, and with the minimum modulus for a Mix C concrete, given on Table No. 23, of 2,000,000 lb. per square inch the compressive stress in the steel would be fifteen times that in the concrete, or generally

$$c_s = mc$$

where c_s = the compressive stress in steel

$$m = \text{the modular ratio} = \frac{E_s}{E_c}$$

and c = compressive stress in the concrete.

In practical calculations it is more convenient to calculate the additional compressive resistance due to the steel; this is $c(m - 1)A_s$ where A_s = area of compressive reinforcement. When this expression is used the concrete resistance must be calculated on the gross sectional area, no deductions being made for the reduction in concrete area to allow for the area occupied by the steel.

3.—Adhesion.

For a reinforcing bar to perform efficiently its task of taking tensile forces it is important that the bar shall be sufficiently anchored. There must be sufficient length of bar beyond any particular section to develop by adhesion between the steel and concrete a bond force equal to the total tensile force in the bar at the section considered. The safe adhesion stress between concrete and round steel bars embedded therein is usually taken as 100 lb. per square inch. Thus a bar stressed to 16,000 lb. per square inch requires a length of forty times its own diameter to develop a safe bond force equal to the tensile force in the bar, irrespective of the diameter of the bar. Similarly a stress of 14,000 lb. per square inch requires thirty-five diameters, a stress of 18,000 lb. per square inch forty-five diameters, and other stresses in proportion; or more generally each 2,000 lb. per square inch of stress requires five diameters of straight bar to provide a secure anchorage.

The anchorage length of bar required beyond any stressed section can be reduced by providing a hook at the end of the bar. A properly-formed semi-circular hook can be considered as providing 50 per cent. of the anchorage, with a maximum value equivalent to twenty diameters. A 60-deg. bend can be considered as half as effective as a hook, that is, as providing 25 per cent. of the total anchorage up to a maximum of ten diameters. A 90-deg. bend can be taken as equivalent to 12½ per cent. of the total with a maximum allowance of five diameters. In order to take advantage of the full anchorage value of such hooks and bends it is important that they shall be properly formed, and suitable dimensions are given on *Table No. 24*.

It will be observed that for general practice, in which stresses of 16,000 lb. per square inch and upwards are employed, a full hook saves a bar length of about ten diameters, and a 60-deg. bend five diameters. Although a 90-deg. bend saves only about two diameters, a bend of this description at each end of a bar reduces the overall length of the bent bar by eight diameters from that required without hooks or bends; it is often useful where space is slightly restricted, and is an economical form of anchorage.

The relative costs of making hooks and bends is roughly in the ratio of 3 to 2 to 1 for hooks, 60-deg. bends, and 90-deg. bends respectively. With the current price of labour and steel the relative economy of the various types of anchorage, allowing for the value of the steel saved by the reduction in length and for the cost of forming the hook or bend, is for various sizes of bars approximately as follows when the tensile stress is 16,000 lb. per square inch:

Diameter of bar	$\frac{1}{2}$ in.	$\frac{3}{4}$ in.	1 in.	1 in.	1½ in.
Straight bar	1·0	1·0	1·0	1·0	1·0
Semicircular hook	2·2	1·4	1·4	1·2	1·2
60-deg. bend	1·6	1·4	1·2	1·2	1·1
90-deg. bend	1·3	1·2	1·1	1·1	1·1

It is a sound rule that every tension bar should be hooked or bent at both ends. As is seen by the above comparison a hook or bend is not necessarily the most economical form of anchorage, but it is nevertheless the most secure.

Therefore in all cases where absolutely refined cutting of costs is not imperative some form of hook or bend should be provided, and for the anchorage of the main bars in important members a semicircular hook should always be provided. In many cases restriction on the length of anchorage available renders the formation of a hook necessary.

The regulations of the London County Council (1915) do not allow the full values of anchorage given above, but require forty diameters of bar in addition to a semicircular hook. If a 90-deg. bend is provided the length of bar required is sixty-seven diameters in addition to the bend. These lengths refer to 16,000 lb. per square inch and are proportionately reduced for lower working stresses.

Table No. 24 gives the various adequate lengths for anchorage at various stresses, and it should be remembered that these lengths must be measured from the point where the bar commences to deviate from the straight. The minimum stress tabulated is 10,000 lb. per square inch, and, although the maximum working stress in a bar may be less than this value, it is recommended that in no case should a less anchorage length than required for this stress be provided, while for straight bars forty diameters should be the minimum. For bars in direct tension members, such as hanging columns, walls of circular tanks, etc., special care must be taken to provide adequate anchorage, and nothing less than forty diameters and a semicircular hook should be provided in these cases; if the working stress exceeds 16,000 lb. per square inch, the length should be increased.

Where a bar is principally in compression a straight anchorage length equal to twenty-four diameters (if the stress does not exceed 9,600 lb. per square inch) should be provided, although, where the bar can be subjected to either tension or compression, anchorage should be provided for whichever is the maximum stress. A compression bond length need not be hooked, although some form of turned end is desirable if the end of the bar is near an outer end face of the member.

Some authorities recommend that the additional moment due to the anchorage provided by a hook should be investigated and provided for. The hook moment for a standard semicircular hook would be computed by multiplying the tension in the bar at the hook by $2\frac{1}{2}$ times the bar diameter.

4.—Detailing Reinforcement.

Careful attention paid to a number of points regarding lengths, sizes, etc., of reinforcing bars is repaid by saving due to ease of fabrication of the bars on the site and by a general rise in the efficiency of construction and design. To this end as few different sizes of bars as possible should be used in any structure or part of a structure, and the largest diameter bars consistent with non-excessive spacing should be used, thus reducing the number of bars to be bent and placed. Also it should be borne in mind that large-diameter bars are cheaper than small. The basic rate is usually quoted for $\frac{5}{8}$ -in. diameter bars and all larger bars are supplied at this rate. Bars under $\frac{5}{8}$ in. are usually charged 5s. per ton more per $\frac{1}{8}$ in. of diameter below $\frac{5}{8}$ in. diameter, and for smaller bars of $\frac{3}{8}$ in. and $\frac{1}{4}$ in. diameter the increased charge is still more.

Generally the longest bar economically obtainable should be used, but due

regard should be paid to the facility with which a long bar can be placed in any particular member. Consideration should also be given to the maximum length that can be handled without being too "whippy." Such maximum lengths are in the order of 20 ft. for bars $\frac{5}{8}$ in. diameter and less, 25 ft. from $\frac{5}{8}$ in. to $\frac{1}{2}$ in., 40 ft. for $\frac{3}{4}$ in. diameter, 60 ft. for 1 in. diameter, and 75 ft. for bars over $1\frac{1}{4}$ in. diameter.

When proposing to use long bars it must be remembered that the basic rate only applies to bars up to 40 ft. long and an extra that may amount to 1s. per ton for every foot over 40 ft. is charged, although bars up to $\frac{3}{4}$ in. diameter can be obtained in long length coils at normal rates and sometimes lower. Above certain lengths it is more economical to lap two bars than to buy long bars, the extra cost of the increase in total length of bars due to overlapping being more than offset by the alternative increased tonnage charge for long lengths. These maximum lengths are in the order of 45 ft. for $\frac{3}{4}$ -in. bars, rising to 55 ft. for $1\frac{1}{4}$ -in. bars. Long bars cannot always be avoided when reinforcing long piles, but bars over 24 ft. and more especially over 40 ft. require special railway wagons, involving transport delays and often additional freight charges.

The careful preparation of bending schedules can result in economies in construction and in material supply. The total length of each bar should for preference be given to a multiple of 3 in. and as many bars as possible should be made to one length, thus keeping the number of different lengths of bars as low as possible.

The method of giving bending dimensions and marking up bars should be uniform throughout a given structure. A well-tried system of standard bending dimensions is illustrated on *Table No. 24.*

5.—Cover of Concrete.

For proper protection, and in order to ensure that the thickness of concrete around a bar is sufficient adequately to develop adhesion, it is necessary to consider the cover of concrete over the bars and the minimum space between adjacent bars. The minimum cover of concrete around a reinforcing bar should be in accordance with the following schedule :

Slabs :	$\frac{1}{2}$ in. minimum and not less than the diameter of the bar.
Beams :	1 in. minimum and not less than the diameter of the bar for top or bottom cover to main bars. $\frac{1}{2}$ in. minimum for binders. 1 in. side cover irrespective of diameter of the bar.
Columns :	1 in. minimum for main bars in columns less than 12 in. square. $1\frac{1}{2}$ in. minimum for main bars in columns 12 in. square or over. $\frac{1}{2}$ in. minimum for binders.
Piles :	$1\frac{1}{2}$ in. minimum to main bars.
Sea Work :	$1\frac{1}{2}$ in. to 2 in. over main bars.

The minimum clear distance between the bars in any one layer in beams should be not less than the diameter of the bar or $\frac{3}{4}$ in. (the maximum size of

the aggregate), whichever is the greater. The minimum clear distance between successive layers should be $\frac{3}{4}$ in. and this distance should be maintained by the provision of $\frac{3}{4}$ -in. diameter spacer bars placed every three or four feet throughout the length of the beam wherever two or more layers of reinforcement occur. Where the bars from transverse beams thread between the layers, special spacer bars are of course neither necessary nor effective.

6.—Steel Areas and Weights.

The following data with reference to the areas and weights of round mild steel bars from $\frac{3}{16}$ in. to $1\frac{1}{2}$ in. diameter are given on *Tables Nos. 25 and 26* :

- (a) Cross-sectional area of any number of bars up to 20.
- (b) Cross-sectional area per foot width for bars spaced at 3-in. to 24-in. centres.
- (c) Weight per foot of each size of bar.
- (d) Length of each bar in one ton weight.
- (e) Weights of any number of bars one foot in length.
- (f) Weight per square yard for bars at different spacings.

For estimating purposes the following rules give the approximate weight of steel per lineal foot of member when designed in accordance with normal practice :

Let A = maximum area of tension steel per beam or column or per foot width of slab as required by the calculations ; for slabs spanning in two directions A = sum of areas required in each direction.

W = approximate weight of steel per foot run of beam or column and per square foot of slab, including all hooks, laps, binders, distribution steel, spacer bars, shear bars, etc.

$$W = KA$$

where K has the following values :

Continuous slabs (spanning in one direction),	$5\frac{1}{4}$ to $8\frac{1}{2}$
Continuous slabs (spanning in two directions),	$4\frac{3}{4}$ to $7\frac{1}{2}$
Cantilevers,	4 to $5\frac{1}{2}$
Free beams,	$3\frac{1}{2}$ to 6
Continuous beams,	$6\frac{1}{2}$ to 8
Columns,	5 to 6

For slabs and beams the minimum figures apply to cases where there is no steel in compression or no special shear reinforcement. The higher figures apply when there is equal steel in compression and tension or where heavy shear steel is provided. The lower figures for columns apply for minimum binding and the upper for heavy binding or when loose splice bars are employed. For large beams or special members the weights should be computed after roughly sketching out the disposition of the reinforcement.

CHAPTER XI

BEAMS AND SLABS

1.—Formulæ for Rectangular Sections reinforced for Tension only.

THE basis of design of structural members subject to flexure is that the internal resisting couple shall equal the bending moment produced by the externally applied load. For members of reinforced concrete it is assumed that the strain at any point in a section is proportional to the distance of the point from the neutral axis, and the resistance of the concrete below the neutral axis is neglected. From these fundamentals the general formulæ given below can be deduced. Referring to *Fig. 37(a)* the notation adopted is as follows, all units being in pounds and inches :

- c = maximum compressive stress on the concrete.
- t = maximum tensile stress on the steel.
- b = breadth of beam.
- d = effective depth of beam or slab.
- D = total depth of beam or slab.
- n = depth of neutral axis = $n_1 d$.
- a = lever arm of section = $a_1 d$.
- A_T = area of tensile reinforcement.
- r = ratio of maximum stresses = $\frac{t}{c}$.
- m = modular ratio.
- T = total tension = $A_T t$.
- C = total compression = $0.5 b n c$.
- M = external bending moment.
- $R.M.$ = safe resistance moment of a section.
- p = percentage of tensile reinforcement = $\frac{100 A_T}{b d}$.

The depth of the neutral axis expressed in terms of the stress ratio is given by

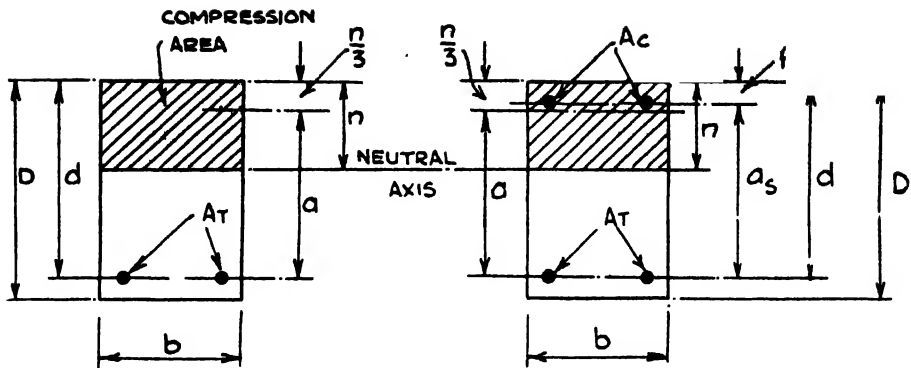
$$n_1 = \frac{1}{1 + \frac{r}{m}}$$

and in terms of the percentage of steel by

$$n_1 = \sqrt{(0.01mp)^2 + 0.02mp} - 0.01mp.$$

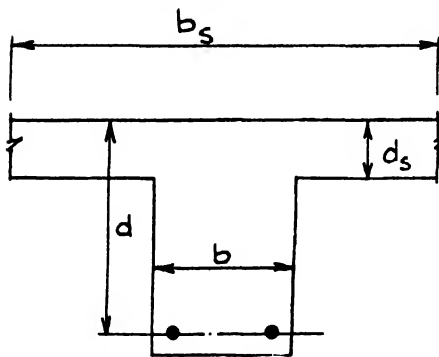
The lever arm factor in relation to the depth of the neutral axis is

$$a_1 = 1 - \frac{n_1}{3}.$$

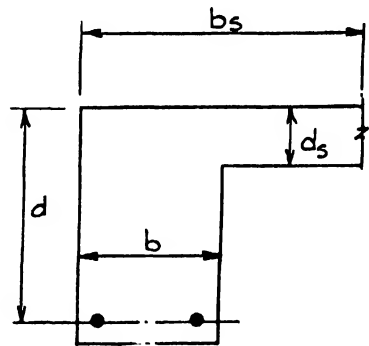


(a) RECTANGULAR SECTION REINFORCED IN TENSION ONLY

(b) RECTANGULAR SECTION REINFORCED IN TENSION AND COMPRESSION



TEE BEAM



ELL BEAM

(c) FLANGED BEAMS

Fig. 37.—Rectangular and Flanged Beams.

The internal resisting couple $= Ta = Ca = R.M.$, which should not be less than the applied bending moment :

$$\begin{aligned} R.M. &= A_T a_1 d \\ &\text{or} = 0.5 b n_1 c a_1 d^2 = Q b d^2 \\ &\text{where } Q = 0.5 n_1 c a_1. \end{aligned}$$

2.—Tabulated Factors.

Much arithmetical labour can be saved by the use of tables giving the relative values of the factors involved in the design of beams and slabs, and to this end

Tables Nos. 27 to 32 inclusive have been compiled. Thus Table No. 27 gives values of n_1 , a_1 , and p for various values of r together with values of Q for concrete stresses from 100 to 1,000 lb. per square inch and for steel stresses from 12,000 to 22,400 lb. per square inch. On Tables Nos. 28 and 29 the resistance moments of a wide range of rectangular beams 1 in. wide for various combinations of maximum stresses are given, and Table No. 31 is a similar table for slabs 12 in. wide.

Table No. 30 is also applicable to slabs and gives factors that when combined with the applied bending moment give the requisite depth of slab and the amount of reinforcement necessary. A certain amount of miscellaneous data relative to the proportions, etc., of rectangular beams and tee beams is included on Table No. 32. On this table also are data for determining the points along a beam at which bars can be stopped off or bent up consistent with providing adequate tensile resistance to bending.

3.—Design of Rectangular Beams.

Apart from considerations of shear (see next Chapter) the scantlings of a rectangular beam should be in accordance with the following rules: the effective depth should not be less than one-twentieth of the span when the maximum stress is 16,000 lb. per square inch in the tensile reinforcement. With steel stresses more or less than 16,000, the minimum effective depth should be increased or decreased to a value represented by

$$\frac{\text{span} \times t}{320,000} \quad (\text{see Table No. 32}).$$

A good working rule is that the total depth of a rectangular or tee beam in inches shall equal the clear span in feet. The breadth of rectangular beams and the breadth of the web of tee beams usually vary from one-third to once the total depth; for rectangular beams a good average figure is one-half the total depth. Much, however, depends upon the particular circumstances controlling a given design, more especially such factors as headroom and the area required for shear. The breadth of beams should also be determined to conform with commercial widths of timber, and therefore 4 in., 5 in., 7 in., 8 in., and 11 in., or widths made up by combining these values, should be employed wherever possible.

The various processes of stress determination and design are as follows:

TO DESIGN A SECTION FOR A GIVEN BENDING MOMENT AND FOR GIVEN STRESSES.

Method (a).—For the given stresses find Q from Table No. 27 and find bd^2 from $\frac{M}{Q}$. Select suitable values of b and d (possibly from consideration of shear) to give the required value of bd^2 . For the ratio of the given stresses find a_1 from Table No. 27 and the amount of tensile steel required from

$$A_T = \frac{M}{a_1 d t}.$$

Method (b).—Select a suitable effective depth and read off from Table No. 28

or 29 the resistance moment corresponding to this depth. The applied bending moment divided by this resistance moment gives the breadth of beam required. If the relative values of d and b thus derived are unsuitable, select another value of d and repeat. From the same tables look up the area of steel for the appropriate value of d and multiply this area by b to give A_T .

TO DETERMINE THE STRESSES ON A GIVEN SECTION SUBJECT TO A GIVEN BENDING MOMENT.

Determine p for the given section and from *Table No. 27* find the corresponding values of a_1 and r . Then the maximum stresses are given directly

$$t = \frac{M}{a_1 d A_T}$$

and $c = \frac{t}{r}$.

TO DETERMINE THE RESISTANCE MOMENT OF A GIVEN SECTION WITH GIVEN MAXIMUM STRESSES.

Determine p , and from *Table No. 27* find the corresponding values of a_1 and n_1 . The resistance moment based on the steel is given by $A_T t a_1 d_1$, and based on the concrete by $\frac{n_1 a_1 b d^2 c}{2}$. The maximum safe resistance moment of the section is then the smaller of the two calculated resistances.

4. Rectangular Beams with Compressive Reinforcement.

When neither sufficient depth nor breadth can be obtained to provide sufficient compressive resistance in a section, compressive reinforcement has to be provided. The provision of this extra reinforcement does not generally lead to economy, although a certain amount of concrete is saved thereby, but in some cases, such as support sections of continuous beams, convenience of steel arrangement allows double reinforcement to be provided cheaply.

The moment of resistance of a section doubly reinforced is the sum of the moments of resistance of the concrete and compression steel. The concrete moment would be calculated as for singly-reinforced beams, and the steel moment would be equal to $A_c f_c a_s$,

where A_c = area of compressive reinforcement

f_c = stress in the compressive reinforcement

a_s = distance from centroid of tension steel to centroid of compressive steel.

If f equals the distance from the compressed edge of the section to the centroid of the compression steel (see *Fig. 37(b)*), then

$$a_s = d - f$$

$$f_c = \left(\frac{n - f}{n} \right) (m - 1) c.$$

The method of designing a rectangular section with given maximum stresses and limiting values of b and d , to take a specified bending moment, is as follows :

Find Q from *Table No. 27* for the appropriate maximum stresses, and find from the same *Table* the value of n_1 for the stress ratio. Then the required moment of resistance of the compression steel is given by $R_s = M - Qbd^2$. Determine $n = n_1d$ and evaluate a_s and f_c from the formulæ already given and substitute in

$$A_c = \frac{R_s}{f_c a_s}$$

to find the area of compression steel required. Next evaluate the resultant lever arm from

$$a_r = \frac{M}{A_c f_c + 0.5bnc}$$

and find the area of tensile steel required from

$$A_t = \frac{M}{a_r f_t}$$

If A_t is less than A_c the "steel-beam theory" is applicable and both A_t and A_c should be proportioned thus

$$A_t = A_c = \frac{M}{a_s f_t}$$

When this method is adopted binders at 6 in. centres should be provided.

Certain engineers are not in favour of the indiscriminate application of the "steel-beam theory" to reinforced concrete sections, and at first sight it would seem that a beam of any size can be designed to resist almost any bending moment irrespective of the compressive stress in the concrete. In fact with a theoretical steel stress of 17,000 lb. per square inch the maximum fibre stress in the concrete would, in accordance with the usual assumptions, be more than 1,150 lb. per square inch, and, although this gives a factor of safety of at least two on the ultimate strength of average concretes, there is little margin for such factors as accidental overloading, differences between theoretical and actual moments and stresses, poor workmanship, etc., such factors being those usually covered by the generally accepted factor of safety of four. Automatic partial safeguards against unreasonable use of the "steel-beam theory" include the provision of sufficient concrete area for shear, the available space for the number of top and bottom bars required, and the reduction of lever arm consequent upon large numbers requiring more than one layer.

In those cases where excessively high stressing of the concrete is particularly undesirable, a rational compromise is to provide a section with equal tensile and compressive reinforcement, the area of which is determined so as to limit the concrete stress to 25 to 33½ per cent. above the normal maximum working stress. Thus for a beam normally designed for 16,000 and 600, sections requiring A_c in excess of A_t to maintain the stresses within these limits could be designed with $A_c = A_t$ and the concrete stress not exceeding, say, 800 lb. per square inch. For normal stresses of 17,000 and 700 the increased concrete stress might be 850 lb. per square inch. On *Tables Nos. 28 and 29* the resistance moments and area of steel per inch width with $A_c = A_t$ for the normal stresses of 16,000 and 600 and 17,000 and 700, and for increased stresses of 16,000 and 800 and 17,000 and 850, are tabulated, from which suitable beam sections to resist given moments can be conveniently selected.

5.—Design of Slabs.

It is usual to design slabs for a strip one foot in width and hence a slab is equivalent to a rectangular beam with $b = 12$ in., with the bending moment expressed per foot width. In general the formulæ developed for rectangular beams apply also to slabs, but they can be modified to facilitate computation; for example, the effective depth required

$$= d = k_1 \sqrt{M}$$

and the area of steel

$$= A_T = k_2 \sqrt{M}.$$

Values of k_1 and k_2 are given on *Table No. 30* for various stresses, and when using these values it should be remembered that M should in this instance be in foot pounds. The method of using this Table is to find the value of k_1 for the given permissible stresses and find the effective depth from the formula already given. If this depth is adopted then the value of k_2 applicable to the permissible stresses should be used to determine the area of reinforcement required per foot width of slab. If any depth greater than that determined as above is adopted, find the value of

$$k_1 = \frac{\text{effective depth adopted}}{\sqrt{M}}$$

and with the given steel stress find the corresponding value of k_2 . This is the value that should be multiplied by \sqrt{M} to give A_T . If any depth less than that first calculated is adopted find the value of

$$\bullet \quad k_1 = \frac{\text{effective depth adopted}}{\sqrt{M}}$$

and with the given concrete stress find the corresponding value of k_2 , which should then be multiplied by \sqrt{M} to find the required steel area.

It is not common to reinforce slabs in compression unless a convenient arrangement of reinforcement is obtained thereby, but where it is necessary to do so the method of calculation would be identical with that for rectangular beams.

When the maximum prescribed stresses are 16,000 and 600, or 16,000 and 700, or in the same ratio as either of these pairs of stresses, suitable sections can be readily selected from *Table No. 31*, which gives the moments of resistance of slabs 12 in. wide without compression steel, with compression steel equal to one-half the area of the tension steel, and with compression steel equal to the area of the tension steel. The latter case is computed on the basis that the specified concrete stresses are not exceeded, and not on the "steel-beam theory." The second case ($A_C = 0.5A_T$) often leads to a convenient design when the moments at midspan and support are equal. In *Fig. 38* are illustrated alternative ways of arranging slab reinforcement.

In detailing slabs certain points should be borne in mind. Independently of the amount of steel required to take the bending moment, the minimum amount of tensile reinforcement should be one-half per cent. of the section, and distribution steel normal to the principal reinforcement should be provided. This distribution steel should not be less than 0.1 per cent. of the section, and for

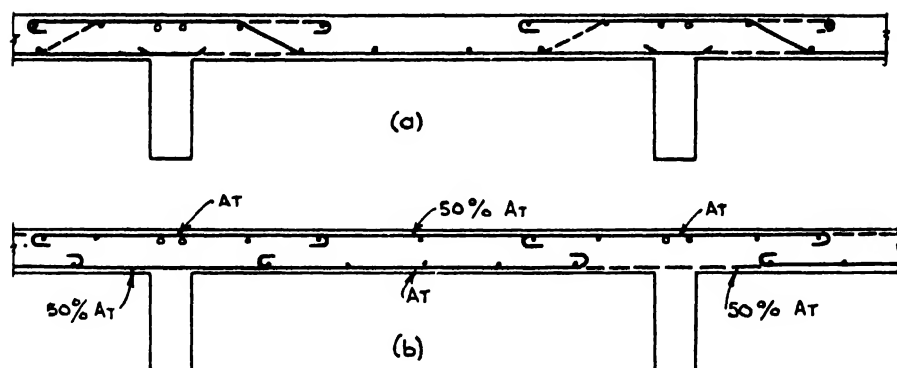


Fig. 38.—Slab Details.

slabs subject to point loads and not designed as rectangular slabs it is advocated that the longitudinal distribution steel should be equal to one-sixth of the principal tensile steel. The maximum spacing of the bars should not exceed twice the effective depth, and recommended maximum spacings are

Slab thickness	Maximum spacing
3 in.	6 in.
6 in.	9 in.
12 in.	12 in.
24 in.	18 in.

For slabs designed with reinforcement in two directions the reinforcement spanning across the shorter equivalent span should be placed at midspan, under and at the supports, over the reinforcement running normal to it.

Shear reinforcement is not usually necessary in slabs, and generally shear need not be considered unless the load exceeds one ton per square foot.

Although slabs as thin as $2\frac{1}{4}$ in. can be satisfactorily constructed given a rich mix and constructional care, for general work 3 in. should be considered as the minimum thickness. The minimum effective depth of the slab should not be less than one-twentieth of the span for a slab reinforced in one direction, nor less than one-thirtieth of the span for slabs designed to span in two directions.

6.—Flanged Beams.

Where beams are constructed monolithic with a slab, the slab can be considered as forming the compression flange of the beam so long as the moment is such as to produce compression on the same face of the beam as is the slab. The treatment of such beams is similar to that for rectangular beams, there being, however, two cases to consider:

- (a) When the neutral axis falls within the slab.
- (b) When the neutral axis falls below the slab.

Referring to Fig. 37(c) the additional notation is

- d_s = total depth of slab
- b_s = effective breadth of flange.

The breadth of slab forming the effective flange of the beam should not exceed the least of the following :

- (a) The distance between centres of adjacent beams, or the extreme width of the available slab.
- (b) One-third the span of the beam.
- (c) Twelve times the thickness of the slab plus the width of the rib of the beam.

If the slab only extends on one side of the rib and the effective section is an ell section instead of a T section, the allowable width of the flange should not be greater than that given in (a) or (b) above, nor greater than four times the slab thickness plus the rib breadth.

The depth of the rib is determined by the effective depth required and is controlled by the same considerations as obtain for rectangular beams. The breadth of the rib is usually controlled by the area required for shear (see next Chapter) but consideration should also be given to the accommodation of the steel; unless circumstances dictate otherwise a rough rule for determining the minimum rib breadth in inches is $2\frac{1}{2}$ times the number of bars in one layer.

When the neutral axis falls within the slab thickness the design of a tee or ell beam is identical with the design of a rectangular beam in which b_s is substituted for b .

When conditions are such that the neutral axis falls below the slab — that is, when $n > d_s$ — it is usual to neglect the small amount of compressive resistance afforded by the concrete lying between the neutral axis and the underside of the slab, and the lever arm is considered as

$$a = d - \frac{d_s}{2}.$$

This is only approximately correct, but the approximation is on the safe side.

The moment of resistance of compression is given by $0.5b_s d_s c_a$ approximately, although if d_s is very much less than n it is more economical to take the less approximate value of $b_s d_s c_s a$, where c_s equals the mean stress in the slab and is given by

$$c_s = \frac{c}{2} \left(1 + \frac{n - d_s}{n} \right).$$

The moment of resistance to tension is given by $A_T t a$, or the area of reinforcement required is found from

$$A_T = \frac{M}{t a}.$$

It is not common for tee beams to require compression reinforcement, but where unavoidable the same principles as for rectangular beams will apply.

Typical details of the arrangement of reinforcement in beams and slabs and the method of designing a floor panel, beam and slab construction, are given in the "Additional Examples" following Table No. 40.

CHAPTER XII

SHEAR RESISTANCE

1.—Limiting Unit Shear Stresses.

It is essential to pay as much attention to the provision of ample resistance to the shearing forces to which a structural member is subjected as to the provision of an adequate resistance moment. Shear stresses produce diagonal tensile stresses in the concrete; if the latter exceed the safe tensile stress on the concrete, reinforcement should be provided either in the form of diagonal bars or binders to augment the shear resistance of the section. Thus the shear resistance is provided by any one or any combination of the following components:

- (a) Concrete.
- (b) Steel binders.
- (c) Diagonal bars.

The maximum shear stress on a section is determined by the maximum shear force at that section divided by an area represented by the product of the lever arm and breadth of the section,

$$\text{that is, } S = \frac{F}{ab}$$

where a = lever arm of section

b = breadth of section

F = maximum total shear force.

The value of b for tee beams should be the breadth of the rib.

Without assistance from shear reinforcement the concrete alone can safely take the shear F when S is not greater than the safe shearing stress C_s specified on *Table No. 23* for the particular concrete mix employed. The value of S may exceed C_s if reinforcement is provided, but for general purposes a limit of $S = 3C_s$ should not be exceeded.

For main beams, if the full loading has been taken, and no reduction made as provided for in Chapter II, the maximum value of S can be taken as $4C_s$. For example, with concrete Mix C the tabulated value of C_s is 60 lb. per square inch; therefore no shear reinforcement is required if the maximum unit shear stress on the section is less than 60 lb. per square inch, and the value of S should never exceed 180 lb. per square inch except for main beams when, subject to the proviso already stipulated, the maximum value of S is 240 lb. per square inch.

2.—Shear taken by Concrete.

In the most conservative practice it is considered that if the value of S exceeds C_s the concrete must be entirely neglected in calculating the shear resistance of a section. Among other engineers the other extreme is practised, and irrespective of the value of S the concrete is always considered as capable of taking C_s lb. per square inch of section.

Rational design would seem to lie between these two extremes, and instead of disregarding the concrete altogether when S is greater than C_s a diminishing effective value of C_s should be taken as S increases, so that the value of the concrete when S equals C_s is C_s lb. per square inch and is nil when S is equal to or greater than $2C_s$. Thus for a concrete Mix C, when S does not exceed 60 lb. per square inch the concrete stress can be taken as 60 lb. per square inch and the concrete takes all the shear; when $S = 90$ lb. per square inch the concrete stress can be taken as 45 lb. per square inch and the concrete takes half of the total shear leaving half to be taken by the shear reinforcement; when S exceeds 120 lb. per square inch the concrete should be neglected and all the shear resisted by shear reinforcement. Generally, therefore, if rF equals the shear to be taken by the steel, between the limits of $S = C_s$ and $S = 2C_s$,

$$r = \frac{S}{C_s} - 1.$$

Values of r are given on *Table No. 33* for various values of S for various mixes.

3.—Shear taken by Binders.

The shear taken by vertical binders is given by

$$V = \frac{A t a}{p} \quad V a$$

where A = cross-sectional area of binder taking into account the number of vertical arms.

t = allowable tensile stress in binders.

a = lever arm of section.

p = pitch or spacing of binders.

V = shear value of binders (taking into account the diameter of the bars from which the binders are made, the allowable stress, and the pitch) = $\frac{A t}{p}$.

Values of V for various stresses, spacings, and sizes of bar for two-armed binders are given on *Table No. 33*, and the shear resistance of any system of binders can be found by multiplying the appropriate value of V by the lever arm of a given section.

The safe value of the maximum working stress to be adopted in calculating the shear resistance of binders should depend on the tensile or shear strength of the concrete, and for concrete of Mix C a general value of 14,000 lb. per square inch is recommended. For Mixes E and F the values of t should not exceed 15,000 lb. and 16,000 lb. per square inch respectively.

In certain cases, such as beams subjected to vibration and impact, the stresses should be less and should be of the order of 12,000 lb. per square inch.

For such beams, also, small diameter binders at close pitches are preferable to larger diameter bars at wider spacings.

The spacing of binders to take any given shear force F is given by

$$p = \frac{Ata}{F}$$

To be effective as shear reinforcement the value of p should not exceed the lever arm, and if the calculated value is greater than a the diameter of the binders can be reduced until a suitable pitch is attained. Although binders may not be required for shear they should always be provided in a beam (except in simple lintel beams), and the maximum spacing should not exceed the effective depth. The minimum diameter of bar suitable for binders is $\frac{1}{4}$ in., and bars over $\frac{1}{2}$ in.

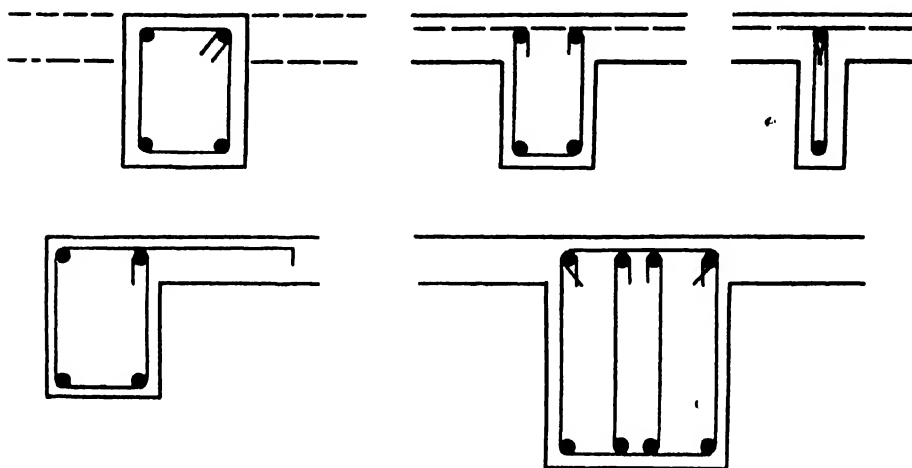


Fig. 39.—Beam Binders.

diameter are usually difficult and costly to provide satisfactorily. When compression steel equal in area to the tension steel is provided in a beam and the calculation for resistance moment is based on the "steel-beam theory," the pitch of the binders should not exceed 6 in. and the diameter should be arranged accordingly.

All binding should be effectively anchored to the top bars in a beam; Fig. 39 indicates a number of approved shapes commonly adopted.

4.—Shear taken by Diagonal Bars.

The shear-resisting value of a bar diagonally crossing a section is equal to the vertical component of the direct tension in the bar; that is,

$$F = At \sin \theta$$

where A = normal cross sectional area of bar

t = maximum allowable tensile stress

θ = angle made with the horizontal.

If the bars are arranged as indicated in Fig. 40(a), this expression evaluates the shear at any vertical section in the length L . If the diagonal bars are dis-

posed as shown in *Fig. 40(b)*, where the topmost bend of any bar is immediately above or to the left of the point of bending up the preceding bar, in effect there are two systems of the diagonal bars shown in *Fig. 40(a)*; hence the shear resistance of any vertical section in the length L_1 is equal to $2F$, and a resistance of F in the length L_2 .

If, as in *Fig. 40(c)*, the line ST drawn at 45° to the horizontal through the midpoint of any section XY intersects any number of diagonal bars, the shear resistance at this section due to these bars alone equals the sum of the shear resistance of each bar.

Tests indicate that diagonal bars are more effective than binders in resisting shear, and therefore a somewhat higher stress can be allowed than for vertical binders. Irrespective of the stress in the principal tensile reinforcement, the

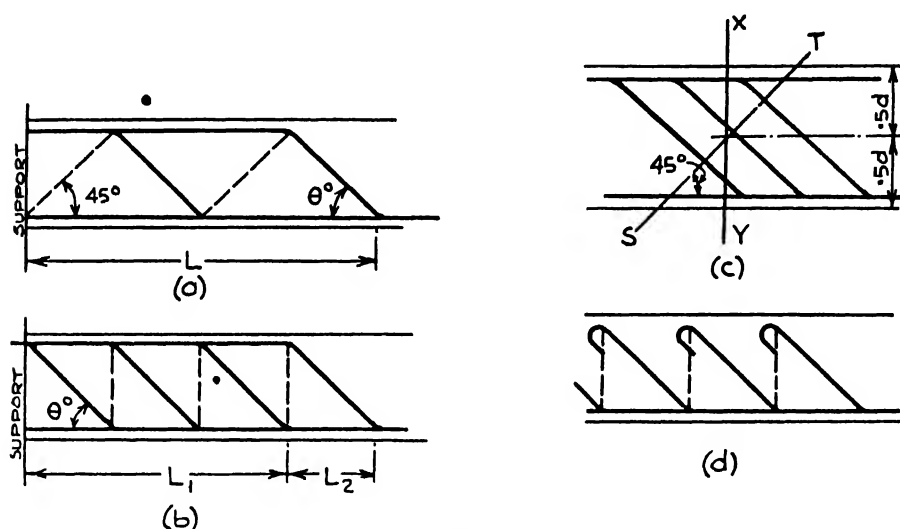


Fig. 40.—Shear Bars.

stress in diagonal shear bars should not exceed 16,000 lb. per square inch for Mix C concrete, 17,000 lb. per square inch for Mix E, and 18,000 lb. per square inch for Mix F; irrespective of the richness of the mix the stress should not exceed the maximum tensile stress in the principal reinforcement. Generally it is preferable to work to stresses less than those stipulated; this is always the case with beams subject to impact and vibration.

Whatever the stress adopted, shear bars should always be anchored horizontally beyond the bends sufficiently to develop the stress adopted. If a bar is simply hooked at the top, as in *Fig. 40(d)*, instead of being bent horizontally, the allowable stress should not exceed 8,000 lb. per square inch and the value of such a system is only equal to F . This design is not advocated for any members except those of little importance.

On *Table No. 33* are tabulated the values of F for bars from $\frac{5}{8}$ in. diameter to $1\frac{1}{2}$ in. bent up at 45° for stresses from 8,000 lb. to 18,000 lb. per square inch.

Diagonal shear bars are usually provided by simply bending up the main

tensile reinforcement, but in so doing inspection must be made to ensure that beyond the bending-up point the bar is not required to assist in providing resistance moment. The limiting points at which bars can be dispensed with as regards bending are tabulated on *Table No. 32*, which applies to beams having up to eight bars in the principal tensile reinforcement. (Although bars can be bent up at the points indicated it does not imply that if they are not bent up they can necessarily be stopped off at these points, since they may not have sufficient bond length from their point of critical stresses. This aspect depends on the rate of change of bending moment, and should be investigated for any particular beam.)

5.—Design Procedure.

Examples of the calculation of the shear resistance at any section of a beam and the design of a section to take a certain shear force are given on page 238. In general the procedure would be as follows:

To calculate the shear resistance at any section, the factor V for the binders is read from *Table No. 33* and by multiplying by the lever arm the shear resistance of the binders is obtained. The resistance of any diagonal bars will be the sum of resistances of each bar as read off the Table, and the total resistance due to the steel, say F_s , is the sum of the resistance of the binders and the diagonal bar resistance. The total shear value will be F_s plus the concrete value, which depends on the magnitude of the applied shear.

In designing a section the first step is to determine S from $S = \frac{F}{ab}$ and then allocate to one of the three following cases:

Case 1, S less than C_s .

Case 2, S greater than C_s and less than $2C_s$.

Case 3, S greater than $2C_s$.

In the first case no shear reinforcement is required. If the problem falls in Case 2, first look up on *Table No. 33* the value of r and find $F_s = rI$. From inspection, decide whether the shear reinforcement shall consist of binders or diagonal bars, or both. If binders alone, then $\frac{F_s}{a}$ gives the value of V required,

and a suitable pitch and diameter can be selected from the Table. If diagonal bars form the principal reinforcement there would generally be some binders, if only a nominal amount, and the value of V for these binders can be found from the Table. Diagonal bars should then be provided to take the shear ($F_s - Va$). If both diagonal bars and binders are provided, the procedure is to combine and adjust the values of Va and ($F_s - Va$) as best suits the problem in hand.

If Case 3 applies all the shear will be resisted by shear reinforcement, and the determination will be as for Case 2, considering $F_s = F$.

In important beams it is advisable to plot the shear resistance diagram for the whole beam on the same base and to the same scale as the shear force diagram, and be assured that the latter is amply covered by the former. For normal beams it is usually sufficient to determine the point at which no shear reinforcement is required and calculate the reinforcement required at the point of maximum shear. Between these two points the intermediate shear reinforcement can generally be allocated by judgment.

CHAPTER XIII

COLUMNS

1.—Column Loads.

THE working stress for which a column should be designed depends among other factors on the precision with which the loading is estimated and upon the method adopted for calculating the maximum stresses. Generally the standard stresses given on *Table* No. 23 should be worked to, but the modifications given in Chapter IX should be taken into account where applicable to column design.

If the loads are approximately ascertained, as when column loading calculations are based on the relative floor areas without reference to the beam arrangement, the standard stress should not be exceeded. When the column loads are calculated carefully from the beam reactions (care being taken that the total of all the beam reactions represents the total load on the floor or other part of the structure in question) the standard stress could be exceeded, especially if the reactions have been judiciously computed from elastic theories, or if the column is in a tall building in which it has been considered that all floors are fully loaded simultaneously. If the usual load reduction factors for multiple-floor buildings (see Chapter II) have been applied, or if the column supports a water tank or other container, then the standard stress should not be exceeded; in fact, if the loads are those likely to be normally experienced but occasionally the column may be subjected to higher loads, or if the bulk of the total load is dead load, the value of the working stress should be maintained low. When the application of the live load is accompanied by impact or vibration, and if the effect of these factors has not already been allowed for in the loading calculations, the working stress should be below the standard stress, the amount of the reduction being dependent on the estimated relative magnitude of the load and the impact effect.

If secondary stresses due to the fixity of beams or to bending due to wind pressure, to minor eccentricities of loading, or to other causes, are ignored, then the standard stress should certainly not be exceeded, but in all cases of single-lift columns the wind moment should be calculated. If all calculable factors are allowed for in determining the stresses, the maximum stress could be 20 per cent. in excess of the standard stress.

For columns carrying direct loads only, the stresses computed on the whole section should not exceed the standard stress irrespective of any of the foregoing conditions allowing increased stresses being fulfilled, since these conditions generally apply only to column stresses calculated on the core section. For columns where the inclusion of a bending moment determines the maximum stress, the whole section should be considered effective.

As will be discussed later, modifications to the standard stresses should also

be made in the case of slender columns or columns with more than the minimum amount of binders or with continuous helical binders.

2.—Columns with Independent Binders.

The effective area of a column section is the area of the concrete plus m times the area (A_c) of the longitudinal reinforcement ; for a rectangular column

$$A = BD + (m - 1)A_c$$

where B and D are the breadth and width of either the core or of the whole section (see *Fig. 41*) depending on which is considered as the effective load-carrying section. The value of m depends upon the mix, and common practice accepts those values given on *Table No. 23* for ordinary calculations notwithstanding the

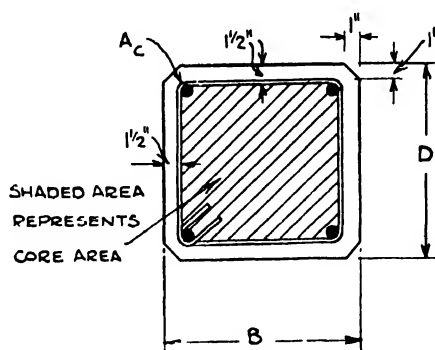


Fig. 41.—Column Section.

fact that such values underestimate the modulus for normally made concrete and give values of m in excess of true values.

The safe axial load carried by a short column is

$$W = cA$$

and thus the concrete stress in a short column carrying a concentric load is given by $c = \frac{W}{A}$, but when the column is subjected to bending due to the eccentricity of the load or to other causes the stress should be calculated by the appropriate method described in Chapter XIV.

In a normal column the amount of reinforcement should be reasonable compared with the size of the column, and the following limits are generally suitable :

Minimum area of longitudinal

reinforcement

= 0.8 per cent. of whole concrete area.

Maximum „ „ „

= 4 per cent. of core area.

Minimum volume of binders

= 0.2 per cent. of the whole concrete volume.

Maximum „ „ „

= 5 per cent. of core volume.

Maximum spacing „ „

= Sixteen times the diameter of the smallest longitudinal bar or three-quarters of the width of the column, whichever is least.

Some general properties of longitudinal reinforcement and binders in columns are given on *Table No. 34*.

When the volume of binders in the form of independent rectangular hoops equals or exceeds 0.5 per cent. of the core volume of the column, the allowable compressive stress calculated on the core section can be increased to c_1 where

$$c_1 = c(1 + 0.1 p),$$

c = standard stress or maximum permissible stress, and

p = volume of binders expressed as a percentage of the core volume.

In no case should c_1 be 50 per cent. in excess of c ; thus the limiting useful percentage of binders would be 5 per cent. of the core section, this value of p giving $c_1 = 1.5c$. Values of c_1 for intermediate values of p are given on *Table No. 34*, together with the corresponding stress increase values allowed by the 1915 London County Council Regulations. The latter allow increased stresses up to a maximum of one-third above the ordinary allowable stress when the volume of independent rectangular binders exceeds 0.5 per cent. of the core volume. The amount of increase for any given percentage depends on the spacing of the binders and conforms to the expression:

$$c_1 = c(1 + ap).$$

When the spacing equals or exceeds 0.6 of the least core width, the value of a is zero. For high percentages of binders the spacing is more or less settled by the limit in convenient diameter of the bar from which the binder is made, and therefore it is not probable that the ratio of spacing to core width would be as high as 0.6 when 5 per cent. of binders is provided. The largest convenient diameter of binder is $\frac{1}{2}$ in., but larger diameters can be used; usually a much smaller bar is all that is necessary, and $\frac{5}{16}$ in. is a very convenient size. The arrangement, and therefore the diameter and spacing, of independent binders depends on the number of longitudinal bars in the column; *Fig. 42* illustrates a variety of arrangements. Those arrangements in which the binder is in the form of links provide almost twice the length per layer (and therefore almost twice the volume for a given diameter) than the simpler types, and are therefore useful for providing large percentages with small diameter binders.

When the spacing of binders varies throughout the length of a column, calculation of the volume of binders should be made in relation to the maximum spacing. Immediately above and below a joint in the longitudinal reinforcement the binders should be spaced at closer centres than elsewhere required in the column. Such joints should only be made at floor levels or beam intersections, unless unavoidably otherwise, as in that position a more secure and more

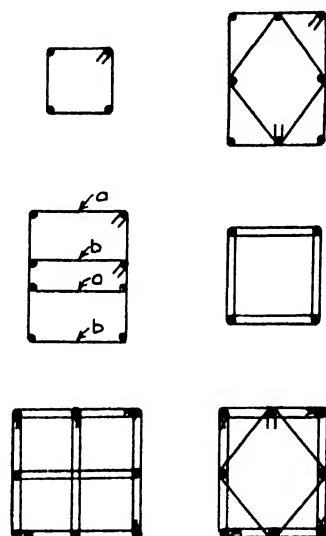


Fig. 42.
Typical Column Binders.

conveniently constructed junction is obtained. Two types of column splices are indicated in *Fig. 43*.

The range of maximum safe loads carried by square columns of various mixes and with maximum and minimum percentages of binders and longitudinal reinforcement is tabulated on *Table No. 35*. These loads have been calculated

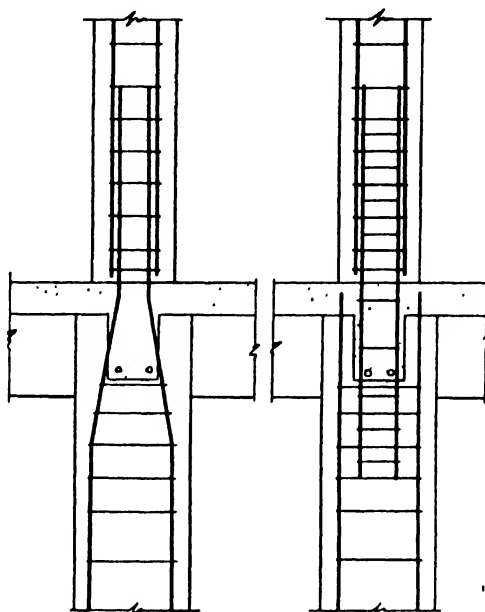


Fig. 43. Alternative Details of Column Joints.

on the standard stresses for the specified mixes, and if these stresses are modified a proportionate modification should be made to the safe load. From this Table a suitable design for a square column can readily be selected to carry a given load.

3.--Slender Columns.

When the ratio of the "free height" of a column to its least overall width exceeds 12 the working stress must be reduced below the standard stress to allow for secondary bending stresses due to slenderness. The value of the reduced stress can be calculated from the expression

$$= c \left(1.4 - \frac{R}{30} \right) \quad (1)$$

where $R = \frac{h}{D}$ = maximum ratio of free height to overall width.

In general the value of R should never exceed about 40, but it will be observed from the values of $\frac{f_c}{c}$ derived from this expression and given on *Table No. 34* that the allowable stress is less than half the standard stress when R exceeds 27 and is zero when $R = 42$. The "free height" of a column depends

upon the end-fixing conditions of the column as well as upon the clear height of column between lateral supports. The end conditions of columns have already been discussed in Chapter V, and when complete fixity or hinged ends are realised the equivalent values of h can be considered as equal to kH , where H = actual clear height of column between lateral supports and k has the following values :

One end fixed and one end hinged,	$k = 1.0$
Both ends hinged,	$k = 1.4$
One end fixed and one end free,	$k = 2.8$
Both ends fixed,	$k = 0.7$

From these values intermediate or indeterminate cases of fixity can be gauged, but usually in a reinforced concrete structure the end conditions are more or less equivalent to one end fixed and one end hinged. For this reason k is taken as unity for these conditions, and formula (1) takes account of R being calculated from $\frac{H}{d}$ for this case. The London County Council Regulations (1915) take "both ends fixed" as the basis (but maintain the relative proportion given above for other conditions) and base the consideration of slenderness of square columns on

$$R_1 = \frac{\text{free height with both ends fixed}}{\text{width of core}}.$$

Alternatively the safe stress on a slender column can be computed by one of the many slender column formulae recommended by various authorities. One such formula in general use reduces to

$$c_2 = \frac{c}{1 + 0.0012R^2} \quad \dots \quad (2)$$

and in this form is applicable to reinforced columns of rectangular section with the general indeterminate end fixing conditions. This formula is derived from the general expression applicable to compression members of any section where the ratio of height to radius of gyration exceeds 20

$$c_2 = \frac{c}{1 + 0.0001 \frac{h}{r}} \quad \dots \quad (2a)$$

where

$$\frac{h}{r} = \frac{\text{free height } (kH)}{\text{radius of gyration}}.$$

The value of $\frac{h}{r}$ should in no case exceed 160, in other words R should not exceed 46 approximately for square or rectangular columns.

From formula (2a) the reduced stress for circular and octagonal slender columns can be calculated; values of $\frac{c_2}{c}$ for various values of $\frac{h}{r}$ are given on Table No. 34. For convenience, the radii of gyration for such sections are also given on this Table, together with the values of $\frac{c_2}{c}$ allowed by the London County Council Regulations (1915).

The German Regulations used to incorporate formula (2a), but now they adopt a simpler relation that expressed in the same symbols is as follows:

Type of Column	Without Helically Bound core	With Helically Bound core
$R = 15$	$\frac{c_1}{c} = 1.00$	1.00
$= 20$	$= 0.80$	0.59
$= 25$	$= 0.57$	0.37

For other values of R the stress reduction factor can be interpolated.

4.—Helically-bound Columns.

By making the binding in a column in a continuous helical form instead of in independent layers the stress to which the bound core can be safely subjected is much increased. Most building regulations give methods of calculating the increase in allowable stress, and for general practice the following conditions are applicable.

The form of the binding must be circular, or practically so, on plan, and the area of concrete effectively stressed must be limited to that contained within the bound core. The minimum volume of helical binding must be 0.5 per cent. of the volume of this bound core before any stress increase can be considered, and the maximum increase should be limited to 100 per cent. more than the safe stress on the column if designed in the ordinary way. To give this maximum increase the amount of binding has to be 3.13 per cent. of the core volume. The increased stress for any less percentages, p , is given by

$$c_1 = c(1 + 0.32p) \quad (3)$$

Values of $\frac{c_1}{c}$ for various percentages are given on Table No. 34. The maximum safe load which a helically-bound column can carry is given by

$$W = [0.7854D^2 + A_c(m - 1)](1 + 0.32p)c \quad (4),$$

where m = the modular ratio as given on Table No. 23 for various mixes

c = standard or maximum permissible stress, and

A_c = area of longitudinal reinforcement which should not be less than 0.8 per cent. of the hooped core nor more than 4 per cent.

The value of c must be adjusted when, because of slenderness or other reasons the standard stress cannot be permitted. On Table No. 35 are tabulated the maximum safe loads on helically-bound columns of various sizes, with the maximum and minimum percentages of longitudinal reinforcement and binding, for various mixes, assuming that the standard stresses can be used. Any adjustment to the permissible stress would produce proportionate adjustment in the maximum safe load.

The allowable increased stress in accordance with the London County Council Regulations (1915) is identical with formula (3) when the spacing of the binders is not more than one-fifth of the core width; for wider spacing the allowable increase is less, and the maximum increase is limited to 50 per cent. of the ordinary permissible compressive stress. The permissible load carried by the column is

found from formula (4), but the value of m must be taken as $\frac{9,000}{c_1}$ which is less than the generally accepted values.

5.—Economical Column Design.

As so many variants enter into the problem of designing a column, it is not easy at first sight to decide which combination leads to the most economical member. Given a constant slenderness factor, and any specified load over 100 tons, consideration of varying richness of mix, of maximum and minimum percentages of binders and longitudinal reinforcement, and of independent and helical binders, would lead to the following general conclusions.

Other things being equal the richer the mix the more economical the column becomes. In square columns the minimum volume of binding and the minimum amount of longitudinal reinforcement produce the cheapest design for a given mix. Also for any one mix a square column is generally less costly than an octagonal column with a helically-bound core.

Taking eight alternative methods of design for columns taking loads from 100 tons to 500 tons, the economical order is as given below; the most efficient type heads the list, and the most costly is about 50 per cent. more expensive than the cheapest.

- Mix F, square with minimum volume of independent binders and minimum area of vertical steel.
- Mix F, octagonal with maximum volume of independent binders and minimum area of vertical steel.
- Mix E, square with minimum volume of independent binders and minimum area of vertical steel.
- Mix E, octagonal with maximum volume of helical binders and minimum area of vertical steel.
- Mix C, square with minimum volume of independent binders and minimum area of vertical steel.
- Mix C, octagonal with maximum volume of helical binders and maximum area of vertical steel.
- Mix C, octagonal with maximum volume of helical binders and minimum area of vertical steel.
- Mix C, square with minimum volume of independent binders and maximum area of vertical steel.

Columns with helically-bound cores are either square, octagonal, or circular; usually an octagonal section is the most economical of these, since the shuttering is less costly than for a circular column and there is less ineffective concrete in the corners than in a square column. The minimum outside size of the column should be 3 in. more than the diameter of the bound core.

Although helically-bound columns are not necessarily the most economical form of column construction, the extra cost is usually offset by the advantages arising from the extra available floor space, especially in multifloor buildings of the warehouse class.

CHAPTER XIV

COMBINED STRESSES

1.—General Principles.

THE maximum stresses in certain structural members, such as arches, walls of rectangular silos and bunkers, tie beams, columns subject to eccentric loads, chimneys, etc., are due to the combined effect of bending and direct force. The latter may be either a pull or a thrust, and the method of determining the magnitude and distribution of the stress on the section depends on the nature of the direct force and the relative magnitude of the moment and the force. There are three principal cases to consider :

(i) When the direct force is a thrust and the resultant stresses on the section are wholly compressive.

(ii) When the direct force is a pull and the resultant stresses on the section are wholly tensile.

(iii) When the direct force is either a pull or a thrust and both tensile and compressive forces are produced on the section.

The effect of a moment M and a direct force N acting simultaneously on a section is equivalent to a direct force acting at a distance e from the centroid of the stressed area where $e = \frac{M}{N}$. For convenience it is usual to consider that this eccentricity e is measured from the centroid of the concrete area, and the error involved in this approximation is negligible. In some problems the actual eccentricity of the load on the member is given, in which case, if the resultant moment is required, it can be determined from the equation $M = eN$.

The value of e relative to the dimensions of the section determines into which of the three cases the problem falls. For problems coming within the limits of Case (i) the maximum and minimum stresses are calculated by adding and subtracting respectively the stresses due to the direct force alone and to the moment alone. The limit of applicability of this case is reached when the tensile stress that would be produced by the bending moment alone (considering the whole concrete section as well as the steel fully effective) equals, or slightly exceeds in some cases, the compressive stress due to a concentric load N . With rectangular sections with normal percentages of reinforcement these limiting conditions are approached when the ratio of $\frac{e}{D}$ lies between 0.167 and 0.5 where D equals the total depth of the section. For any section the limiting value of e equals $\frac{Z}{A}$, where A is the effective area of the section expressed in concrete

units and Z the modulus of the effective section (also expressed in concrete units) measured about an axis passing through the centroid of the section.

When N is a pull the stresses on the section are entirely tensile and the problem falls under Case (ii), when the ratio $\frac{e}{f}$ is less than 0.5 where f is the distance between the centroids of the reinforcement on opposite faces of the section. In this case the tensile resistance of the concrete is entirely neglected.

When Case (i) is applied to a particular problem with N a thrust and excessive tensile stress is produced in the concrete, or when Case (ii) is applied to a problem with N a pull and compressive stresses are produced, the problem should be considered as coming within the limits of Case (iii), and various methods of calculation of the stresses for this case have been devised by various engineers. Any direct method of solution of such problems becomes complicated as an

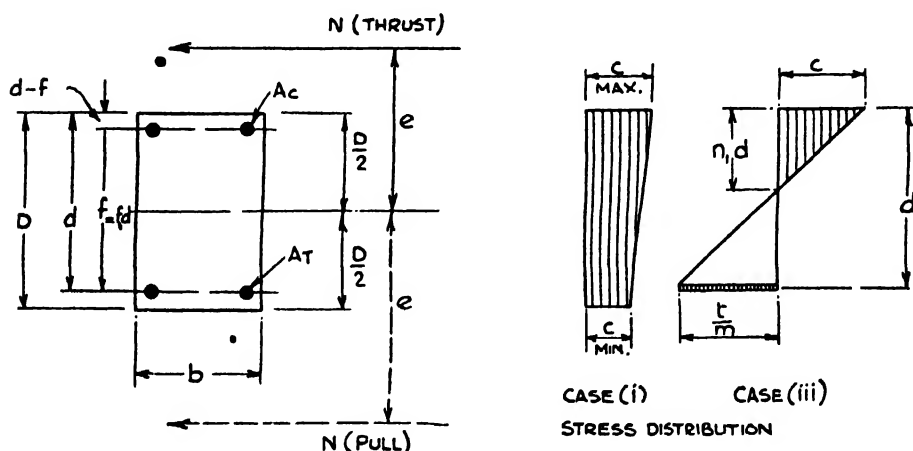


Fig. 44.—Rectangular Section.

exact analysis involves the solution of a cumbersome cubic equation, and any ready means of computation would necessitate the possession of an impracticably large number of curves if all the probable variations of the components of the equation are to be allowed for. The comprehensive method advocated in this Chapter involves the assumption of a trial factor that can be checked and adjusted, and in common with all other methods this method is based on the two conditions of equilibrium that

- (a) the algebraic sum of all forces acting on the section equals zero, and
- (b) the algebraic sum of the moments of all forces acting on the section equals zero.

For rectangular sections, or sections reducible to equivalent rectangles, the notation will be in accordance with that indicated on Fig. 44, and for the consideration of any irregular section the notation will be as shown on Figs. 45 and 46. When there is no reinforcement on the compression face of the section,

the factor A_c in the formulæ will be written zero and consequent simplifications will follow.

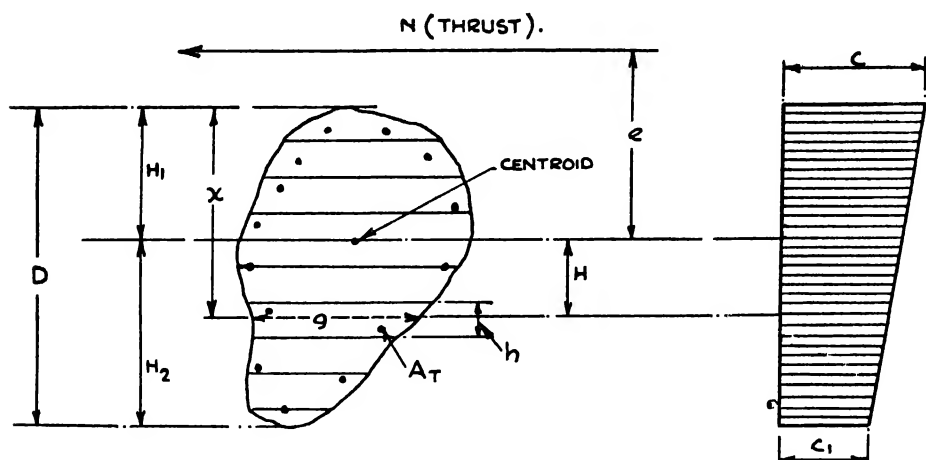


Fig. 45.—Irregular Section.

For convenience an abstract of the methods of stress determination for any given rectangular section subject to combined bending and direct pulls or thrusts is given on *Table No. 36*, together with values of certain of the factors involved

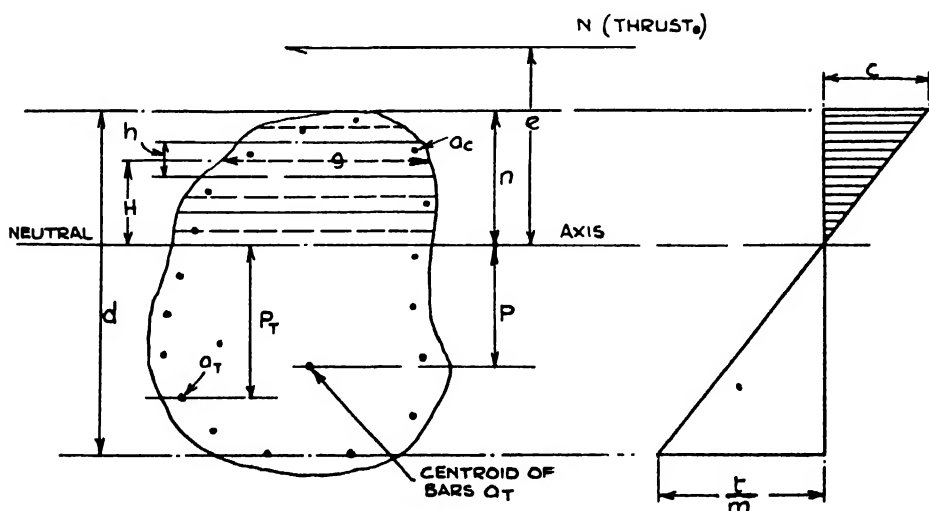


Fig. 46.—Irregular Section.

in the calculation. For large values of $\frac{e}{D}$ (say exceeding 1.5) an approximate method can be followed that gives the resultant stresses on the section with a reasonable degree of accuracy.

2.—Rectangular Section subject to Bending and Compression where e is less than $0.167D$.

In this case, with any proportion of reinforcement, compressive stresses only are developed on the section and the maximum and minimum values are given by

$$c = \frac{N}{A} \pm \frac{M}{Z}.$$

For a rectangular section the values of A and Z are given by

$$\begin{aligned} A &= A_1 + A_2 \\ \text{where } A_1 &= (m - 1)(A_c + A_T) \\ \text{and } A_2 &= bD. \end{aligned}$$

$$\text{In this case } Z = \frac{2A_1(0.5D - d + f)^2}{D} + \frac{A_2 D}{6}.$$

The design of sections for this case involves the assumption of trial scantlings and reinforcement areas.

3.—Rectangular Section subject to Bending and Compression where e is greater than $0.167D$ and less than $0.5d$.

With no reinforcement (a plain concrete section) tension would be developed in one face of the section when e exceeds $0.167D$, but with increase in the proportion of reinforcement the ratio of e to D can be considerably increased before tensile stresses are developed. The actual limiting value of $\frac{e}{D}$ depends on the amount of A_c and A_T and the relative values of f , d , and D . Cases where $\frac{e}{D}$

lies between 0.167 and 0.5 should be first treated as if $\frac{e}{D}$ were less than 0.167 , and if no tensile stress is developed the stresses calculated by this method will be the actual stresses developed. Even if a small amount of tensile stress is developed, treatment as in Paragraph 2 is justified so long as the tensile stress in the concrete for the worst combination of M and N does not exceed about one-sixth of the allowable compressive stress.

If the stress calculated in accordance with Paragraph 2 exceeds this limit the tensional resistance of the concrete must be ignored, and the stresses calculated as in Paragraph 4.

4.—Determination of Stresses for a Rectangular Section subject to Bending and Compression where e is greater than $0.5d$ and less than $1.5d$.

This is the general case, when tension on the concrete is ignored, and is applicable to sections with and without compression reinforcement and with any cover ratio or value of $\frac{d-f}{D}$ and any ratio of elastic moduli.

The first steps are to take a trial position for the neutral axis by assuming n_1

the neutral axis factor (see observations in Paragraph 13) and calculate the maximum stresses c and t from the formulæ

$$c = \frac{NF}{Gbd + Kf_1} \quad (1)$$

$$t = \frac{c(J + K) - N}{A_T} \quad (2)$$

where the factors F , G , K , and J have the following signification

$$F = \frac{e - 0.5D}{d} + 1$$

$$G = \frac{n_1}{2} \left(1 - \frac{n_1}{3} \right)$$

$$K = HA_c \text{ where } H = \frac{(m - 1)(n_1 + f_1 - 1)}{n_1}$$

$$J = 0.5bdn_1.$$

The ratio $\frac{t}{c}$ calculated from formulæ (1) and (2) should coincide with or be very nearly equal to the stress ratio corresponding to the trial value of n_1 , and the appropriate modular ratio as determined from

$$n_1 = \frac{1}{1 + \frac{t}{cm}}$$

In the first trial there may be some discrepancy between the two values of n_1 , in the event of which the factors G , H , and J should be amended and a second calculation made for c and t ; this should give a satisfactory value of n_1 . On Table No. 36 values of n_1 for various values of $\frac{t}{c}$ for various modular ratios are tabulated, together with values of G and H . The value of F is constant for a given section with given ratio of $\frac{M}{N}$.

When the section is only reinforced in tension formulæ (1) and (2) become simplified to

$$c = \frac{NF}{Gbd}$$

$$t = \frac{cJ - N}{A_T}$$

since K equals zero.

5.—Design of Rectangular Section subject to Bending and Compression where e is greater than $0.5d$.

If the member being designed is a slab, mid-span section of beam, etc., and need not have compression steel, suitable concrete sizes and tension steel area can be determined by first assuming an appropriate value of d (and therefore D)

and finding the minimum breadth of section required to keep the concrete stress within the specified limit from the relation

$$b = \frac{NF}{cdG}.$$

In this instance G is calculated (or read off *Table No. 36*) from the permissible values of t and c . The area of tension steel required is given by

$$A_T = \frac{cJ - N}{t}.$$

If the value of b thus obtained is unsatisfactory, an adjustment to the assumed value of d might give a reasonable section; but if no suitable value of b is obtained within rational or permissible limits of d , then a convenient section can be found (a) by reducing the working stress in the tensile reinforcement, thereby increasing the area of concrete in compression; (b) by adding compression steel; or (c) by combining (a) and (b).

If reinforcement is added to aid the compressive resistance of the section, or if the member is such that conventional design or other considerations require the provision of compression steel (for example, columns, piles, support section of beams, sections subject to reversal of flexure), it is necessary to assume (or fix from other considerations) suitable values of b as well as d . With these values, and with the ratio of the allowable stresses in tensile steel and concrete, the factors F , H , J , and G can be calculated or read off *Table No. 36*. The amount of compression steel required is given by

$$A_C = \frac{P}{Hf_1}$$

$$\text{where } P = \frac{NF}{c} - bdG.$$

The area of tension steel can be found from

$$A_T = \frac{c(J + HA_C) - N}{t}.$$

If A_C should exceed A_T both values should be adjusted by reducing the value of tensile stress or by modifying the concrete dimensions.

6.—Rectangular Section subject to Bending and Compression where e is greater than $1.5d$.

When the eccentricity of the thrust is large compared with the dimensions of the sections the stresses will be primarily determined by the bending moment, the thrust only producing a secondary modification. In this case the stresses should first be calculated for the bending moment on the section alone as described in Chapter XI. The actual stresses can then be determined by adding c_1 to the maximum compressive stress in the concrete, and by deducting mc_1 from the maximum tensile steel stress, where

$$c_1 = \frac{N}{A_T m + bn + A_C(m - 1)}.$$

The design of sections that are within the limits of $\frac{e}{d}$ applicable to this case can be readily carried out by the method outlined in Paragraph 5.

7.—Any Section subject to Bending and Compression where e is less than $\frac{Z}{A}$.

Given any reinforced concrete section subject to a direct force and a moment, the relative magnitude of $\frac{M}{N}$ being small, the first step in the determination of the stresses is to find the effective area of the section and the modulus Z of the section about an axis passing through the centroid of the section. When the section is symmetrically reinforced about the axis of bending, and is either rectangular (bending about a diagonal), circular, octagonal, or annular, etc., the values of A and Z for the concrete section are readily obtained from the data given on *Table No. 39*. The additional area A_A and additional modulus Z_A due to the reinforcement are given by

$$A_A = (m - 1)\Sigma A_T$$

$$Z_A = \frac{2(m - 1)}{D} \Sigma A_T x^2$$

where A_T = area of a bar or group of bars placed at a distance x from the axis of bending.

When the section is irregular, as in *Fig. 45*, the values of A and Z can be determined by dividing the section into a number of narrow horizontal strips and by calculating the position of the centroid of the section, the effective area of the section, and the moment of inertia of the section. Thus, referring to *Fig. 45*

$$\text{Effective area of each strip} = A_T(m - 1) + gh = a$$

$$\text{Total effective area of section} = \Sigma a = A$$

$$\text{Position of centroid: } H_1 = \frac{\Sigma ax}{\Sigma a}$$

$$\text{Moment of inertia: } I = \Sigma a(0.083h^2 + H^2).$$

If h is small compared with g the term $0.083h^2$ can be neglected.

The maximum compressive stress is given by

$$c = \frac{N}{A} + \frac{H_1 M}{I}$$

and the minimum compressive stress is given by

$$c_1 = \frac{N}{A} - \frac{H_2 M}{I}.$$

In extreme cases c_1 may be negative; this is permissible if the magnitude of this tensile stress on the concrete does not exceed say one-sixth of the allowable compression stress.

8.—Any Section subject to Bending and Compression where e is greater than $\frac{Z}{A}$.

When the value of c_1 as determined in the preceding paragraph, exceeds the permissible negative value, or in those cases where e is so large compared with D

that the simultaneous production of compressive and tensile stresses can be assumed at the outset, the tensile stress on the concrete should be neglected and the total tension should be resisted by reinforcement only. In this case it is necessary to select a position for the neutral axis, either after consideration of the maximum permissible stresses or otherwise, and to plot this axis parallel to the line of action of the direct force on a diagram of the given section drawn to scale, as indicated in *Fig. 46*. Next find the centre of gravity of the forces represented by the tension in the reinforcement below the neutral axis :

Total area of tensile reinforcement, $A_T = \Sigma a_i$

Position of centre of gravity, $p = \frac{\Sigma a_i p_i^2}{\Sigma a_i p_i}$

where a_i = sectional area of individual bars or groups of bars.

If all the bars are equal in diameter, then

$$p = \frac{\Sigma p_i^2}{\Sigma p_i}$$

The next stage is to divide the compression area above the neutral axis into a number of narrow horizontal strips. The depth h of each of these strips need not be the same, as any regularity in the conformation of the section may suggest convenient subdivisions. When the strips are of equal depth, or when the section is symmetrical or is hollow, the necessary modifications and simplifications should be readily perceivable.

For each strip determine the factors Q and S , where

$$Q = H[gh + (m - 1)a_c]$$

$$\text{and } S = H + p$$

a_c being the total area of reinforcement in each strip.

In previous cases the eccentricity of the direct force has been measured about the centroid of the concrete area ; this method can also be adopted in the case under review, but the centroid calculation can be avoided without serious sacrifice of accuracy if e is taken to represent the distance from the neutral axis to the line of action of the direct force. With this approximation the calculated maximum stresses are given by

$$c_1 = \frac{Nn(e + p)}{\Sigma QS}$$

$$t_1 = \frac{d - n}{pA_T} \left(\frac{c_1}{n} \Sigma Q - N \right)$$

The value of n corresponding to these stresses should be compared with the assumed value, and adjustment made if necessary (see Paragraph 13).

The design of sections falling within the limits of this case is most readily determined by assuming (or otherwise determining) the scantlings of the section and a disposition for the reinforcement. The position of the neutral axis would generally be such as would correspond to the permissible maximum stresses, and c_1 and t_1 would be found by the method and formulæ already given. If c_1 is not appreciably less than the permissible concrete stress c , the maximum

compared with the trial value of 13 in., and considering the magnitude of the stresses a second trial should not be necessary as any reasonable increase in n will only slightly modify the concrete stress and will still less affect the steel stress. However, the problem will be re-worked with $n = 14.5$ (that is, $1\frac{1}{2}$ in. more than previously) to indicate a ready method of adjustment. As before

$$A_T = 2.77 \text{ sq. in.}$$

$$\text{and } p = \frac{2(3\frac{1}{2}^2 + 9\frac{1}{2}^2 + 15^2 + 18.5^2) + 19.5^2}{2(3\frac{1}{2} + 9\frac{1}{2} + 15 + 18.5) + 19.5} = 15.2 \text{ in.}$$

Add the additional strip E ; the values of $\frac{Q}{H} = gh + (m - 1)a_c$ for the original strips A, B, C , and D will remain, and therefore the revised tabulation becomes

Strip	g	h	gh	$(m-1)a_c$	$\frac{Q}{H}$	H	Q	S	QS
A	.	.			51	13	663	28.2	18,700
B	.	.			76	10	760	25.2	19,150
C	.	.			57	8	456	23.2	10,580
D	.	.			59	$3\frac{1}{2}$	207	18.7	2,870
E	$12\frac{1}{2}$ in.	$1\frac{1}{2}$ in.	18	nil	18	0.75	14	15.95	223
							$\Sigma O = 2,100$	$\Sigma OS = 51,523$	

$$c_1 = \frac{50,000 \times 14.5 \times 35.2}{51,523} = 495 \text{ lb. per square inch.}$$

$$t_1 = \frac{19.5}{15.2 \times 2.77} \left(\frac{495}{14.5} \times 2,100 - 50,000 \right) = 10,000 \text{ lb. per square inch.}$$

The value of n corresponding to these stresses is 14.5 in., the value assumed. Therefore these second values of c_1 and t_1 will be the stresses produced by the given moment and thrust.

In Paragraph 8, Chapter VIII, some factors are given for the direct design of annular sections such as that just investigated.

9.—Any Section subject to Bending and Tension where e is less than $0.5f$.

If f is the distance between the centroids of the reinforcement on opposite faces of a given section, and if e is measured about an axis midway between the two centroids, then if e is less than $0.5f$ the stresses are wholly tensile. The average stress in the reinforcement on the face nearer the line of action of N is given by

$$t_1 = \frac{N(e + 0.5f)}{A_T}$$

where A_T = total area of reinforcement on this face.

The average stress in the reinforcement on the face remote from the line of action N is given by

$$t_2 = \frac{N(0.5f - e)}{A_G}$$

where A_G = total area of reinforcement on this face.

The maximum stress depends upon the distance of the farthest bars in any group from the centroid of that group.

These two expressions can be readily rearranged to give the area of reinforcement required in design problems.

10.—Rectangular Section subject to Bending and Tension where e is greater than $0.5f$ and less than $1.5d$.

This is the general case, and the method of treatment is similar to that given in Paragraphs (4) and (5), modifications being introduced to allow for the difference between a direct thrust and a direct pull.

In determining the stresses produced by given values of M and N on a certain section, first assume a neutral axis factor (see Paragraph 13) and calculate the maximum stresses from

$$c = \frac{NL}{bdG + Kf_1} \quad (1a)$$

$$t = \frac{c(J + K) + N}{A_T} \quad (2a)$$

where the factors L , G , K , and J have the following signification:

$$L = \frac{e + 0.5D}{d} - 1$$

$$G = 0.5n_1 \left(1 - \frac{n_1}{3} \right)$$

$$K = HA_c \text{ where } H = \frac{(m - 1)(n_1 + f_1 + 1)}{n_1}$$

$$J = 0.5bdn_1$$

The ratio of $\frac{t}{c}$ as derived from (1a) and (2a) should be reasonably equal to that corresponding to the trial value of n_1 , and in the event of appreciable discrepancy a second trial should be made and the factors G , H , and J recalculated. On Table No. 36 values of the various factors involved have been tabulated.

When the section is reinforced in tension only ($A_c = 0$), the formulæ (1a) and (2a) reduce to

$$c = \frac{NL}{bdG}$$

$$t = \frac{cJ + N}{A_T}$$

When designing sections to take a moment and pull the procedure should be as follows: If the provision of compression steel is not anticipated, assume d (and D) and determine the minimum breadth from

$$b = \frac{NL}{cdG}$$

where G is calculated (or read off Table No. 36) from the permissible stresses. If the value of b is unsatisfactory, d should be adjusted or compression steel added. The area of the tension steel required is given by

$$A_T = \frac{cJ + N}{t}$$

In the case of singly-reinforced slabs subject to a moment and direct pull (walls and bottoms of tanks and bunkers, etc.) a simple procedure is the following.

Determine the eccentricity $e = \frac{M}{N}$ and evaluate $e_s = e + \frac{D}{2} - d$.

Find $A_{T_1} = \frac{Ne_s}{at}$ and $A_{T_2} = \frac{N}{t}$. The total tensile reinforcement required $= A_{T_1} + A_{T_2}$. The value of d (and D) is that required to resist M acting alone, and the value of the lever arm, a , is approximately that corresponding to the maximum permissible stresses.

In designing sections in which compressive reinforcement is required (or is usual) first assume or otherwise determine suitable values of b and d , and with these values and the allowable maximum stresses determine

$$A_c = \frac{Q}{Hf_1}$$

$$\text{where } Q = \frac{NL}{c} - bdG.$$

The tensile reinforcement is found from

$$A_T = \frac{c(J + HA_c)}{t} + \frac{N}{t}.$$

If it is necessary to reduce the amount of compression steel, this can usually be effected by reducing t and thus increasing n . Usually in problems of bending and direct pull the tension is the deciding factor, and a more economical section can be arrived at by decreasing the working stress on the concrete.

11.—Rectangular Section subject to Bending and Tension where e is greater than $1.5d$.

Stress determination for a given section falling within the limits of this case is similar to that given in Paragraph 6, and the stresses are computed as if the section were subjected to the moment acting alone. Evaluate

$$c_1 = \frac{N}{A_{Tm} + bn + A_c(m-1)}$$

and deduct c_1 from the maximum concrete stress, and add mc_1 to the steel tension stress to find the actual maximum stresses in the concrete and steel.

Design problems are best treated by an adaption of the approximate method for singly reinforced slabs outlined in Paragraph 10, in which the depth, breadth, and amount of compression steel (if any) are determined as if the moment were acting alone, and

$$e_s = e + 0.5D - d$$

is evaluated and the area of tension steel required found from

$$A_T = \frac{N}{t} \left(\frac{e_s}{a} + 1 \right)$$

where a = lever arm of section designed for pure bending.

12.—Any Section subject to Bending and Tension where Compressive and Tensile stresses are simultaneously produced.

With modifications to allow for N being a pull instead of a thrust, the method given in Paragraph 8 can be applied to problems of stress determination on any given section that cannot be treated as rectangular. A trial position for the neutral axis is taken and the part of the section above the neutral axis is divided into a number of narrow horizontal strips. The value of A_T and p and the summations ΣQ and ΣQS are determined and substitution made in the modified formulæ

$$c_1 = \frac{Nn(e - p)}{\Sigma QS}$$

$$t_1 = \frac{d - n}{pA_T} \left(\frac{c_1}{n} \Sigma Q + N \right).$$

If the value of n corresponding to the stresses c_1 and t_1 is approximately equal to that assumed, these stresses are approximately the maximum stresses produced by the applied moment and pull. If the discrepancy between the calculated and trial values of n is serious, a second trial value of n must be selected and the summations revised by taking in a greater or less number of strips to accord with the revised n .

13.—Position of Neutral Axis.

The accuracy of the results of certain of the methods advocated in the foregoing paragraphs and the labour entailed in arriving at these results depend upon the accuracy with which the position of the neutral axis is selected. From a preliminary consideration of the section and forces acting thereon it is possible to assume a value for n very close to that corresponding to the calculated stresses. The maximum stresses for which the section has been ostensibly designed may indicate a reasonable value of n for the first trial, or consideration can be given to the stress ratio for pure bending as determined by the percentage of tension steel. The value of n selected should differ from the pure bending value thus:

Bending and compression: the selected value of n should be greater than the pure bending value, and will increase with decrease in the value of $\frac{e}{D}$.

Bending and tension: the selected value of n should be less than the pure bending value, and will decrease with increase in the value of $\frac{e}{D}$.

If there is such a discrepancy between the assumed value of n ($=n_i$) and that corresponding to the calculated stresses ($=n_e$) that it is necessary to select another value ($=n_s$) intermediate between the n_i and n_e , the following considerations should be borne in mind:

Bending and compression: the value of n_s should be nearer n_e than it is to n_i .

Bending and tension: the value of n_s should be nearer n_i than it is to n_e .

If the values of n_t and n_c do not differ by more than 10 per cent. it is unnecessary to recalculate the stresses with a second trial value of n , since more direct adjustments can be made. If n_c is within 5 per cent. of n_t the calculated stresses are well within 5 per cent. of the actual stresses, and if n_c is between 5 per cent. and 20 per cent. of n_t the actual concrete stress c can be safely approximated thus:

$$c = 0.5c_1 \left(1 + \frac{n_t}{n_c} \right)$$

where c_1 is the calculated concrete stress.

The stress in the steel is affected to a less degree by small variations in the value of n .

By way of illustration, applying this adjustment to the example worked in Paragraph 8, we get

$$n_t = 13 \text{ in.}; \quad n_c = 15.2 \text{ in. } (= 117\% \text{ of } n_t);$$

$$c_1 = 569 \text{ lb. per square inch}$$

$$\therefore c = 0.5 \times 569 \left(1 + \frac{13}{15.2} \right) = 528 \text{ lb. per square inch compared}$$

with 495 lb. per square inch by the more involved calculation.

Applying this method of adjustment to example (c) given on the page facing Table No. 36, we get

$$n_t = 0.55 \times 16.5 = 9.1 \text{ in.}; \quad n_c = 0.49 \times 16.5 = 8.1 \text{ in.}$$

$$(= 89\% \text{ of } n_t)$$

$$c_1 = 412 \text{ lb. per square inch.}$$

$$\therefore c = 0.5 \times 412 \left(1 + \frac{9.1}{8.1} \right) = 436 \text{ lb. per square inch compared with}$$

426 lb. per square inch by the second trial position calculation.

CHAPTER XV

SPECIFICATIONS, QUANTITIES, AND COST ESTIMATING

1.—Associated Materials.

It was stated in the opening Chapter that it was necessary for the concrete engineer to be conversant with the properties of building materials other than those in which he specialises, and it is here emphasised that where it is expedient to incorporate such other materials in the production of an economical structure the engineer should be in a position to specify the quality of material he requires. The leading features of some of the more common structural materials are given in the succeeding paragraphs, while the essential properties of cement, stone, sand, and steel have been discussed in some detail in Chapters IX and X.

There are many special proprietary materials in use in up-to-date building practice, and particulars of various floor coverings, roofing materials, partitions, walling, ceiling materials, and glazing are best obtained from the literature provided by the manufacturers.

2.—Timber : Shuttering, Joinery, and Piles.

Timber enters into concrete construction mainly in the form of temporary shuttering for moulding the concrete to the required shape, but it also has to be considered in excavation work, temporary shoring, and in the forms of joinery, piles, and fenders. All timber should be of the best quality obtainable of the particular kind required, and should be sound and straight grown, free from sap, shakes, loose knots, wormholes, or other defects. Seasoning of wood is of great importance, whether for use in permanent or temporary work, and all timber should be kept in air six weeks before being made up; oak should have been felled for twelve months before use. Pitch-pine should weigh not less than 45 lb. per cubic foot when dry.

Shuttering is generally exposed to the weather for some length of time, especially when repeatedly used for exterior walls or for slabs, but this exposure combined with contact with wet concrete should not produce warping if the material is properly seasoned. The scantlings of the various boards, props, cleats, etc., that are assembled to form a unit of any part of a cast-in-situ or pre-cast structure should be sufficiently generous to provide a safe and rigid construction that will not deform when filled with wet concrete. All timber in contact with concrete should be specified as wrought on two edges and one face, the material being assembled with the unwrought face outwards. The thickness of boarding for beam and column sides, walls, and slabs should be not less than $1\frac{1}{4}$ in., but for walls, if there is much repetition, the extra cost of providing $\frac{3}{4}$ -in.

wrought tongued and grooved boarding is sometimes justified by the production of a face free from defects due to badly aligned boards.

Wall panel forms and column boxes should be constructed to allow the concrete to be placed in lifts not exceeding 3 ft. to 4 ft., depending on the thickness of the work. Column boxes are usually built the full height of the column on three sides and so arranged that the fourth side can be built up in 3-ft. increments as concreting proceeds. The outside surfaces of the forms for columns and walls should be constantly subjected to sharp hammer taps during the placing of the concrete, especially if the depth being concreted exceeds 3 ft.

The bottom boards of beams are usually given a slight camber to counteract the slight deflection due to the weight of the wet concrete; one inch per 20 ft. of span is ample for this camber.

The various parts of the timber falsework should be so assembled as to allow striking to proceed in the recognised manner. For floors, the beam sides should be stripped first, followed by the slab soffit boards, and last by the beam-bottom boards and props. Particular care should be taken to ensure that the young concrete is not jarred in any way during the process of striking the shuttering; aids to this end include coating the inside of the forms with mould oil and leaving all nails "proud" of the wood to assist in easy withdrawal. A proper interval should be allowed to elapse between pouring the concrete for any particular member and striking the shuttering for that member. The usual specified times when using normal-hardening cements are as follows.

Beam sides	3 days
„ bottoms	3 weeks
„ props	4 weeks
Slab bottom boards	2 weeks
„ props	3 weeks
Column sides	2 days
Pile sides	2 days

Piles should not be moved off their bottom boards for 14 days. Props under suspended bunker bottoms should not be removed before three weeks after the pouring of the walls and beams from which the bottoms hang. These times should be considered as normal periods, but reductions are possible with discretion if precautions are taken to ensure proper curing. If cold weather has been experienced during the curing period the stated times should be increased. If rapid-hardening cement has been employed the periods can be reduced to a third or a quarter of those stated, with a minimum of 24 hours.

Having regard to the permanence of concrete, the use of timber for joinery work incorporated in the structure should be governed by the best practice in that trade. Generally Baltic redwood is a satisfactory medium-priced material for doors, partitions, etc., of non-residential buildings and for structures where decorative woodwork is not required. Waney edges should be prohibited, and all joints and ends should be treated with two coats of red lead and boiled oil. Wrought work should be knotted and primed, nails well punched in, nail holes topped with the best oil putty, and the work thoroughly rubbed down, and given three coats of paint.

The most suitable timbers for fenders and rubbing-pieces on wharf walls and

jetties are greenheart, Oregon pine, or elm. Such members are attached to the concrete framework by wrought-iron straps and mild-steel bolts, with a piece of hardwood packing between the concrete and the softer timber. Longitudinal rubbing pieces are usually attached to the vertical fender piles by oak trenails, and all exposed corners should be rounded off.

Timber piles for wharf and jetty construction and for foundation work have certain advantages over reinforced concrete piles, but when they are in such positions that they are alternately wet and dry, as in structures in tidal water, these advantages are often offset by the rapid and serious deterioration that takes place. In the first instance timber piles of moderate lengths are cheaper than reinforced concrete piles, and, even for long piles, short lengths can be readily spliced together with a minimum of delay. The delay that occurs while reinforced concrete piles are maturing is also avoided, although with rapid-hardening cements and with a well-planned construction programme this cause of delay is almost eliminated in modern practice. Although more resilient when subjected to blows from vessels, a timber pile wears away more rapidly due to abrasion by shingle, and the action of worms and rot render the life of timber short and uncertain. On the other hand a blow from a vessel may cause undetected cracks in a reinforced concrete pile and thus allow sea water to reach the reinforcement and cause corrosion.

Just as creosoting assists in retarding deterioration of timber, so the application of a coating of tar is sometimes added to concrete piles with the object of preservation. Timber piles must be bound at the head with an iron ring and, unless driven through soft mud only, the lower end should be pointed and shod. The timber dollies for fitting into pile helmets are usually of elm and should be so placed that the blows are received along the grain. •

3.—Brickwork and Masonry.

Brickwork may be incorporated in a reinforced concrete structure either for load-carrying purposes or to act merely as a facing or filling material. The nature of the facing and the importance of the ultimate appearance of the structure will determine the excellence or otherwise of the materials and workmanship of brickwork filling for wall panels in a concrete framework; but when the brickwork is carrying loads, providing supports for floor slabs, beams, or stairs, it is essential that certain standards should be maintained. These standards, the purposes of which are fairly obvious, include the stipulation that the bricks should be of the best quality of the type specified, new, of good shape, and uniformly hard. They should be free from stones and flaws and should be well burnt. The texture should be such that the bricks do not absorb more than 10 per cent. of their weight of moisture when saturated, and immediately before laying they should be well soaked in water. It is essential that they should be set in Portland cement mortar; the joints should be $\frac{1}{4}$ in. thick and be either "struck," or, for external work, "raked and pointed." Old English or Flemish bonds are the most suitable, and the face of walls and piers should be plumb. The type of brick used depends upon its purpose. For sustaining high pressures blue bricks are the best, but for ordinary wall work good quality London stocks are satisfactory unless special facing work is required.

Natural stone used for constructing supports for reinforced concrete structures should be fine grained, hard, sound, free from excessive discoloration marks, rust veins, and from heads and shakes and other imperfections. Stones cut from the lower beds of quarries are superior. Successive courses of stones in the work should be well bedded and laid on their natural bed. Exterior work should be neatly pointed, and one-sixth of the area should be composed of "headers" that penetrate three feet or right through the wall or pier being constructed. For purely facing work it is sufficient if the minimum thickness is 5 in. with one-third of the area 9 in. thick with sufficient ties into the body of the concrete backing.

4.—Iron, Steel, and other Metals.

Apart from its use as reinforcement mild steel may be used in conjunction with concrete in composite structures. If in the form of joists, stanchions, plate or lattice girders, the quality of the material should be in accordance with the British Standard Specification for mild steel for girder bridges. The sections should be well and cleanly rolled and straight, and be free from scales, blisters, laminations, cracked edges, and defects of any sort. The edges of plates of girder flanges should be planed, and the ends of sections should be cut square and neatly dressed. All rivet holes should be drilled, although if the piece is less than $\frac{3}{4}$ in. thick punching and reamering is sometimes permitted. The B.S.S. provides for rivets to be made from "A" steel, and tension tests should show an ultimate strength of 25 to 30 tons per square inch and should show a tough and silky fracture. The rivets should be capable of having the head flattened out to $2\frac{1}{2}$ diameters when hot, and should be able to be bent double when cold without fracture. It is usual to specify structural steelwork to be painted one coat before erection, one coat before leaving the works, and two or three coats erection.

A use for cast steel in reinforced concrete work is found in its employment for hinges in arch bridges and for pile helmets. The castings should be thoroughly annealed, and in order to prove freedom from cracks they should be hung in chains and hammered. Cast steel for any permanent work should generally be in accordance with the B.S.S. for this material, the tensile strength of which should be 26 tons per square inch with elongation not less than 20 per cent. A cold bend test should be made on a 1-in. diameter piece 9 in. long which should be bent through 120 deg. to a radius of $2\frac{1}{2}$ in. without signs of fracture.

Wrought iron should be well rolled, tough, fibrous, and uniform in character. The use of this material in concrete work is limited to its employment for the straps of pile shoes, for the straps connecting timber fenders, etc., to concrete wharves and similar purposes. The B.S.S. stipulates a minimum tensile strength of 21 tons per square inch for Grade C material with an elongation of 15 per cent. The fracture should be fibrous, and the cold bend test provides for the bar to be bent through 90 deg. to a radius equal to one to one and a half diameters.

The quality of cast iron for pipes, pile shoes, pile forks, etc., should be in accordance with the appropriate B.S.S. Failure to comply with the specified tests condemns all castings made from the pouring represented by the test piece,

The material should be tough grey metal cast from approved pig iron or from a mixture of pig and scrap. Castings should be clean and sound, free from air and sand holes and from lumps and flaws. Cast-iron pipes should be specified to be cast vertically with the socket downwards, and the allowable variation of thickness should be stated. Pipes should be coated with Dr. Angus Smith's solution, and if required for pressure work should be subjected to an hydraulic test of 200-ft. head of water for gas pipes and 400-ft. to 800-ft. head for water and sewage pipes.

Usually pipes that have to be built into reinforced concrete tanks are made of brass, phosphor bronze, or other copper alloy, since deterioration due to corrosion is much less than for ferrous metals and replacements that may affect the watertightness of the structure are thus obviated. Pieces that have to be built into tank walls should have an additional intermediate flange cast in such a position that it will be buried in the thickness of the wall and thus form a type of waterbar.

5. -- Specifications.

The principal clauses in a specification for a reinforced concrete structure would cover the quality of the sand, stone, cement, steel, and any other material involved, the proportions of the concrete and the strength required, and the standard of workmanship as affecting the steel bending and placing, mixing and placing of concrete, erecting and striking of shuttering, and such special labours as surfacing, finishing, waterproofing, etc. The storage of materials, the curing of concrete, the testing of the structure upon completion, the method of measuring-up, assessing cost of extra work, allowing for inspection facilities, and other relevant matters would also be covered.

A clause concerning maintenance should also be included, and this should provide that the contractor at his own cost shall uphold and maintain in good and perfect order, repair, and condition the whole of the works executed under the contract during the whole period up to the date of the granting of the completion certificate and for a further period of, say, six months during which period the contractor shall at his own cost restore and make good or renew such portions of the work as the engineer may consider to be unsound or defective.

A general clause should be included to the effect that all materials used in the particular structure should be of British production (if necessary or convenient). They should be new and of the best quality and description of their several kinds, and in every respect subject to the engineer's approval. They should be subjected to such tests as specified and to such other tests as the engineer may deem advisable. Any materials rejected shall be replaced by the contractor free of charge and shall be immediately removed from the site of the works.

A similar general clause should be inserted to cover workmanship. The contractor's employees should be all skilled men in their respective positions, and the engineer should reserve the right to demand the dismissal from the work of any employee who appears to be incompetent or negligent. The contractor should be called upon to pay the standard rate of wages in force in the district in which the works or manufacture of parts are carried out.

The provision of an office for the clerk of works, and details of arrangements regarding the method of payment and with reference to retention moneys and to detail drawings, should also be specified in terms suitable to the work under consideration.

6.—Quantities.

The methods of taking-off quantities and setting up a schedule of quantities vary considerably, although certain standard methods are in vogue. Concrete is usually measured in cubic feet or cubic yards, and the items should be divided up according to the mix and further subdivided into beams, slabs, columns, walls, and special work. Pre-cast work should be kept separate and specified as such. It is important to give the sizes of the various members, especially the thicknesses of walls and slabs, and often these items are given in superficial measurement of specified thickness. The level of the work to which any item applies should be indicated.

Shuttering is taken off in superficial measurement, and for general building work it is usually sufficient to lump all beams and slabs and other horizontal work into one item and all walls and columns and other vertical work in another item. Otherwise all beams, slabs, walls and columns are kept separate and a separate item given for each size of member. In wall work it should be definitely stated whether measurement is made of both faces or only one face. Plane work should be kept separate from curved work, and spherical work should also be separately billed. All special and pre-cast work should be given separate items. Sometimes shuttering items are omitted from the schedule altogether and a stipulation made that the concrete prices should allow for all necessary shuttering. This saves labour in preparing the quantities and in measuring up the completed work, but may lead to trouble over unit prices if any variations are made from the drawings from which the contractor prepares his tender.

It is usually sufficient if reinforcement is scheduled to allow for the supply of the material, handling, bending, placing, and provision of tying wire, separate items being given for bars $\frac{5}{8}$ in. diameter and over and for each diameter below $\frac{5}{8}$ in. It assists the contractor in preparing his price if typical details of the reinforcement in various parts of the structure are provided in addition to general drawings, but if such details are provided care must be taken in the subsequent detailing to ensure that deviations from the typical arrangement are not extensive; otherwise the contractor has usually an indisputable claim for an extra. It is sometimes sufficient to specify the maximum, minimum, and average length of the bars of each diameter, or alternatively to state that the bars would be in commercial lengths.

Separate items should be given for making complete and curing, for handling and driving to a specified depth, and for stripping the heads of pre-cast reinforced concrete piles of a specified section and length. A price per foot should be given for cutting off surplus lengths, and prices per foot for variations in driven length. The cost of making the piles should include the provision of concrete, reinforcement, moulds, shoes, forks, etc., and the probable number of piles required should be specified. If any variation in number is anticipated alternative items should be allowed.

Excavation should be measured net in the solid and the quantities should be stated as such ; this allows the contractor to make his own allowances for extra to be taken out, bulking, etc. It is usual to specify that the price for excavation should include for any necessary timbering, pumping, removal of surplus spoil, and part return fill and ram. In fairness to the tenderer the engineer should give some indication of the probable extent of these factors if possible. Separate items should be given for soils of markedly different qualities, and sometimes the amount of excavation between certain levels is scheduled. This is preferable if the extent of the excavation is uncertain ; otherwise it is sufficient to state the total depth of the excavation.

Special labours and materials such as surfacings, fittings, etc., should be scheduled separately, and the aim should be to make the bill of quantities as comprehensive as possible in order that the liability of over-charging for extras or omitted items should be eliminated. It should be borne in mind that few structures are erected exactly as originally planned, and much economy can be effected if possible avenues of variations are anticipated and care taken that any variations in these directions are covered by the wording not only of the specification but also by that of the bill of quantities."

Accuracy and speed in taking off quantities are only attained if a method of taking off is decided upon and adhered to strictly. The usual method is to write down in the second of three columns the length, breadth, and height (in this order) of a part of the work, these particulars being placed one under the other. The number of identical parts is written in the first column and the product of this number and the three items in the second column, that is, the total volume, is entered in the third column. If area (as for shuttering) is being measured, only two items for each product would appear in the second column. Thus, if three rectangular blocks of concrete each measuring 4 ft. long by 2 ft. 6 in. broad by 5 ft. deep are being taken off, the entry would appear as shown in the left-hand columns, and the shuttering for the four vertical faces would appear as shown in the right-hand columns :

Concrete.				Shuttering			
			cu ft				ft sup.
Blocks . .	3/	4 ft 0 in.		3/2/	4 ft 0 in		
		2 ft 6 in			5 ft 0 in.		120
		5 ft. 0 in.	150		2 ft 6 in.		
				3/2/	5 ft. 0 in.		75

The method of taking off the quantities for a floor panel is given in detail in the pages following *Table No. 40.*

7.—Cost Estimating.

A reinforced concrete engineer has often to provide an estimate of the cost of any particular structure he has designed. If he has connections with a contracting organisation the necessary data and methods for making up the estimate

would be available, but if he is not in the position to be able to call upon such expert estimating services he can adopt one of two courses. The first is to send his bill of quantities and drawings to one or more contractors asking for a competitive tender. Unless there is a reasonable chance of one of these tenders being accepted this is not a commendable procedure, and, further, such a procedure usually takes up more time than can be spared in the preliminary stages of a job.

His second course is to make up an estimate himself, and it is generally sufficient for him to price out each item in his bill of quantities at the average prevailing rates for similar construction. Unless local rates are known, however, this may lead to misleading results. If a priced bill of quantities for previous work in the locality is not at hand, the rates given in technical periodicals can be adopted for the purpose of arriving at an approximate cost.

The preferable method to adopt is to calculate the probable rates, basing the computation on current wages and material costs. There are three factors to consider, (i) Labour charges, (ii) Material costs, and (iii) General costs.

Labour charges include all wages paid to workmen (exclusive of staff) on the site. The rate of wage payable to different classes of workers is given in most technical periodicals, and the man-hours required to perform any given piece of work are given in the many text-books on this subject. Present-day net labour costs vary between the following limits:

<i>Concrete</i> —including unloading, storing, and handling materials, mixing, transporting, placing, ramming, screeding, etc.	4s. to 12s. 6d. per cubic yard
<i>Reinforcement</i> —including unloading, stacking, sorting, bending, placing, etc.	2s. to 7s. per cwt.
<i>Shuttering</i> —including making, erecting, striking, or including cleaning, withdrawing nails, repairing, re-erecting, striking, etc.	2s. to 5s. per square yard
<i>Excavating</i> —for each time the material is handled (but excluding cost of transport of surplus, timbering, pumping or ramming) measured in: solid (does not allow for rock excavation)	1s. to 2s. per cubic yard

The factors that cause the labour costs to tend towards the higher limits are

- Small quantity of concrete in job.
- Necessity of washing materials on the site.
- Long distance from point of unloading to position in work, whether horizontally or vertically.
- Exceptionally dry mix of concrete.
- Concrete placed in small quantities, in narrow widths, or under water.

Slab work where large amount of screeding per cubic yard of concrete required.

Small or short bars.

Lack of repetition of bar bending or of shuttering.

Curved work for shuttering.

Hard ground to be excavated.

Excavation within confined spaces.

Deep excavation.

Waterlogged ground.

Tidal work, etc.

Similarly the absence or the opposite of the foregoing factors will tend to bring the net labour costs towards the lower limit.

The material costs can be obtained from current quotations, and the amount of each kind of material required can be determined from the schedule of quantities. On *Table No. 39* are given the quantities of stone, sand, and cement required for a cubic yard of concrete of any mix. To these net quantities must be added an allowance for waste; an increase of 10 per cent. on the net quantities of sand and stone is sufficient for measurement tolerances and waste, and 5 per cent. added to the net quantities of cement will usually allow for waste and for material used in making cement grout, etc. If applicable the cost of cleaning and returning cement bags should be allowed. If material is not quoted "delivered on site," the transport costs from the point of supply to the site must be included.

An exact estimation of the general costs involved in any contract depends principally upon the contract time, the plant it is proposed to employ, and the general planning of the work. These general costs include overhead charges, head-office costs, drawings, site staff (foreman, engineers, surveyors, clerks, watchmen, etc.), lighting and hoardings, insurance, licences, legal charges, loss of interest on retention money, contingencies, profits, and plant charges. The plant charges include the first cost (or depreciation) of all mixers, cranes, hoists, chutes, bending machines, circular saws, lorries or wagons and tracks, etc., that may be brought on to the site, in addition to the costs of carriage, erection, dismantling, removing, spares, etc., that may be applicable to these pieces of plants. Such items as small tools, consumable stores, fuels, oils, timber, scaffolding, barrows, ropes, bolts, offices, stores, etc., are also included in this item. In detailed estimating it is usual to allocate the charges of each particular piece of plant to those items in the schedule to which it applies, but for an approximate estimate of a normal type of reinforced concrete structure in which profits do not exceed 10 per cent. a most convenient and fairly accurate method of allowing for all the charges under the heading of general costs is to increase the net material costs by 15 per cent. and the net labour cost by 40 per cent. For jobs under, say, £1,000 or for very large jobs this method is not dependable, and such work should be estimated on a more precise basis.

The following example will illustrate an application of the method advocated:

To find the cost per cubic yard of concrete Mix C (1 : 2 : 4) placed in beams and slabs on the upper floor of a building in London—

	s. d.
<i>Materials</i> (quantities from <i>Table</i> No. 39) :	
510 lb. cement at 47s. per ton delivered	10 8
11½ cb. ft. sand at 11s. 6d. per cubic yard delivered	4 10
22½ cb. ft. Thames ballast at 10s. per cubic yard delivered	8 5
Add 5 per cent. to cement cost	0 6
Add 10 per cent. to sand and stone.	1 4
Cleaning and return cement sacks	1 0
Total materials cost	<hr/> 26 9
<i>Labour</i> , including handling materials, mixing, hoisting, and placing, say	9 0
<i>General costs</i> : 15 per cent. on materials	4 0
40 per cent. on labour	3 9
Estimated total cost per cb. yd.	<hr/> 43 6

The method of making up comparative cost estimates from published unit rates is given in the pages following *Table* No. 40.

8.—Miscellaneous Data.

On *Tables* Nos. 37, 38, and 39 are given data of a miscellaneous kind. The former *Tables* give fractions of an inch expressed in decimals of a foot and metric equivalents respectively, while on *Table* No. 39 the quantities of materials required in one cubic yard of concrete are given. This *Table* also gives the standard loading gauge for British railways, and the list of areas, section moduli, and moments of inertia of certain regular sections as mentioned in Chapter V.

Table No. 40 gives the trigonometrical functions of angles to a degree of accuracy sufficient for design purposes, and in conjunction with this *Table* essential trigonometrical formulæ are given.

PART II

TABLES AND EXAMPLES

EXAMPLES OF USE OF TABLE No. 1.

(a) To find the intensity of total load on a $4\frac{1}{2}$ -in. flat roof slab, with an average of $1\frac{1}{2}$ -in. screeding and $\frac{1}{2}$ -in. asphalt:

Superload	=	30	lb. per sq. ft.
$\frac{1}{2}$ -in. Asphalt	= 0.5×12	= 6	" " "
Screeding	= 1.5×10	= 15	" " "
Slab	= $4\frac{1}{2} \times 12$	= 54	" " "
		<hr/>	
		105 lb. per sq. ft.	

(b) To find the live and dead load per ft. run of secondary beam in office floor construction; concrete slab 5 in. thick surfaced with $\frac{3}{4}$ -in. granolithic and plastered $\frac{1}{2}$ in. thick on underside. Beams 10 in. deep by 6 in. wide, at 10-ft. centres.

Live load from 10-ft. width of slab (assuming entrance floor at 80 lb and 20 lb for partitions) = 10 ft. \times 100 lb. = 1,000 lb. per ft. run of beam.
Dead load per sq. ft. of slab:

Granolithic	= $\frac{3}{4} \times 12$	= 9	lb.
Slab	= 12×5	= 60	"
Plaster	= $\frac{1}{2} \times 9$	= 5	"
		<hr/>	
		74 "	

Dead load on beam from slab = 10 \times 74 = 740 lb. per ft.

Weight of rib = 10 in. by 6 in. = 60 " "

Plaster on sides of ribs = $2 \times 0.83 \times \frac{1}{2} \times 9$ = 8 " "

Total dead load = 808 " "

(c) To find the total load on a 9-in. by 9-in. lintel spanning a clear opening of 7 ft. and supporting a 9-in. brick wall (built in cement); height of wall exceeds 5 ft.

Height of 60 deg. triangle on 7 ft. base = $\frac{7}{2} \sqrt{3}$ = 4.04 ft.

Weight of wall at 10 lb. per inch thickness; = 90 lb. per sq. ft.

Do. ' carried on lintel = $\frac{4.04 \times 7}{2} \times 90$ = 1,270 lb.

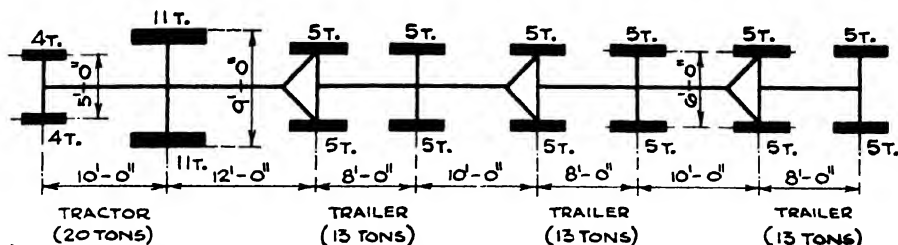
Weight of lintel = $9 \times 9 \times 7$ = 567 "

Total load = 1,837 "

NOTES.

LOADINGS.

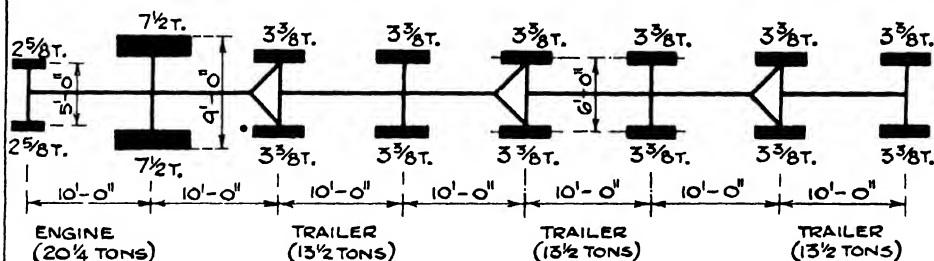
TABLE No 2.



NOTE:- THE BRIDGE SHALL BE ASSUMED TO BE LOADED IN SUCH A WAY AS WILL PRODUCE THE MAXIMUM STRESS, THERE SHALL NOT BE MORE THAN ONE ENGINE PER 75'-0" OF THE SPAN OF THE BRIDGE, AND EACH STANDARD TRAIN SHALL OCCUPY A WIDTH OF 10'-0". WHERE THE WIDTH OF THE CARRIAGEWAY EXCEEDS A MULTIPLE OF 10'-0" SUCH EXCESS SHALL BE ASSUMED TO BE LOADED WITH A FRACTION OF THE AXLE LOADS EQUAL TO THE EXCESS WIDTH IN FEET DIVIDED BY TEN. (APRIL 1927).

THE APPLICATION OF THE ABOVE LOADING IS SIMPLIFIED BY ASSUMING A UNIFORMLY DISTRIBUTED LOAD THE MAGNITUDE OF WHICH DEPENDS ON THE "LOADED LENGTH" OF THE MEMBER (SEE FIG 1, CHAP 2). IN ADDITION TO THE DISTRIBUTED LOAD A "KNIFE EDGE" LOAD OF 2700 LBS PER FT IN ANY POSITION ON THE STRUCTURE MUST BE ALLOWED FOR (1931).

MINISTRY OF TRANSPORT STANDARD LOADING FOR ROAD BRIDGES.



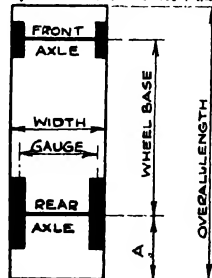
NOTES:- WHEEL LOADS DO NOT INCLUDE FOR IMPACT. (SEE TABLE No 4).

BRITISH STANDARD LOADING FOR ROAD BRIDGES.

[WITH AXLE LOAD MULTIPLE OF 15 UNITS. - B. E. S. A. No 153. APP. No 1 (1925)]

WEIGHTS OF ROAD VEHICLES.

APPROX. MAXIMUM AXLE LOADS IN TONS (WITHOUT IMPACT).



DESCRIPTION.	AXLE LOADS.		DIMENSIONS.				
	FRONT.	REAR.	WHEEL BASE.	GAUGE.	WIDTH.	OVERALL LENGTH.	A.
5TON STEAM WAGON.	4	8	15'-3"	6'-0"	7'-0"	26'-0"	8'-3"
16TON TRACTOR.	5	11	10'-0"	6'-8"	7'-0"	20'-0"	-
20TON STEAM ROLLER	8	12	10'-0"	5'-9"	9'-0"	-	-
MOTOR COACH.	3 1/4	4 3/4	10'-0"	6'-0"	7'-0"	27'-0"	-
LIGHT MOTOR BUS.	2	4	13'-0"	5'-9"	6'-6"	23'-0"	7'-0"
DOUBLE DECK BUS.	4	5 1/2	MAXIMUM PERMISSIBLE LOADS. (M. OF T.)				
4 WHEELED LORRY.	4	8					
6 WHEELED LORRY.	4	7 1/2					
	CENTRE AXLE 7 1/2						

EXAMPLES OF USE OF TABLES Nos. 3 AND 4.

(a) To find the forces acting on a crane beam carrying a 10-ton overhead travelling crane; span of crane 40 ft.:

Total load on pair of wheels $16\frac{1}{2}$ tons \times 125 per cent. = 46,800 lb.

Hence beam is subjected to moments and shears produced by two moving point loads of 23,400 lb. each (allowing for impact and vibration) at 10 ft. centre to centre. For refinement of shear calculation, each wheel load can be considered as distributed over a length of say 2 ft (6 times depth of rail) plus twice the effective depth of beam. Dead weight of beam, crane rail, etc., to be allowed for in moment and shear calculation.

Longitudinal thrust on beam due to braking = $0.2 \times 16\frac{1}{2} \times 2,240 = 7,500$ lb

Side thrust on beam due to traversing = $0.1 \times 25 \times 2,240 = 5,600$ lb. which can act anywhere on the beam as two horizontal thrusts at 10-ft. centres.

(b) To find the maximum total load on a 12-in. concrete slab supporting a single line of standard railway over which can pass at slow speeds any size of locomotive:

Load from ballast (12 in. thick), sleepers, chairs, rails,

etc. (Table No. 1) = 100 + 175

= 275 lb. per sq. ft.

Concrete slab = 12×12

= 144 " " "

Total dead load = 419 " " "

Dispersion of wheel loads (Table No. 4):

D = say, 12-in. ballast + $10\frac{1}{2}$ -in. effective depth of slab = 1 ft. $10\frac{1}{2}$ in.

$A = 2D$ + distance over all two sleepers (= min. at joints) = 3 ft. 9 in. + 3 ft. = 6 ft. 9 in. (This dimension should not exceed distance between axles of locomotive.)

$B = 2D$ + length of sleeper = 3 ft. 9 in. + 9 ft. = 12 ft. 9 in.

Axle load of 20 tons (Table No. 3) = $\frac{20 \times 2,240}{12.75 \times 6.75} = 521$ lb. per sq. ft.

For infrequent passage and slow speed of such a heavy axle load, allow 25 per cent. impact.

Total load = $419 + (1.25 \times 521) = 1,070$ lb. per sq. ft.

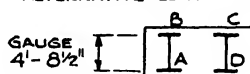
Alternative: Total static load (= 419 + 521) plus 15 per cent. for vibration = 1,080 lb. per sq. ft.

(In normal railway bridge construction, longitudinal rail beams would be provided, thereby relieving slab of traffic load.)

TABLE No 3.

JIB CRANES:- WHEEL LOADS DEPEND ON DISPOSITION OF JIB, ETC.
ESSENTIAL DATA SHOULD BE OBTAINED FROM MAKERS.

TYPICAL LOADING FOR 3 TON



(1) CRANE STATIONARY, UNLOADED, JIB PARALLEL TO TRACK, STRONG WIND ON SIDE :-	11	13	10 ^{5/8}	8 ^{5/8}
(2) CRANE TRAVELLING UNLOADED, JIB PARALLEL TO TRACK, LIGHT WIND, IMPACT :-	15 ^{1/2}	16 ^{1/2}	13 ^{1/2}	12 ^{1/2}
(3) CRANE STATIONARY, JIB AT RIGHT ANGLES TO TRACK LIFTING LOAD ON LEeward SIDE :-	2 ^{7/8}	20	20	2 ^{7/8}
(4) AS (3) BUT DISCHARGING :-	10 ^{3/4}	12 ^{1/8}	12 ^{1/8}	10 ^{3/4}
(5) CRANE STATIONARY, JIB OBLIQUE TO TRACK AND SLEWING WITH LOAD :-	12 ^{5/8}	24 ^{1/3}	13 ^{3/4}	2 ^{1/2}

MAX. LOADS ON PAIR OF WHEELS (IN TONS).

CAPACITY OF CRANE.	SPAN				WHEEL BASE	TOTAL WEIGHT APPROX.	ALLOWANCES.
	30'	40'	50'	60'			
2 TONS	5½	6	6½	7½	8'-6"	10 TONS	BRAKING : 20% OF WHEEL LOAD.
5 "	9¾	10½	11½	12¼	8'-6"	17 "	IMPACT AND VIBRATION " " = 25% OF TOTAL LOAD.
10 "	15½	16¾	18¼	19¼	10'-0"	25 "	SIDE RACKING = 10% " " "
20 "	-	28¾	30¼	31½	10'-6"	40 "	DISTRIBUTION OF WHEEL LOAD = 4 TO 6 (DEPTH OF RAIL).
30 "	-	40½	43	45	12'-0"	55 "	
50 "	-	64	67½	70¼	13'-0"	84 "	

4 WHEELED TRAMCAR :- 3 TO 4 TONS PER WHEEL; WHEEL BASE 7'-0"; OVERALL LENGTH 30'-0".

20 TON RAILWAY TRUCK:- $7\frac{1}{2}$ " " " " " 12'-0"; " " 24'-6"

MAIN LINE LOCOMOTIVES :- MAXIMUM AXLE LOADS 20 TONS APPROX.

COLLIERY TUBE :- TOTAL LOAD LOADED : $1\frac{1}{2}$ TONS; GAUGE : $1'-11\frac{1}{2}"$; OVERALL LENGTH $4'-9"$
MIN. TURNING RADIUS : $7'-0"$; HEIGHT : $3'-2"$

GRAIN	40 TO 50 LBS. PER CU. FT.	WATER (FRESH)	62.4 LBS. PER CU. FT.
CEMENT (NORMAL)	90 " " " "	SEA WATER	64 " " " "
COAL (VARIES)	50 " " " "	FERMENTING BEER	64 " " " "
COKE	28 " " " "	TAR	64 " " " "
DRY SAND	40 " " " "	SULPHURIC ACID	115 " " " "
CRUSHED STONE	100 " " " "	AMMONIA (28%)	55 " " " "
COMPACT SNOW	20 " " " "	BITUMEN	53 " " " "
LOOSE SNOW	5 " " " "	PETROLEUM	35 " " " "
ASHES	40 " " " "	WINE	67 " " " "

MATERIALS IN WATER :-

LIGHTER THAN WATER: PRESSURE ON BOTTOM OF CONTAINER = 62.4 (HEAD OF WATER)

HEAVIER THAN WATER : " " " " " = WT. OF DRY MATERIAL
MEASURED IN BULK + WT. OF WATER IN VOIDS + WT. OF WATER ABOVE TOP OF
SUBMERGED MATERIAL.

EXAMPLES OF USE OF TABLE No. 4.

(See also Page 178.)

(a) To find the overturning moment due to wind on an exposed water tower consisting of a 20-ft. diameter tank 12 ft. deep, supported on an enclosed square tower, average width 14 ft. The distance from ground level to the underside of the tank is 50 ft., and 3 ft. from ground level to foundation level:

Wind pressure on whole of exposed tower (from Table No. 4) = 25 lb. per sq. ft. of projected area.

Wind on Tank: Total pressure = $20 \times 12 \times 25 \times 0.6 = 3,600$ lb.
(0.6 = reduction factor due to circular shape.)

Centre of pressure = $3 + 50 + \frac{12}{2} = 59$ ft. above foundation level.

Wind on sub-structure:

Normal to face: = $12 \times 50 \times 25 = 15,000$ lb.

Normal to diagonal: = $12\sqrt{2} \times 50 \times 25 \times 0.67 = 14,200$ lb. approx.
(0.67 = reduction factor for wind normal to diagonal.)

Centre of pressure = $3 + \frac{50}{2} = 28$ ft. above foundation level.

Overturning moment (about axis parallel to face of tower) at foundation level: = $(3,600 \times 59) + (15,000 \times 28) = 632,000$ ft. lb.

Overturning moment (about diagonal) = $(3,600 \times 59) + (14,200 \times 28) = 609,000$ ft. lb.

(b) To find width of deck slab of a road bridge assumed to carry a tractor wheel load; slab 10 in. thick with 4-in. road metalling; span of slab 7 ft. between longitudinal beams (i.e. $L = 7$ ft.):

Longitudinal dispersion:

$a =$ say, 12 in.; $D = 4$ in. + say, $8\frac{1}{2}$ in. = 1 ft. 0½ in.

$A = a + 2D = 1$ ft. + 2 ft. 1 in. = 3 ft. 1 in.

Transverse dispersion:

$b =$ say, 1 ft. 6 in.; $B = 1$ ft. 6 in. + 2 ft. 1 in. = 3 ft. 7 in.

According to an American rule: $S = 0.67 (L - B) + A$
= $0.67 (7 \text{ ft.} - 3 \text{ ft. } 7 \text{ in.}) + 3 \text{ ft. } 1 \text{ in.}$
= 5 ft. 4 in.

LOADINGS.

TABLE No 4.

IMPACT ALLOWANCE.

IN ACCORDANCE WITH B.S.I. No 153 - PART No 3. (1923).

LENGTH L.	VALUES OF η .			
	1.	2.	3.	4.
20 FT.	.70	.67	.62	.57
30 "	.67	.59	.53	.48
40 "	.62	.53	.47	.42
50 "	.57	.48	.42	.37
60 "	.53	.44	.38	.33
70 "	.50	.41	.35	.30
80 "	.47	.38	.32	.28
100 "	.42	.33	.28	.23
150 "	.33	.25	.21	.17
200 "	.28	.21	.16	.14

$$I = \frac{80}{90 + \left(\frac{\eta+1}{2}\right)L} \quad \text{FOR ROAD BRIDGES.}$$

WHERE L = LOADED LENGTH IN FEET OF TRACK OR TRACKS PRODUCING MAX. STRESS IN MEMBER CONSIDERED.

η = NUMBER OF LINES OF ROLLING LOAD MEMBER SUPPORTS OR ASSISTS IN SUPPORTING.

I = IMPACT FACTOR. (MAX. VALUE = .7)

FOR RAILWAY BRIDGES INCREASE I BY 50%.

M.O.F.T. LOADING INCLUDES 50% FOR IMPACT. SUITABLE FACTOR FOR SLOW MOVING OR INFREQUENT LOADING = 25%.

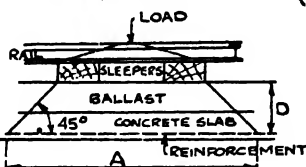
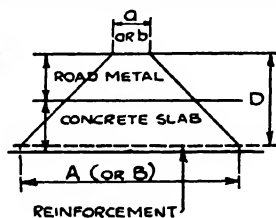
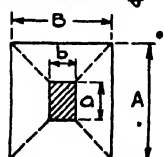
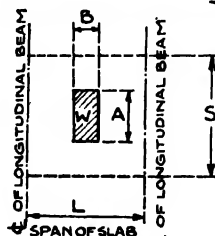
WIND PRESSURES.

DESCRIPTION.	LBS./ FT. ²	NOTES
BUILDINGS:-		
UPPER TWO THIRDS	15	NEGLECT WIND IF HEIGHT IS LESS THAN TWICE WIDTH
PROJECTIONS ABOVE ROOF	25	
EXPOSED STRUCTURES (TOTAL) ON LOWER 80'-0" HEIGHT. ON PORTION ABOVE 80'-0".	25 40	FOR CHIMNEYS SEE CHAPTER II
EXPOSED STRUCTURES (PANELS) ON LOWER 80'-0" HEIGHT. ON PORTION ABOVE 80'-0".	40 40	
BRIDGES :- (PER B.S.5)		FOR DETAILS OF EFF. AREA CHAP. II
UNLOADED.	50	
LOADED (ROAD BRIDGE). (RAILWAY BRIDGE).	20 30	

GENERAL FORMULA :- $P = 0.003 V^2$

VELOCITY OF WIND IN M. P. H. V.	DESCRIPTION OF WIND.	PRESSURE IN LBS./ FT. ² P.
20	LIGHT BREEZE	1.2
30	MODERATE	2.7
40	HIGH WIND	4.8
50	GALE	7.5
60	STORM	10.8
90	HURRICANE	24.3
100	VIOLENT DITTO.	30.0

DISPERSION OF LOADS.



FOR WHOLE AXLE LOAD :-
 $B = 2D + \text{LENGTH OF SLEEPER.}$

- (1). WHEEL CONTACT AREA,
 a = LENGTH OF CONTACT (ZERO TO 12").
 b = WIDTH OF TYRE (6" TO 1'-0").

- (2). DISPERSION THROUGH METALLING,
SLAB, ETC.

$$A = a + 2D \quad (\text{FOR ROAD WHEELS})$$

$$B = b + 2D \quad (\text{FOR LOADS ON RAILS SEE DIAGRAM.})$$

- (3). WIDTH OF SLAB ASSUMED TO

$$\text{CARRY LOAD} = S.$$

$$\text{AMERICAN :- } S = .67(L-B) + A$$

$$\text{E.A. SCOTT :- } S = .25L + A$$

$$\text{E.O. WILLIAMS :- } S = .67L + a^2 \quad (\text{MAX.} = 7'-0").$$

MIN. OF TRANSPORT :- SEE FIG. 1. IN CHAPTER II.

EXAMPLES OF USE OF TABLE No. 5.

(a) To find total horizontal pressure on the back of a vertical wall 15 ft high retaining ordinary filled earth; level fill subject to a surcharge of 2 cwt. per sq. ft. ($\therefore W$):

Assume for earth: $w = 100$ lb. per cb. ft. Angle of repose $= \theta = 35$ deg.

From Table No. 5: $k_2 = 0.271$.

Intensity of pressure at base of wall due to earth backing and surcharge

(Case 3): $p_2 = k_2(wh + W) = 0.271(100 \times 15 + 224) = 467$ lb. per sq. ft.

Intensity of pressure at top of wall due to surcharge: $p_2 = 0.271 \times 224 = 60.6$ lb. per sq. ft.

Total pressure $= \frac{h}{2}(467 + 60.6) = 0.5 \times 15 \times 528 = 3960$ lb.

(b) A vertical wall 20 ft high retains a heap of dry coal, the top surface of which is sloped downwards from the wall at the natural slope. Find the intensity of horizontal pressure at the base of wall, (a) neglecting friction on the back of wall and (b) allowing friction:

From Table No. 5, $w = 50$; $\theta = 35$ deg.

(a) Neglecting friction: $k_3 = 0.205$

$p_3 = k_3wh = 0.205 \times 50 \times 20 = 205$ lb. per sq. ft.

(b) Allowing for friction: $K_3 = 7$

$p_3 = K_3h = 7 \times 20 = 140$ lb. per sq. ft.

NOTES.

PRESSURE OF CONTAINED MATERIALS.

TABLE N°5.

MATERIAL .	ANGLE OF REPOSE		FACTORS FOR VERTICAL WALL .		
	DEGREE	GRADIENT	k ₁	k ₂	k ₃
PERFECT FLUIDS.	0°	—	1.000	1.000	1.000
	5°	1 IN 11.4	.992	.840	.786
FINE CEMENT. (MIN. VALUE) (MAX. = 17°)	10°	1 IN 5.7	.970	.702	.625
	11° 20'	1 IN 5	.962	.675	.591
VERY WET PEAT.	14° 0'	1 IN 4	.942	.610	.522
WET CLAY.	15°	1 IN 3.7	.934	.589	.500
VERY WET EARTH.	18° 30'	1 IN 3	.900	.520	.428
	20°	1 IN 2.7	.884	.490	.403
WET SAND; GRAIN.	25°	1 IN 2.1	.821	.406	.320
SANDY GRAVEL.	26° 35'	1 IN 2	.800	.382	.301
DRY CLAY; DRY EARTH.	30°	1 IN 1.7	.750	.333	.256
AVSE DAMP SAND.	33° 40'	1 IN 1.5	.692	.286	.216
FINE DRY SAND; COAL,	35°	1 IN 1.4	.670	.271	.205
SHINGLE; ASHES; COKE.	40°	1 IN 1.2	.586	.218	.161
DAMP WELL DRAINED CLAY, MOIST EARTH.	45°	1 IN 1	.500	.172	.125
CLEAN GRAVEL (NATURAL BED)	50°	1 IN .84	.414	.132	.095
DRY COMPACT PEAT.	55°	1 IN .7	.329	.100	.071
	60°	1 IN .58	.250	.072	.050
PUNNED EARTH.	65°	1 IN .47	.179	.049	.035
	70°	1 IN .36	.117	.031	.022
WELL PUNNED EARTH.	75°	1 IN .27	.067	.017	.012
	80°	1 IN .18	.030	.008	.005
	85°	1 IN .09	.008	.002	.003

NOTES :-

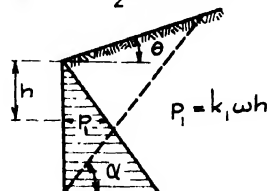
FRICITION NEGLECTED.

p = INTENSITY OF HORIZ. PRESS.

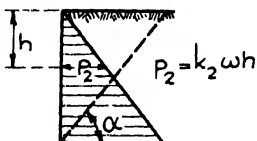
w = UNIT WT. OF MATERIAL - SEE TABLES N°1 & 3 FOR VALUE

θ = ANGLE OF REPOSE.

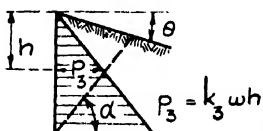
α = 45° + $\frac{\theta}{2}$ (APPROX.)



MAXIMUM POSITIVE SURCHARGE.



LEVEL FILL.



MAXIMUM NEGATIVE SURCHARGE.

DRY MATERIAL.	w.	θ	μ	K ₁	K ₂	K ₃
CEMENT	90	17°	29°	82	37	29
WHEAT.	48	25°	24°	39	15.5	12
ANTHRACITE.	52	27°	27°	41	15.4	12
SAND.	90	35°	30°	60	19.2	14
BITUMINOUS COAL.	50	35°	35°	33	9.7	7
ASHES.	40	40°	40°	23	6.4	4.5

ALLOWING FOR FRICTION ON VERTICAL WALL

μ = ANGLE OF FRICTION B'T'N MATERIAL & CONCRETE.

$p_1 = K_1 h$ $p_2 = K_2 h$

$p_3 = K_3 h$

FOR FORMULÆ FOR ALL CONDITIONS OF VERTICAL AND INCLINED WALLS SEE CHAP. III.

SURCHARGE.

(APPROXIMATE TREATMENT)

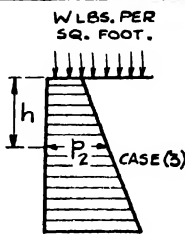
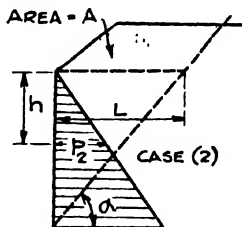
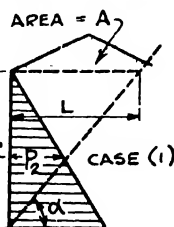
CASES (1) & (2) :-

$$p_2 = c k_2 w h$$

$$c = 1 + \frac{4.5A}{HL}$$

CASE (3)

$$p_2 = k_2 (w h + w)$$



EXAMPLES OF USE OF TABLE No. 6.

To find the horizontal pressures at various depths in a 20-ft. diameter bin 60 ft. deep containing grain ; and also the total load carried on the walls due to the filling :

For grain take $w = 48$ lb. per cb. ft.

$$k = 0.5$$

$$\tan \mu = 0.444.$$

Therefore $p (= wC) = 48C$.

The values of C are obtained from the table for

$$Q = 0.25 \times 20 = 5.0.$$

At 10 ft. depth : $p = 4.0 \times 48 = 192$ lb. per sq. ft.

„ 20 ft. „ „ $6.7 \times 48 = 322$ „ „

„ 30 ft. „ „ $8.4 \times 48 = 403$ „ „

„ 40 ft. „ „ $9.4 \times 48 = 452$ „ „

„ 60 ft. „ „ $10.6 \times 48 = 498$ „ „

The load transferred to the walls of the bin by friction (use formula given in Paragraph 5, Chapter III, for $w = 48$ lb. per cb. ft.)

$$= (12H - 0.5p)D$$

$$= (12 \times 60 - 0.5 \times 498)20$$

$$= 9,420 \text{ lb. per lineal foot of circumference.}$$

NOTES.

DEEP SILOS.

TABLE NO. 6.

JANSSSEN'S FORMULA.

$$P = \frac{WQ}{\tan \mu} \left[1 - \frac{1}{N} \right] = kV$$

WHERE

P = HORIZONTAL PRESSURE IN LBS. PER SQ. FT. AT DEPTH H FT.

W = WEIGHT OF FILLING IN LBS. PER CU. FT. (SEE TABLE NO. 5)

Q = $\frac{\text{PLAN AREA OF BIN IN SQ. FT.}}{\text{PERIMETER OF BIN IN FT.}}$

μ = ANGLE OF FRICTION BETWEEN WALL AND FILLING (SEE TABLE NO. 5)

V = VERTICAL PRESSURE IN LBS. PER SQ. FT. AT DEPTH H FT.

k = $\frac{\text{HORIZONTAL PRESSURE}}{\text{VERTICAL PRESSURE}}$ (= .5 FOR GRANULAR MATERIALS)

N = NUMBER WHOSE COMMON LOG. = $\frac{HK \tan \mu}{2.303Q}$

H = DEPTH OF FILLING IN FEET ABOVE POINT BEING CONSIDERED.

VALUES OF $C = \frac{P}{W}$

FOR $\tan \mu = .444$; $k = .5$

VALUES OF H FEET	VALUES OF Q								
	10	7.5	5.0	4.0	3.0	2.5	2.0	1.5	1.0
10	4.5	4.0	4.0	3.9	3.5	3.3	3.1	2.5	2.2
15	6.5	5.9	5.5	5.1	4.5	4.1	3.6	3.0	2.3
20	8.1	7.5	6.7	6.0	5.2	4.7	4.0	3.2	2.4
25	9.5	8.8	7.7	6.7	5.7	5.0	4.2	3.3	-
30	10.9	9.9	8.4	7.1	6.0	5.3	4.4	3.4	-
35	12.2	10.9	8.9	7.7	6.3	5.4	4.4	-	-
40	13.2	11.8	9.4	8.1	6.5	5.5	4.5	-	-
45	14.2	12.3	9.8	8.3	6.6	5.6	-	-	-
50	15.1	13.0	10.1	8.6	6.7	5.6	-	-	-
55	16.0	13.5	10.3	8.7	6.7	-	-	-	-
60	16.5	14.0	10.6	8.8	6.7	-	-	-	-
65	17.2	14.3	10.7	8.9	<p>VALUES OF Q.</p> <p>SQUARE BIN; $D' \times D'$; $Q = .25D$.</p> <p>RECTANGULAR BIN; $B' \times D'$; $Q = \frac{BD}{2(B+D)}$</p> <p>HEXAGONAL BIN; $Q = .25$ (DISTANCE ACROSS FLATS)</p> <p>OCTAGONAL BIN; $Q = .25$ (DISTANCE ACROSS FLATS)</p> <p>CIRCULAR BIN; $Q = .25$ DIAMETER.</p>				
70	17.7	14.7	10.9	9.0					
75	18.2	15.0	11.0						
80	18.7	15.3	11.1						
90	19.4	15.7	11.3						
100	20.0	16.0	11.4						

INCREASE IN LATERAL PRESSURE IS NEGLIGIBLE BELOW THE DEPTHS TABULATED ABOVE.

NOTES.

CANTILEVERS & SINGLE SPAN BEAMS. TABLE No 7.

BEAM TYPE		MOMENT COEFFICIENTS			REACTION COEFFS.		MAXIMUM DEFLECTION COEFFICIENT	
		AT A	AT B	AT C	AT A	AT C		
CANTILEVERS		-1.0	-	ZERO	1.0	-	1/3	
		-1/2	-	ZERO	1.0	-	1/8	
		-(a+b/2)	-	ZERO	1.0	-	-	
		-1/3	-	ZERO	1.0	-	1/15	
		-1/2	-	ZERO	1.0	-	11/96	
SINGLE SPAN BEAMS	BOTH ENDS FREE		ZERO	a(1-a)	ZERO	1-a	a	a^2(1-a)^2/3
			ZERO	1/4	ZERO	1/2	1/2	1/48
			ZERO	1/8	ZERO	1/2	1/2	5/384
			ZERO	1/7.81 MAX.	ZERO	2/3	1/3	1/76.75
			ZERO	1/6	ZERO	1/2	1/2	1/60
	ONE END FIXED ONE END FREE		MINUS a^2(1-a) x 2(2-a)	a^2(1-a) x 2(3-a)	ZERO	(1-a) x (1+a-a^2/2)	a^2(3-a)/2	-
			-3/16	5/32	ZERO	11/16	5/16	1/107.3
			-1/8	5/128 MAX.	ZERO	5/8	3/8	1/185
			-1/7.5	1/16.7 MAX.	ZERO	4/5	1/5	1/210
			-5/32	1/10.5	ZERO	21/32	11/32	1/139.5
	BOTH ENDS FIXED		-a(1-a)^2	2a^2(1-a)^2	-a^2(1-a)	(1-a)^2 x (1+2a)	a^2(3-2a)	-
			-1/8	1/8	-1/8	1/2	1/2	1/42
			-1/12	1/24	-1/12	1/2	1/2	1/384
			-1/10	1/20.2 MAX.	-1/15	7/10	3/10	1/382
			-5/48	1/16	-5/48	1/2	1/2	7/1920

ALL MOMENTS = COEFFICIENT X TOTAL LOAD X SPAN = K₁WL
REACTIONS = COEFFICIENT X TOTAL LOAD = K₂W. DEFLECTION = COEFFICIENT X (WL³/EI)

ALL MOMENTS = COEFFICIENT X TOTAL LOAD X SPAN = $K_1 WL$

REACTIONS = COEFFICIENT X TOTAL LOAD = $K_2 W$.

DEFLECTION = COEFFICIENT $\times (\frac{WL^3}{EI})$

EXAMPLES OF USE OF TABLE No. 8.

(a) To find the maximum moments at the middles of the end span and centre span, and at the penultimate and interior supports of a five-span continuous beam; all spans = 15 ft.; dead load = 600 lb. per ft. run and live load of 1,200 lb. per ft. run.

(i) From Table No. 8.

Penultimate support:

$$\text{Dead load: } 0.105 \times 600 \times 15^2 = 14,200 \text{ ft. lb.}$$

$$\text{Live load: } 0.120 \times 1,200 \times 15^2 = 32,300 \text{ ,,}$$

$$\text{Total: } = 46,500 \text{ ,, (Neg.)}$$

Interior support:

$$\text{Dead load: } 0.080 \times 600 \times 15^2 = 10,400 \text{ ft. lb.}$$

$$\text{Live load: } 0.111 \times 1,200 \times 15^2 = 29,900 \text{ ,,}$$

$$\text{Total: } = 40,300 \text{ ,, (Neg.)}$$

Middle of end span:

$$\text{Dead load: } 0.078 \times 600 \times 15^2 = 10,250 \text{ ft. lb.}$$

$$\text{Live load: } 0.100 \times 1,200 \times 15^2 = 27,000 \text{ ,,}$$

$$\text{Total: } = 37,250 \text{ ,, (Pos.)}$$

Middle of interior span:

$$\text{Dead load: } 0.046 \times 600 \times 15^2 = 6,200 \text{ ft. lb.}$$

$$\text{Live load: } 0.086 \times 1,200 \times 15^2 = 23,200 \text{ ,,}$$

$$\text{Total: } = 29,400 \text{ ,, (Pos.)}$$

(ii) From Table No. 10.

$$\text{Ratio of live to dead load} = \frac{1,200}{600} = 2$$

$$\text{Total load} = 1,200 + 600 = 1,800 \text{ lb.}$$

$$\text{Penultimate support: } 0.107 \times 1,800 \times 15^2 = 43,100 \text{ ft. lb. (Neg.)}$$

$$\text{Interior support: } 0.102 \times 1,800 \times 15^2 = 41,100 \text{ ,, (Neg.)}$$

$$\text{Middle of end span: } 0.094 \times 1,800 \times 15^2 = 38,100 \text{ ,, (Pos.)}$$

$$\text{Middle of centre span: } 0.069 \times 1,800 \times 15^2 = 28,000 \text{ ,, (Pos.)}$$

(b) Same problem as Example (a) opposite Table No. 9:

Penultimate support:

$$\text{Dead load: } 0.107 \times 1,000 \times 15^2 = 24,100 \text{ ft. lb.}$$

$$\text{Live load: } 0.181 \times 10,000 \times 15 = 27,200 \text{ ,,}$$

$$51,300$$

NOTES.

CONTINUOUS BEAMS.

EQUAL LOADING ON
EQUAL SPANS.

TABLE N^o 8.

	LOAD	ALL BEAMS FREELY SUPPORTED AT END SUPPORTS .	
		ALL SPANS LOADED	INCIDENTAL LOADING
MAXIMUM BENDING MOMENTS	UNIFORMLY DISTRIBUTED		
MAXIMUM BENDING MOMENTS	CENTRE POINT LOAD		
MAXIMUM BENDING MOMENTS	POINT LOADS AT 1/3RD. POINTS		
MAXIMUM SHEARS	UNIFORMLY DISTRIBUTED		

B.M. COEFFICIENTS :- MULTIPLY BY (SPAN X TOTAL LOAD ON SPAN). COEFFICIENTS ABOVE THE LINE ARE FOR NEGATIVE B.M.s. AT SUPPORTS; THOSE UNDER LINE ARE FOR POSITIVE MID-SPAN B.M.s.
SHEAR COEFFICIENTS :- MULTIPLY BY (TOTAL LOAD ON SPAN).

EXAMPLES OF USE OF TABLE No. 9.

(a) To find the maximum moment at the penultimate support of a beam continuous over four equal spans of 15 ft. carrying a uniformly distributed load of 1,000 lb. per ft. run; a live point load of 10,000 lb. can occur at the centre of one or more of the spans simultaneously.

Maximum B.M. occurs when 1st, 2nd, and 4th spans are loaded with point load. Dividing the total loading into single-span loads:

Load - W .	Span.	Load Factor = F .	Support Moment Coefficient = Q .	Product = FQW .
Uniformly distributed $W = 1,000 \times 15 = 15,000$ lb.	All spans	1.00	- 0.107	- 1,605
Point load $W = 10,000$ lb.	1st span	1.50	- 0.067	- 1,000
Ditto	2nd span	1.50	- 0.049	- 740
Ditto	4th span	1.50	- 0.004	- 60
$\Sigma FQW =$				- 3,405

$$\text{B.M.} = L \times \Sigma FQW = - 3,405 \times 15 = - 51,075 \text{ ft. lb.}$$

It can be seen from the Table that these results are the maximum possible, because if the 3rd span were loaded a positive moment would be introduced that would reduce the total negative moment.

(b) To find the moment at the middle of the centre span of three continuous spans, 10 ft., 15 ft., and 10 ft. respectively. The load on the centre span is a uniform dead load of 7,000 lb. per ft. run and on each of the end spans the load is triangularly distributed being 6,000 lb. per ft. at the inner supports and zero at the outer supports, at which the beam is simply supported. (This is the type of loading that would be encountered in the design of a rectangular tank when wall counterforts—tied at their extremities—are continuous with the beams of the suspended bottom.)

First determine the B.M. at the inner supports by dividing the loading and evaluating the corresponding products. Owing to symmetry the moments at each interior support will be equal.

$$k_1 = k_2 = \frac{\text{End span}}{\text{Base span}} = \frac{10}{15} = 0.67$$

$$x = y = 0.67 \div 1 = 1.67$$

$$H = \frac{5}{1 \times 1.67 \times 1.67} = 0.493.$$

$$\text{For 1st span loaded} \quad 3 \times 0.67^2 \times 0.493 = 0.663$$

$$q = 0.5 \times 1.67 \times 0.663 = 0.555$$

$$\text{For 2nd span loaded} \quad (U_B \text{ owing to symmetry}) \div 0.493 (1.67 + 0.67) = 1.15$$

$$\text{For 3rd span loaded} \quad \text{Owing to symmetry } U_C = 0.555; U_B = 0.663.$$

Load - W .	Span - L .	(Load Factor) \times (Support Mt. Coef) = FQ	Moment Multiplier = U .	Product = $FQUWL$.
Triangularly distributed (Apex at left-hand support) $W = 6,000 \times 10 \times 0.5$ $= 30,000$ lb.	3rd $L = 10$ ft.	+ 0.018	$U_B = 0.663$	+ 3,580 ft. lb.
Uniformly distributed $W = 7,000 \times 15$ $= 105,000$ lb.	2nd $L = 15$ ft.	$1.00 \times (-0.050)$	$U_B = 1.15$	- 90,500 ..
Triangularly distributed (Apex at right-hand support; therefore reverse system) $W = 30,000$ lb.	1st $L = 10$ ft.	- 0.071	U_C for 3rd span loaded = 0.555	- 11,800 ..

$$\text{Net B.M. at inner support} = - 98,720 \text{ ft. lb.}$$

$$\text{Free moment at mid-span} = \frac{7,000 \times 15^2}{8} = 197,000 \text{ ft. lb.}$$

$$\text{Less negative B.M. at mid-span} = 98,720 \text{ ..}$$

$$\text{Net positive B.M. at mid-span} = 98,280$$

CONTINUOUS BEAMS

CONSTANT M.O.F 1.
ANY SPAN AND LOAD

TABLE No 9.

NOTES: DIVIDE GIVEN BEAM SYSTEM UP INTO A NUMBER OF SIMILAR SYSTEMS EACH HAVING ONE SPAN LOADED WITH A PARTICULAR TYPE OF LOAD. TO FIND THE B.M. AT ANY SUPPORT DUE TO ANY ONE OF THESE LOADS, EVALUATE THE FOLLOWING FACTORS FOR THE PARTICULAR SUPPORT AND LOAD TYPE :-

LOAD FACTOR = F (=UNITY FOR DISTRIBUTED LOAD).

SUPPORT MOMENT COEFFICIENT = Q.

MOMENT MULTIPLIER = U (=UNITY FOR EQUAL SPANS).

B.M. AT SUPPORT = FQUXW X BASE SPAN.

NOTE RE TRIANGULARLY DISTRIBUTED LOADS: FOR FQ USE THE VALUE GIVEN IN BRACKETS IN COLUMN HEADED SUPPORT MOMENT COEFFICIENTS WHEN APEX IS AT L.H. SUPPORT. REVERSE SYSTEM WHEN APEX IS AT R.H.S.

LOAD TYPE (W = TOTAL LOAD ON SPAN)	LOAD FACTOR = F	MAX. FREE B.M.
	1.00	.125WL
	1.25	.167WL
	SEE NOTE	.128WL
	1.50	.250WL
	1.33	.167WL

No OF SPANS	LOADED SPAN	EQUAL SPANS SUPPT. M.T. COEFF TS			UNEQUAL SPANS.	
		Q _A	Q _B	Q _C	MOMENT MULTIPLIERS = U	
2.		-	-.063 (-.058)	-	$U_B = \frac{2}{1 + K_1}$	-
		-	-.063 (-.067)	-	$U_B = \frac{2K_1^2}{1 + K_1}$	
•	BOTH SPANS LOADED WITH IDENTICAL LOAD.	-	-.125 (-.125)	-	SEE NOTE BELOW FOR UNEQUAL SPANS SIMULTANEOUSLY LOADED.	
3.		-	-.067 (-.056)	+.017 (+.016)	$U_B = .54U_C$ $U_C = 3K_1^2H$	$X = K_1 + 1$ $Y = K_2 + 1$ $H = \frac{5}{4xy - 1}$
		-	-.050 (-.056)	-.050 (-.044)	$U_B = H(Y + K_2)$ $U_C = H(X + K_1)$	
		-	+.017 (+.018)	-.067 (-.071)	$U_B = 3K_2^2H$ $U_C = .5XU_B$	
	ALL SPANS LOADED WITH IDENTICAL LOAD.	-	-.100 (-.092)	-.100 (-.101)	FOR TWO, THREE, OR FOUR UNEQUAL SPANS LOADED SIMULTANEOUSLY, DETERMINE B.Ms. FOR EACH SPAN LOADED SEPARATELY AND ADD.	
4.		-.067 (-.063)	+.018 (+.017)	-.004 (-.004)	$U_A = \frac{.133}{X} (14 + K_1H_1Z)$ $U_B = 2H_1$ $U_C = 2K_2H_1$	$X = K_1 + 1$ $Y = K_1 + K_2$ $Z = K_2 + K_3$ $H_1 = 14K_1Y$ $H_2 = 4.67K_1^2Y(X+1)$ $H_3 = 4.67K_2^2Y(K_2+Z)$ $H_4 = 14K_3^2Y$
		-.049 (-.055)	-.054 (-.048)	+.013 (+.012)	$U_A = \frac{.543}{X} K_1(4.67K_1 - U_B)$ $U_B = 2H_2$ $U_C = 2K_2H_2$	
		+.013 (+.015)	-.054 (-.060)	-.049 (-.043)	$U_A = 2K_1H_3$ $U_B = XH_3$ $U_C = \frac{.543}{Z} K_2(4.67K_2 - U_B)$	
		-.004 (-.005)	+.018 (+.019)	-.067 (-.072)	$U_A = 2K_1H_4K_2$ $U_B = XH_4K_2$ $U_C = .133H_4(4xy - K_1^2)$	
	ALL SPANS LOADED WITH IDENTICAL LOAD.	-.107 (-.107)	-.071 (-.072)	-.107 (-.107)	SEE NOTE ABOVE	
					$Y = \frac{1}{4xyZ - K_1^2Z - K_2^2X}$	

$$Y = \frac{1}{4xyz - K_1^2Z - K_2^2X}$$

NOTES.

(See Examples for *Table* No. 8 for application of moment coefficients.)

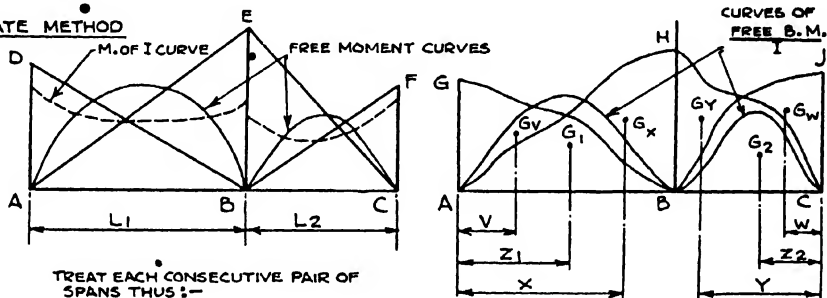
TABLE N° 10.

TABLE N°10.

SECTION	DEAD LOAD	LIVE LOAD	TOTAL LOAD (APPROX)	TOTAL LOAD : RATIO OF LIVE TO DEAD LOAD.							
				·5	1·0	1·5	2·0	2·5	3·0	3·5	4·0
NEAR MIDDLE OF END SPAN. (POSITIVE B.M.)	$\frac{1}{12}$	$\frac{1}{10}$	$\frac{1}{10}$	·089	·092	·095	·094	·095	·096	·096	·097
PENULTIMATE. (SUPPORT NEGATIVE B.M.)	$\frac{1}{10}$	$\frac{1}{9}$	$\frac{1}{10}$	·104	·106	·107	·107	·108	·108	·109	·109
MIDSPAN INTERIOR SPANS. (POSITIVE B.M.)	$\frac{1}{24}$	$\frac{1}{12}$	$\frac{1}{12}$	·056	·063	·067	·069	·072	·073	·074	·075
INTERIOR. (SUPPORTS NEGATIVE B.M.)	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{10}$ (OR $\frac{1}{12}$)	·093	·097	·100	·102	·103	·104	·105	·105

COEFFICIENTS = $\frac{BM}{WL^2}$ FOR UNIFORMLY DISTRIBUTED LOAD ON APPROXIMATELY EQUAL SPANS.

ACCURATE METHOD



ON THE SPANS DRAWN TO SCALE, CONSTRUCT THE FREE MOMENT CURVES AND THE MOMENT OF INERTIA CURVES. DIVIDE THE ORDINATES OF THE FORMER BY THOSE OF THE LATTER TO ENABLE THE CURVES OF FREE B.M. TO BE DRAWN.

FIND $A_1 = \text{AREA UNDER } \frac{\text{FREE B.M}}{I}$ CURVE FOR SPAN L_1 AND POSITION OF CENTROID G_1
AND $A_2 = \quad " \quad " \quad " \quad " \quad " \quad " \quad L_2 \quad " \quad " \quad " \quad " \quad G_2$

SET UP AD, BE, AND CF TO A SUITABLE SCALE TO REPRESENT ANY ASSUMED VALUES OF THE MOMENTS AT A, B, & C RESPECTIVELY. CONNECT DB, AE, EC, & BF.

LET $M_A = K_A AD$ $M_B = K_B BE$ $M_C = K_C CF.$

DIVIDE THE ORDINATES OF DBF AND AEC BY THE ORDINATES OF THE M. OF I. CURVE.
TO GIVE THE CURVES AHC AND GBJ.

FIND A_v = AREA UNDER CURVE GB AND POSITION OF CENTROID G_v

$A_x = \text{ " " " AH " " " " Gx}$

$$A_Y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad G_Y$$

AND A... = " " " B.I " " " " " G.W

SUBSTITUTE IN $(A_1 z_1 + k_A v_A + k_B x_A) \frac{1}{L_1} = -(A_2 z_2 + k_C A_w w + k_B A_y y) \frac{1}{L_2}$

UNKNOWNs ARE K_A , K_B , & K_C AND REQUISITE NUMBER OF EQUATIONS FOLLOWS FROM CONSECUTIVE PAIRS OF SPANS AND END SUPPORT CONDITIONS.

APPROXIMATE METHOD.

CALCULATE B.M.S. FOR CONSTANT MOMENT OF INERTIA AND INCREASE OR DECREASE BY THE APPROPRIATE PERCENTAGE TABULATED.

$I_s = \text{M.OF I. AT SUPPORT.}$

$$I_c = \text{M.O.F. I. AT MIDSPAN}$$

RATIO OF $I_s:I_c$.25	.50	.75	1.25	1.50
BOTH ENDS FIXED	MIDSPAN	+55%	+26%	+7%	-6%	-13%
	SUPPORT	-28%	-15%	-5%	+3%	+4%
ONE FIXED	MIDSPAN	+27%	+13%	+6%	-2%	-10%
ONE FREE	SUPPORT	-40%	-20%	-8%	+5%	+13%

NOTES.

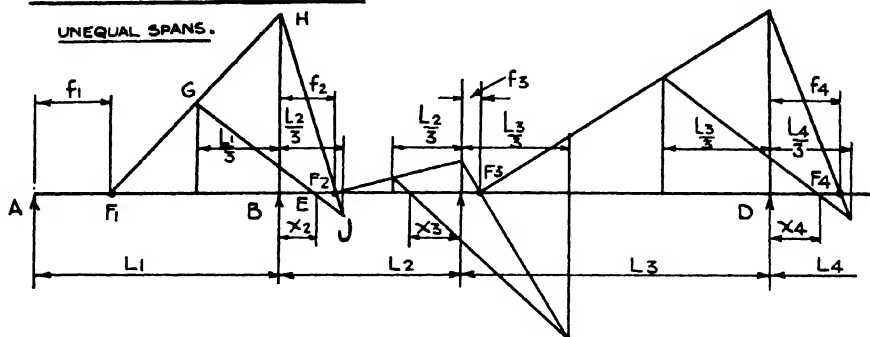
CONTINUOUS BEAMS

TABLE No 11.

"FIXED POINTS" - GRAPHICAL SOLUTION.

(1) TO DETERMINE FIXED POINTS

UNEQUAL SPANS.



ON THE GIVEN BEAM SYSTEM A, B, C, D, ETC, DRAWN TO SCALE, PLOT THE POSITION OF THE FIXED POINT F_1 IN LEFT HAND PORTION OF SPAN AB.

WHEN END A IS FREELY SUPPORTED: $f_1 = \text{ZERO}$.

" " A " FIXED: $f_1 = .333 L_1$

SET OFF $BE = X_2 = .333(L_1 - L_2)$; $X_3 = .333(L_2 - L_3)$; $X_4 = .333(L_3 - L_4)$, ETC.

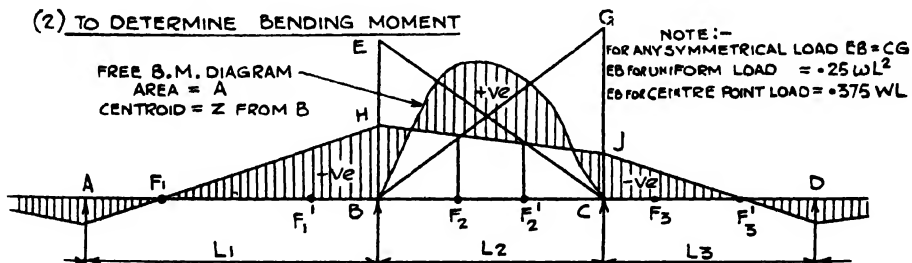
SET UP VERTICALS AT THE THIRD POINTS OF EACH SPAN. AT ANY CONVENIENT ANGLE DRAW GEJ . JOIN F_1G AND PRODUCE TO MEET SUPPORT LINE AT H. JOIN HJ TO INTERSECT BEAM LINE IN F_2 WHICH IS FIXED POINT FOR LEFT HAND PORTION OF SPAN L_2 .

REPEAT SIMILAR CONSTRUCTION TO FIND F_3, F_4 , ETC. TO THE END OF THE SYSTEM, AND, COMMENCING AT THE EXTREME RIGHT HAND SUPPORT, REPEAT THE WHOLE PROCESS WORKING TOWARDS THE LEFT, THUS ESTABLISHING THE FIXED POINTS IN THE RIGHT HAND PORTION OF EACH SPAN.

EQUAL SPANS. (i.e. $L_1 = L_2 = L_3$ ETC. = L)

DISTANCE OF FIXED POINT:	f_1	f_2	f_3	f_4 ETC.
FIXED AT A:	$.3333L$	$.2222L$	$.2121L$	$.2113L$
FREELY SUPPORTED AT A:	0	$.2000L$	$.2105L$	$.2113L$

(2) TO DETERMINE BENDING MOMENT



FREE B.M. DIAGRAM
AREA = A
CENTROID = Z FROM B

NOTE:-

FOR ANY SYMMETRICAL LOAD $EB = CG$
EB FOR UNIFORM LOAD = $.25 \omega L^2$
EB FOR CENTRE POINT LOAD = $.375 \omega L$

DRAW OUT THE BEAM SYSTEM TO SCALE AND PLOT THE POSITIONS OF THE FIXED POINTS, F_1, F_2, F_3 ETC. IN THE LEFT HAND PORTION AND F'_1, F'_2, F'_3 ETC. IN THE RIGHT HAND PORTION OF EACH SPAN. SET UP $BE = \frac{GAZ}{L_2}$ AND $CG = \frac{GA(L_2 - Z)}{L_2}$ (SEE NOTE ABOVE). JOIN BG & CE .

DRAW HJ THROUGH INTERSECTIONS OF VERTICALS FROM F_2 AND F'_2 WITH BG AND CE . TO COMPLETE B.M. DIAGRAM FOR LOAD ON SPAN BC, CONNECT H TO F_1 , J TO F'_1 , ETC. REPEAT FOR OTHER LOADED SPANS AND COMBINE DIAGRAMS TO GIVE TOTAL MOMENTS.

NOTES.

(For examples of the use of influence lines, see the page opposite *Table* No. 13.)

INFLUENCE LINES

TABLE No 12.

DATA FOR CONSTRUCTING INFLUENCE LINES FOR BENDING MOMENTS PRODUCED

AT SUPPORTS AND MID-SPAN BY A SINGLE UNIT POINT LOAD MOVING ALONG A BEAM OF TWO OR THREE SPANS.

B.M. DUE TO ANY LOAD AT ANY POINT = hWL

h = ORDINATE OF APPROPRIATE INFLUENCE LINE AT POINT CONSIDERED.

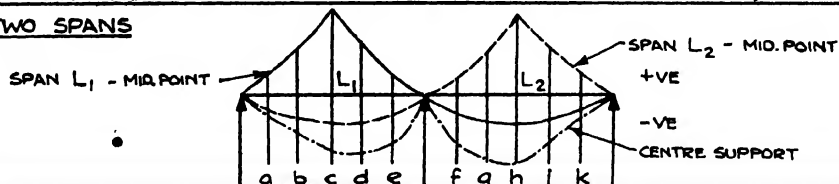
W = TOTAL LOAD.

L = SPAN (WHEN SPANS ARE UNEQUAL L = END SPAN = L_1)

ORDINATES FOR INTERMEDIATE SPAN RATIOS CAN BE INTERPOLATED.

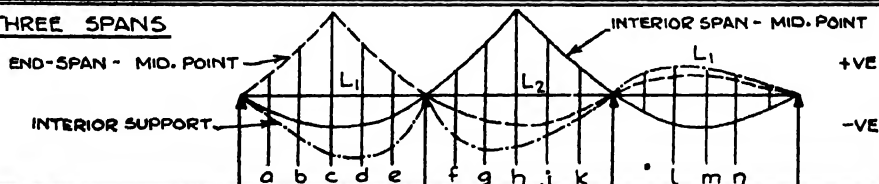
REFERENCE TO THE DIAGRAMS WILL INDICATE WHETHER THE MOMENT DUE TO THE LOAD IN ANY PARTICULAR POSITION IS POSITIVE OR NEGATIVE.

TWO SPANS



	SPAN RATIO $L_1 \cdot L_2$	ORDINATES									
		a	b	c	d	e	f	g	h	j	k
SHORT SPAN MID. POINT	1 · 1	·063	·130	·203	·121	·052	·032	·046	·047	·037	·020
	1 · 1½	·067	·137	·213	·130	·058	·058	·083	·084	·067	·037
	1 · 2	·070	·142	·219	·136	·062	·085	·124	·125	·099	·054
CENTRE SUPPORT	1 · 1	·041	·074	·094	·093	·064	·064	·093	·094	·074	·041
	1 · 1½	·032	·089	·075	·074	·051	·115	·167	·169	·133	·073
	1 · 2	·027	·049	·063	·062	·042	·170	·247	·250	·198	·108
LONG SPAN MID. POINT	1 · 1	·020	·037	·047	·046	·032	·052	·121	·203	·130	·063
	1 · 1½	·016	·030	·038	·037	·025	·067	·167	·291	·183	·088
	1 · 2	·014	·025	·031	·031	·021	·082	·210	·375	·235	·113

THREE SPANS



		SPAN RATIO $L_1 \cdot L_2 \cdot L_3$	ORDINATES													
			a	b	c	d	e	f	g	h	j	k	l	m	n	
END SPAN MID. POINT	1 · 1 · 1	·062	·127	·200	·117	·050	·029	·040	·038	·027	·013	·012	·013	·010		
	1 · 1½ · 1	·066	·134	·209	·126	·056	·051	·070	·065	·046	·021	·012	·012	·010		
	1 · 2 · 1	·068	·139	·215	·132	·060	·075	·102	·094	·065	·029	·012	·012	·009		
INTERIOR SUPPORT	1 · 1 · 1	·043	·079	·100	·099	·068	·057	·079	·075	·054	·026	·025	·025	·020		
	1 · 1½ · 1	·036	·065	·082	·081	·056	·102	·139	·130	·092	·042	·024	·025	·020		
	1 · 2 · 1	·030	·056	·070	·069	·048	·151	·204	·188	·129	·058	·023	·023	·019		
INT. SPAN MID. POINT	1 · 1 · 1	·016	·030	·038	·037	·025	·042	·100	·175	·100	·042	·037	·038	·030		
	1 · 1½ · 1	·013	·023	·029	·028	·020	·053	·135	·245	·135	·053	·028	·029	·023		
	1 · 2 · 1	·010	·019	·023	·023	·016	·063	·167	·313	·167	·063	·023	·023	·019		

EXAMPLES OF USE OF TABLE No. 13.

(a) To determine the bending moment at the penultimate (left-hand end) support of a system of four spans (constant moment of inertia, freely supported on end supports) subject to a central point load of 10 tons on the first and third spans (counting from left-hand end); end spans = 20 ft.; middle spans = 30 ft.; hence span-ratio is $1 : 1\frac{1}{2} : 1\frac{1}{2} : 1$.

ft. lb.

With load on 1st span: B.M. = $-(0.082 \times 10 \times 2,240 \times 20) = 36,800$ (neg.).
(ordinate c)

With load on 3rd span: B.M. = $+(0.035 \times 10 \times 2,240 \times 20) = 15,700$ (pos.)
(ordinate m)

Net B.M. at penultimate support = 21,100 (neg.)

(b) For the determination of the moments at any of the critical sections in a beam system due to a train of point loads in any given position, the procedure is as follows:

Draw the beam system to some convenient linear scale.

With the ordinates given on *Tables Nos. 12 and 13* construct the influence line (for a unit load) for the section to be considered, selecting a convenient scale for the bending moment.

Plot on this diagram the position of the train of loads.

Tabulate the value of (ordinate \times load) for each point load.

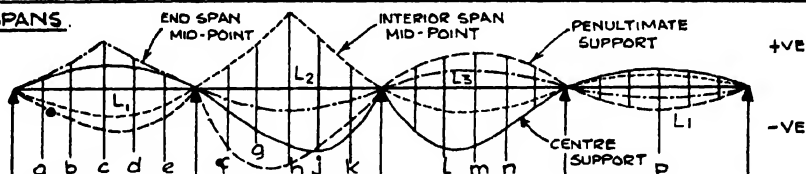
Algebraically add the products (ordinate \times load) to give the resultant bending moment at the section considered.

NOTES.

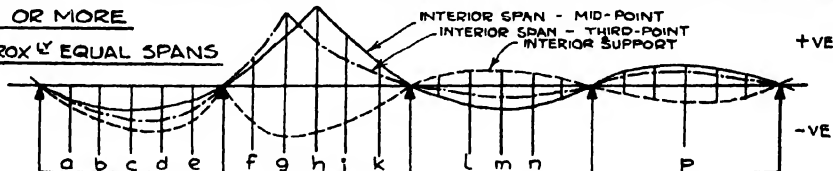
The influence line marked "*Penultimate Support*" on the four span diagram on Table No. 13 refers to the second support from the left-hand side.

TABLE N° 13.

REFERENCE TO THE DIAGRAMS WILL INDICATE WHETHER THE MOMENT DUE TO THE LOAD IN ANY PARTICULAR POSITION IS POSITIVE OR NEGATIVE.



		SPAN RATIO				ORDINATES													
		L ₁	L ₂	L ₃	L ₄	a	b	c	d	e	f	g	h	j	k	l	m	n	p
END SPAN	MID-POINT	1	0	1	0	0.062	0.127	0.200	0.117	0.044	0.028	0.034	0.037	0.027	0.013	0.011	0.010	0.007	0.003
		1	1/2	1/2	0	0.066	0.135	0.209	0.126	0.056	0.052	0.071	0.067	0.048	0.023	0.019	0.017	0.012	0.003
		1	2	2	0	0.069	0.140	0.216	0.133	0.060	0.077	0.106	0.100	0.072	0.034	0.027	0.025	0.017	0.003
PENULTIMATE	SUPPORT	1	0	1	0	0.043	0.079	0.100	0.049	0.068	0.057	0.078	0.074	0.053	0.025	0.021	0.020	0.015	0.007
		1	1/2	1/2	0	0.035	0.065	0.082	0.081	0.055	0.103	0.142	0.134	0.096	0.046	0.037	0.035	0.025	0.007
		1	2	2	0	0.030	0.054	0.069	0.068	0.047	0.155	0.213	0.200	0.143	0.068	0.055	0.050	0.035	0.006
INT. SPAN	MID-POINT	1	0	1	0	0.016	0.029	0.037	0.036	0.025	0.041	0.094	0.173	0.098	0.040	0.032	0.030	0.022	0.010
		1	1/2	1/2	0	0.013	0.024	0.030	0.029	0.020	0.054	0.138	0.250	0.140	0.057	0.043	0.041	0.029	0.007
		1	2	2	0	0.011	0.020	0.025	0.025	0.017	0.066	0.175	0.325	0.180	0.073	0.054	0.050	0.035	0.006
CENTRE	SUPPORT	1	0	1	0	0.012	0.021	0.027	0.027	0.018	0.028	0.058	0.080	0.085	0.061	0.085	0.080	0.058	0.026
		1	1/2	1/2	0	0.010	0.017	0.022	0.022	0.015	0.038	0.082	0.116	0.124	0.091	0.124	0.116	0.082	0.022
		1	2	2	0	0.008	0.015	0.019	0.019	0.013	0.046	0.104	0.150	0.163	0.121	0.163	0.150	0.104	0.019



SECTION	ORDINATES													
	a	b	c	d	e	f	g	h	j	k	l	m	n	p
INT ^L . SPAN : MID-POINT	·010	·021	·029	·031	·022	·039	·096	·171	·096	·039	·031	·029	·021	·008
INT ^L . SPAN : THIRD-POINT	·016	·033	·046	·049	·035	·062	·147	·087	·045	·017	·013	·012	·009	·003
INTERIOR SUPPORT	·023	·057	·079	·084	·061	·061	·084	·079	·057	·027	·023	·021	·015	·006
END SPAN : MID-POINT	ORDINATES FOR END SPAN AND PENULTIMATE SUPPORT OF FOUR EQUAL SPANS SYSTEM CAN BE USED WITH REASONABLE ACCURACY.													
PENULTIMATE SUPPORT														

EXAMPLES OF USE OF TABLE No. 14.

(a) To determine the bending moments in a rectangular slab, $ABCD$, subject to an inclusive load of 300 lb. per sq. ft. The slab is freely supported along the edge AB and continuous over the supports on the remaining three sides; $AB = DC = 10$ ft.; $AD = BC = 12$ ft. (i) Spanning from AD to BC only. (ii) Designed to span in two directions (French Government Rules).

- (i) Span $AD - BC$ is equivalent to an interior span of a system of continuous spans;

$$\text{hence B.M. at mid-span} = \frac{300 \times 10^2}{15} = 2,000 \text{ ft. lb.}$$

$$\text{B.M. at support} = \frac{300 \times 10^2}{12} = 2,500 \text{ ,,}$$

In direction $AB - DC$ only distribution steel would be provided.

- (ii) Spanning in both directions:

$$\text{Equivalent short span} = 0.67 \times 10 = 6.7 \text{ ft.}$$

$$\text{Equivalent long span} = 0.80 \times 12 = 9.6 \text{ ft.}$$

$$k = \frac{9.6}{6.7} = 1.43.$$

From French Government formula: coefficients = 0.69 and 0.10.

Pending moments:

$$\text{Mid-span: } AD - BC: \quad 0.69 \times 300 \times 10^2 \times \frac{1}{15} = 1,380 \text{ ft. lb.}$$

$$\text{Mid-span: } AB - DC: \quad 0.10 \times 300 \times 12^2 \times \frac{1}{11} = 393$$

$$\text{Support: } DC: \quad 0.10 \times 300 \times 12^2 \times \frac{1}{10} = 432$$

$$\text{Supports: } AD \text{ and } BC: \quad 0.69 \times 300 \times 10^2 \times \frac{1}{12} = 1,613$$

$$\text{Support: } AB: \quad \text{nil}$$

(b) For example of "flat slab" design, see "Additional Examples" following Table No. 40.

NOTES.

SLABS

TABLE No. 14.

B. M. COEFFICIENTS.

$$= \frac{M}{WL^2} \quad \text{UNIFORM LOADING. APPROXIMATELY EQUAL SPANS}$$

(SEE ALSO TABLE No. 10)

SECTION	END SPAN MID. SPAN	PENULTIMATE SUPPORT	INT ⁹ SPAN MID. SPAN	INTERIOR SUPPORT
WITHOUT HAUNCHES	$\frac{1}{11}$	$\frac{1}{10}$	$\frac{1}{15}$	$\frac{1}{12}$
WITH HAUNCHES	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{1}{18}$	$\frac{1}{10}$

RECTANGULAR SLABS

(REINFORCED IN TWO DIRECTIONS)

VALUE OF K	GRASHOF AND RANKINE		FRENCH GOV I		PIGEAUD		AMERICAN	
	$\frac{W_B}{W}$	$\frac{W_L}{W}$	$\frac{W_B}{W}$	$\frac{W_L}{W}$	$\frac{W_B}{W}$	$\frac{W_L}{W}$	$\frac{W_B}{W}$	$\frac{W_L}{W}$
1.00	.50	.50	.33	.33	.30	.30	.50	.50
1.05	.55	.45	.38	.29	.33	.27	.55	.45
1.10	.60	.41	.42	.26	.36	.24	.60	.40
1.15	.64	.36	.47	.22	.39	.22	.65	.35
1.20	.67	.33	.51	.19	.43	.19	.70	.30
1.25	.71	.29	.55	.17	.45	.17	.75	.25
1.30	.74	.26	.59	.15	.48	.15	.80	.20
1.35	.77	.23	.63	.13	.51	.13	.85	.15
1.40	.79	.21	.66	.12	.53	.12	.90	.10
1.45	.82	.19	.69	.10	.56	.11	.95	.05
1.50	.84	.17	.72	.09	.58	.10	1.00	0
1.55	.85	.15	.74	.08	.60	.09	-	-
1.60	.87	.13	.77	.07	.63	.08	-	-
1.65	.88	.12	.79	.06	.65	.08	-	-
1.70	.89	.11	.81	.06	.66	.07	-	-
1.75	.90	.10	.83	.05	.68	.06	-	-
1.80	.91	.09	.84	.05	.70	.06	-	-
1.85	.92	.08	.86	.04	.72	.05	-	-
1.90	.93	.07	.87	.04	.74	.05	-	-
1.95	.94	.07	.88	.04	.75	.04	-	-
2.00	.94	.06	.89	.03	.77	.04	-	-

W = TOTAL LOAD ON SLAB

W_B = LOAD CARRIED IN DIRECTION OF SHORTER EQUIVALENT SPAN.

W_L = LOAD CARRIED IN DIRECTION OF LONGER EQUIVALENT SPAN.

k = $\frac{\text{LONGER EQUIVALENT SPAN}}{\text{SHORTER EQUIVALENT SPAN}}$

EQUIVALENT SPAN = f_k (ACTUAL SPAN)

VALUES OF f :-

SPAN FREELY SUPPORTED AT BOTH ENDS $f = 1.00$

SPAN FREELY SUPPORTED AT ONE END AND FIXED AT OTHER END $f = .80$

SPAN FIXED AT BOTH ENDS ... $f = .67$

SPAN FIXED AT ONE END AND UNSUPPORTED AT THE OTHER. (I.E. CANTILEVER) $f = .3.0$

WHEN BOTH SPANS ARE SIMILAR AS REGARDS END CONDITIONS

$k = \frac{\text{LONGER ACTUAL SPAN}}{\text{SHORTER ACTUAL SPAN}}$

GRASHOF AND RANKINE FORMULA:-

$$W_B = \frac{Wk^4}{k^4 + 1}; W_L = W - W_B$$

FRENCH GOVERNMENT FORMULA:-

$$W_B = \frac{Wk^4}{k^4 + 2}; W_L = \frac{W}{1 + 2k^4}$$

AMERICAN :-

$$W_B = W(k - 0.5); W_L = W - W_B$$

(FOR REFERENCES TO FULL STUDY OF PIGEAUD METHOD SEE DESCRIPTIVE BIBLIOGRAPHY).

FLAT SLABS.

("MUSHROOM FLOORS.")

(SEE ALSO CHAP. IV.)

$L < B$ AND $\frac{1}{3}B$

DIA. OF COLUMN SHAFT :- NOT LESS THAN 12", NOR $\frac{H_t}{15}$ BETWEEN FLOORS, NOR $.025(L + B)$

SLAB THICKNESS :- NOT LESS THAN $\frac{1}{16}L$, OR $.016(L + B)$ FOR FLOORS OR $.015(L + B)$ FOR ROOFS,

OR $.001(L + B)\sqrt{W + 1.5}$ WITHOUT DROPPED PANELS.

OR $.00083(L + B)\sqrt{W + 1.0}$ WITH DROPPED PANELS.

W = TOTAL LOAD LBS. PER SQ. FT.

DIA. OF COLUMN CAPITAL = $D < .225L$

SIZE OF DROPPED PANEL $< .4L \times .4B$. EFFECTIVE SPANS: $S_L = L - .67D$; $S_B = B - .67D$

MOMENTS :- TOTAL POSITIVE PARALLEL TO $L = \frac{1}{3600} W L S_B^2$; PARALLEL TO $B = \frac{1}{3600} W B S_L^2$ (INCH LBS.)

TO BE PROVIDED FOR IN CENTRE HALF: 30% WITH DROPPED PANELS, 40% WITHOUT.

TOTAL NEGATIVE PARALLEL TO $L = \frac{1}{2160} W L S_B^2$; PARALLEL TO $B = \frac{1}{2160} W B S_L^2$

TO BE PROVIDED FOR IN CENTRE HALF: 20% WITH DROPPED PANELS, 30% WITHOUT.

MOMENTS IN END SPANS AND AT PENULTIMATE SUPPORTS TO BE INCREASED BY 20%.

EXAMPLES OF USE OF TABLE No. 15.

(See also Chapter V.)

(a) The bases of the columns of a rectangular portal frame, 10 ft. wide and 15 ft. high, are equivalent to "hinged"; the beam and columns are of identical section. To find the moments at the head of the columns and at the centre of the beam, if a point load of 10,000 lb. is placed at the centre of the beam:

$$L = 10.0; H = 15.0; I_c = I_b = I; R = \frac{I}{I} \times \frac{15}{10} = 1.5.$$

From tabulated values, $\frac{A}{L} = \frac{WL}{8}$ for central point load

$$\frac{10,000 \times 10}{8} = 12,500 \text{ ft. lb.}$$

Moment at head of columns (= moment at ends of beam) = $M_B = M_\theta$

$$= \frac{3}{3 + 2R} \frac{A}{L} = \left(\frac{3}{3 + 2 \times 1.5} \right) 12,500 = 6,250 \text{ ft. lb. (Negative).}$$

From tabulated values, free B.M. for central point load

$$= \frac{WL}{4} = 25,000 \text{ ft. lb. (Positive)}$$

Less end moment = 6,250 ,, (Negative)

Mid-span = 18,750 ,, (Positive)

(b) To find the approximate wind moments and forces in the columns of a gantry support; the maximum load on each pair of columns is 100 tons, and the horizontal wind load = 5 tons. The height of the columns is 48 ft. braced at top and bottom and with intermediate braces at 12 ft. centres; distance apart of columns at base = 20 ft.:

$$P = 5 \text{ tons; } W = \frac{100}{4} = 25 \text{ tons per column.}$$

$$H = 48.0; h = 12.0; D = 20.0.$$

$$\text{B.M. in columns at junction with brace} = \frac{5 \times 12.0}{4} = 15 \text{ ft. tons.}$$

$$\text{B.M. in brace at junction with columns} = 2 \times 15 = 30$$

$$\text{The vertical load on the leeward column} = F_B = 50 + \frac{5 \times 48}{20} = 62 \text{ tons}$$

plus dead load of columns and braces.

$$\text{The vertical load on the windward column} = F_A = 50 - \frac{5 \times 48}{20} = 38 \text{ tons}$$

plus dead load of columns and braces.

These loads and moments are combined as explained in Chapter XIV (and Table No. 36) to design the sections or determine the stresses thereon.

NOTES.

FRAMED STRUCTURES.

TABLE N^o 15.

GENERAL FORMULAE FOR NON-ELASTIC SUPPORTS.

CONDITIONS.	M_{AB} .	M_{BA} .	$E = \text{YOUNG'S MODULUS}$
	$2EK(2\theta_A + \theta_B) - P$	$-2EK(2\theta_B + \theta_A) - Q$	$K = \frac{M.O.F. I}{\text{LENGTH}} = \frac{I}{L}$
	$2EK\theta_B - P$	$-4EK\theta_B - Q$	$\theta_A = \text{SLOPE AT A}$ $\theta_B = \text{SLOPE AT B}$
	$-P$	$-Q$	$P = \frac{2A}{L^2}(2L - 3Z)$
	$3EK\theta_A - (P + \frac{Q}{2})$	ZERO	$Q = \frac{2A}{L^2}(3Z - L)$
	$-(P + \frac{Q}{2})$	ZERO	$A = \text{AREA OF FREE MOMENT DIAG.}$ $Z = \text{DISTANCE OF CENTRE OF AREA OF FREE MOM. DIAG. FROM A.}$

WHEN MEMBER IS UNLOADED P AND $Q = \text{ZERO}$.

WHEN LOAD IS SYMMETRICAL $P = Q = \frac{A}{L} (= \text{B.M. AT END OF FIXED BEAM SIMILARLY LOADED})$

VALUES OF $\frac{A}{L}$ FOR SYMMETRICAL LOADS.

$W = \text{TOTAL LOAD.}$
 $L = \text{SPAN.}$

LOAD					
MAX. FREE B.M.	$\frac{WL}{8}$	$\frac{W}{8}(L + 2a)$	$\frac{WL}{4}$	$\frac{Wb}{2}$	$\frac{WL}{6}$
$\frac{A}{L}$	$\frac{WL}{12}$	$\frac{W}{12L}[6a(a+b)+b^2]$	$\frac{WL}{8}$	$\frac{Wb}{2L}(L-a)$	$\frac{5WL}{48}$

PORTAL FRAMES

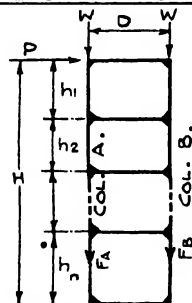
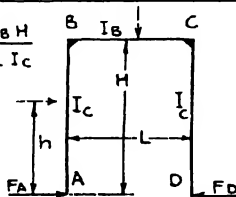
$$F_A = \frac{M_A - M_B}{H}$$

$$F_D = \frac{M_D - M_C}{H}$$

$$R = \frac{I_B H}{L I_C}$$

WHEN FIXED AT A AND D AND WITH LOAD ON AB: $M_C = -M_B$
WHEN HINGED AT A AND D AND WITH LOAD ON BC: $M_C = M_B$

WHEN HINGED AT A AND D AND WITH ANY LOADING: $M_A = M_D = 0$



$$F_A = W - \frac{Ph}{H}$$

$$F_B = W + \frac{Ph}{H}$$

$$\text{B.M. IN COL. AT ANY JOINT} = \frac{Ph}{4}$$

$$\text{SHEAR ON COLUMN} = \frac{P}{2}$$

$$\text{B.M. IN BRACE} = \frac{P}{4}(h_n + h_{n-1})$$

LOADS AND MOMENTS DUE TO DEAD WEIGHT OF COLS. AND BRACES TO ADD

BRACED COLUMNS.

ENDS FIXED	-	HORIZONTAL LOAD ON AB.	VERTICAL LOAD ON BC.
M_B	$M_B = M_B - \frac{Wh}{2} - \frac{3AZ}{H^2} \cdot \frac{R}{3+2R}$	$\frac{3A}{L(3+2R)}$	
M_A	$-X - \frac{A}{H^2}[Y(2+15R) - 3Z - T]$	$\frac{A}{L^2}(U - V)$	$M_C = M_B + \frac{2AV}{L^2}$
M_B	$\frac{3XR - AR}{1+3R} \cdot \frac{1}{H^2}[3Y - T]$	$-\frac{A}{L^2}(2U + V)$	
M_D	$X - \frac{A}{H^2}[Y(2+9R) - 3Z - T]$	$\frac{A}{L^2}(U + V)$	
WHERE	$T = \frac{H - 3Z}{2+R}$ $X = \frac{Wh(1+3R)}{2+12R}$ $Y = \frac{H}{1+6R}$	$U = \frac{L}{2+R}$ $V = \frac{3(L-2Z)}{1+6R}$	

EXAMPLES OF USE OF TABLE No. 16.

(a) To find the bending moment at the head of an exterior column partially fixed at the base, and with $L_c = 15$ ft., $L_B = 25$ ft., and $L_D = \text{nil}$. There is a uniformly distributed load of 1,000 lb. per ft. on the beam AB ; the latter is continuous at B , but cannot be considered rigidly fixed. Given that $I_B = 5,000$ ins.⁴ and $I_c = 6,000$

$$\text{Hence } K_B = \frac{5,000}{25} = 200; \quad K_c = \frac{6,000}{15} = 400;$$

$$\frac{K_c}{K_B} = \frac{400}{200} = 2$$

$$\text{From Table, for } \frac{K_c}{K_B} = 2 \text{ and } \frac{K_D}{K_B} = 0$$

$$C = 0.60 \text{ to } 1.09.$$

Since end conditions both at C and B are intermediate between "hinged" and "fixed," consider a mean value of $C = 0.85$.

From Table No. 15,

$$\frac{A}{I_c} \text{ for distributed load} = \frac{WL_B}{12} = \frac{1,000 \times 25^2}{12} = 52,000 \text{ ft. lb.}$$

$$\text{Hence } M_{AB} = C \cdot \frac{A}{I_B} = 0.85 \times 52,000 = 44,000 \text{ ft. lb.}$$

(b) A beam carrying 2,000 lb. per ft. over a span of 20 ft. is continuous at one end, and, for the purposes of the beam calculation, has been assumed freely supported on a column at the other end. To find the moment for which the column should be designed, assuming monolithic construction:

If the beam is an end span of a series of continuous beams it will have been designed for a mid-span bending moment of $\frac{WL^2}{10} = \frac{2,000 \times 20^2}{10} = 80,000 \text{ ft. lb.}$

The column should be designed for a moment of $\frac{1}{3} \times 80,000 = 26,700 \text{ ft. lb.}$ at its head, falling away to zero if hinged at the bottom, or to a reverse moment of 13,350 ft. lb. if fixed at the bottom.

NOTES.

CONTINUITY BETWEEN COLS. & BEAMS

TABLE No. 16.

EXTERIOR COLUMNS.

$$M_{AB} = -C \cdot \frac{A}{L_B}$$

$$M_{AD} = \frac{M_{AB}}{1 + \frac{K_C}{K_D}}$$

$$M_{AC} = M_{AB} - M_{AD}$$

FOR VALUES OF $\frac{A}{L}$

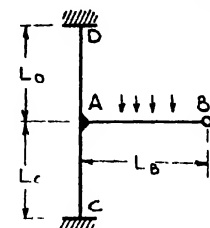
SEE TABLE No. 15.

I = MOM. OF INERTIA.

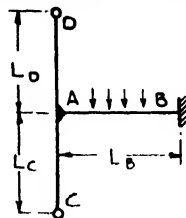
$$C = \frac{6(K_D + K_C)}{3K_B + 4K_C + 4K_D}$$

$$C = \frac{3(K_D + K_C)}{4K_B + 3K_C + 3K_D}$$

ANY SYMMETRICAL LOAD ON BEAM.



MAXIMUM



MINIMUM

$$K_B = \frac{I_B}{L_B}$$

$$K_C = \frac{I_C}{L_C}$$

$$K_D = \frac{I_D}{L_D}$$

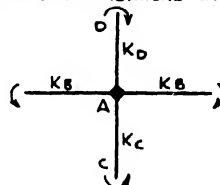
VALUES OF C

$\frac{K_b}{K_B}$	VALUES OF K_c/K_b													
	.25		.5		1.0		1.5		2.0		5		10	
	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.
10	.87	1.40	.89	1.40	.90	1.40	.90	1.40	.90	1.40	.92	1.42	.94	1.45
5	.80	1.31	.81	1.32	.82	1.33	.83	1.34	.84	1.35	.88	1.39	.92	1.42
2	.63	1.12	.65	1.15	.69	1.20	.72	1.23	.75	1.26	.84	1.35	.90	1.41
1.5	.57	1.05	.60	1.09	.65	1.15	.69	1.20	.72	1.24	.83	1.34	.89	1.40
1.0	.48	.94	.53	1.00	.60	1.09	.65	1.15	.69	1.20	.82	1.33	.89	1.40
.5	.36	.75	.43	.86	.53	1.00	.60	1.09	.65	1.15	.81	1.32	.89	1.40
.25	.27	.60	.37	.75	.48	.94	.57	1.05	.63	1.12	.80	1.30	.89	1.39
0	.16	.38	.27	.60	.43	.86	.53	1.00	.60	1.09	.79	1.30	.88	1.39

THIS CASE APPLIES TO A Γ SHAPED FRAME; IF $K_D = 0$ IN ABOVE FORMULAE.

INTERIOR COLUMNS.

ANY SYMMETRICAL LIVE LOAD ON BEAMS.



$$M_{AC} = C \cdot \frac{A}{L}$$

$$M_{AD} = M_{AC} \cdot \frac{K_D}{K_C}$$

B.M. IN BEAM IS IN ALL CASES LESS THAN B.M. AT SUPPORT AS COMPUTED BY THREE MOMENT THEORY.

$$C = \frac{K_C}{K_B + K_C + K_D} \text{ (APPROX.)}$$

VALUE OF $\frac{A}{L}$ SHOULD BE TAKEN FOR "LIVE" LOAD ONLY; SEE TABLE No. 15 FOR VALUES.

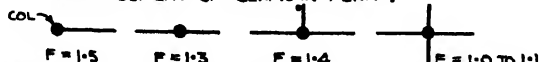
VALUES OF C

$\frac{K_D}{K_C}$	VALUES OF $\frac{K_C}{K_B}$					
	.25	.5	1.0	2.0	5.0	10.0
10	.02	.03	.08	.15	.31	.48
5	.04	.08	.14	.25	.45	.63
2	.08	.14	.25	.40	.63	.77
1.5	.09	.17	.28	.45	.67	.80
1.0	.11	.20	.33	.50	.72	.83
.5	.14	.25	.40	.57	.77	.87
.25	.17	.29	.45	.62	.80	.89
0	.20	.33	.50	.67	.83	.91

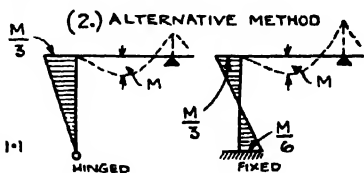
APPROXIMATE TREATMENT.

(1.) LOAD INCREASE FACTORS.

ARRANGEMENT OF BEAMS IN PLAN :-



EQUIVALENT LOAD ON COLUMN = F X ACTUAL LOAD.



EXAMPLES OF USE OF TABLE No. 17.

(a) To determine the breadth of a mass concrete foundation 10 ft. long, 2 ft. thick, carrying a load of 100 tons placed 1 ft. eccentric, founded on ordinary clay :

From Table, safe pressure = 2 tons per sq. ft.

Allowing for eccentric pressure (add 10 per cent.), $p = 2.2$ tons per sq. ft.

$$\text{Weight of base} = \frac{2 \times 130}{2,240} = 0.12 \text{ ton per sq. ft.}$$

$$\text{Available pressure} = 2.2 - 0.12 = 2.08 \text{ tons per sq. ft.}$$

$$\text{Since } \frac{L}{b} = \frac{10}{6} = 1.67 \text{ and } e = 1.0, e < \frac{L}{6}.$$

$$\therefore k = \left(1 + \frac{6 \times 1}{10}\right) = 1.6.$$

Rearranging the appropriate formula :

$$\text{Breadth of foundation} = \frac{1.6 \times 100}{10 \times 2.08} = 7.7 \text{ ft, say, 8 ft.}$$

(b) To find the maximum ground pressure under the base in (a) if base is made 8 ft. wide and load is 2 ft. eccentric :

Determine eccentricity of total load (base + applied load) :

$$\text{Wt. of base} = \frac{130}{2,240} \times 10 \times 8 \times 2 = 9.3 \text{ tons.}$$

$$\text{Moments about short side of base} = (9.3 \times 5) + (100 \times 3) = 346.5 \text{ ft. tons.}$$

$$\text{Total load} = 9.3 + 100 = 109.3 \text{ tons.}$$

$$\text{Eccentricity} = 5 - \frac{346.5}{109.3} = 1.83 \text{ ft. } \left(\because \frac{L}{6}\right).$$

$$p = \frac{4 \times 109.3}{8(10 - 2 \times 1.83)} = 2.88 \text{ tons per sq. ft.}$$

(If this pressure is not permissible the size of the base would have to be altered, or the relative positions of load and base re-arranged to give less eccentricity.)

(c) To find the bending moment at the centre of a base 50 ft. long and 5 ft. wide, carrying five unequal point loads as shown in the diagram on Table No. 17. The loads (in tons) are $W_1 = 50$, $W_2 = 45$, $W_3 = 40$, $W_4 = 35$, and $W_5 = 30$; the distances are $x_1 = 45$, $x_2 = 37$, $x_3 = 28$, $x_4 = 18$, and $x_5 = 5$ ft. Thus $\Sigma W = 200$ and $\Sigma Wx = (50 \times 45) + (45 \times 37) + (40 \times 28) + (35 \times 18) + (30 \times 5) = 5,815$.

$$\therefore e = \frac{5,815}{2,000} - \frac{50}{2} = 4.075 \text{ ft. and is } < \frac{L}{6} = 8\frac{1}{6} \text{ ft.}$$

$$\therefore k = 1 \pm \frac{6 \times 4.075}{50} = 1.489 \text{ and } 0.511.$$

$$\text{Maximum pressure: } p_1 = \frac{1.489 \times 200}{50 \times 5} = 1.19 \text{ tons per sq. ft.}$$

$$\text{Minimum pressure: } p_2 = \frac{0.511 \times 200}{50 \times 5} = 0.41 \text{ " " "}$$

At the middle of the base $a = 25$ ft.

$$\therefore p_x = \left(\frac{50 - 25}{50}\right)(1.19 - 0.41) + 0.41 = 0.80 \text{ ton per sq. ft.}$$

Also $x_1 = 20$, $x_2 = 12$, and $x_3 = 3$

$$\therefore \Sigma Wx = W_1x_1 + W_2x_2 + W_3x_3 = 1,000 + 540 + 120 = 1,660.$$

B.M. at x.x.

$$= 1,660 - \frac{25^2}{6}(2 \times 1.19 + 0.80) = 1,660 - 331 = 1,329 \text{ ft. tons.}$$

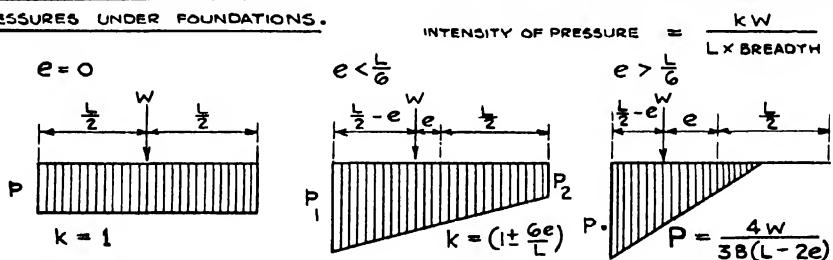
The B.M. at other sections could be found in a similar manner.

FOUNDATIONS.

TABLE No. 17.

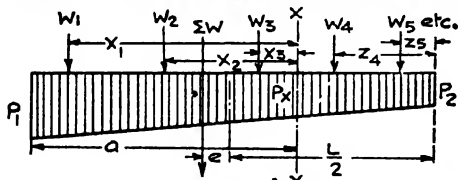
SAFE PRESSURES ON GROUND.				SAFE PRESSURES ON VARIOUS MATERIALS.		
NATURE OF GROUND	SAFE PRESSURE		MINIMUM DEPTH	MATERIAL	SAFE PRESSURE	
	GENERAL	L.C.C.			GENERAL	L.C.C.
MUD, MUDDY CLAYS, ETC.	0 TO 1/4	-	3'-0"	CONCRETE - MIX A	10	-
ALLUVIAL EARTH	1/2 TO 3/4	-	2'-0"	CONCRETE - MIX C	15	12
ARTIFICIAL FILLING	1/4 TO 3/4	-	2'-0"	BLUE BRICK	12	12
CLAY - SOFT	1	1	2'-0"	HARD BRICK (LONDON STOCK)	8	8
" - ORDINARY	2	2	2'-6"	ORDINARY BRICK	6	5
" - DRY	3	-	3'-0"	GRANITE	15	-
" - SOUND YELLOW	4 TO 6	-	3'-0"	PORTLAND STONE	12	-
" - BLUE	5 TO 8	4	3'-0"	LIMESTONE	9	-
SAND - WET OR LOOSE	1	1	4'-0"	ALL BRICKWORK IN P.C. MORTAR.		
" - DRY WITH CLAY	2	-	3'-6"	<p>ALL SAFE BEARING PRESSURES GIVEN IN TONS PER SQ. FOOT.</p> <p>MINIMUM DEPTH OF FOUNDATION LEVEL BELOW NATURAL SURFACE TO PREVENT SPOILING:-</p> $h = \left(\frac{\text{SAFE PRESSURE}}{\text{WT. PER CU. FT.}} \right) K_1$ <p>SEE TABLE No 5 FOR VALUES OF</p> $K_1 = \frac{1 - \sin \theta}{1 + \sin \theta}$ <p>WHEN MINIMUM DEPTHS TABULATED ARE EXCEEDED, EXTRA SAFE PRESSURE = EXTRA DEPTH X WT. PER CU. FT.</p> <p>FOR ECCENTRIC LOADS, MAX. SAFE PRESSURE = 1.1 X TABULATED PRESSURES.</p>		
" - DRY OR CONFINED	3	2	3'-6"			
" - IN RIVER BED	3 1/2 TO 4	-	25'-0"			
GRAVEL - COMPACT	4 TO 7	4	3'-0"			
" - BOULDER	5 TO 7	-	3'-0"			
" - WITH SAND	6	-	20'-0"			
MARL, FIRM SHALE	5 TO 7	-	-			
CHALK - SOFT	1 1/2	-	-			
" - HARD WHITE	2 1/2 TO 4	4	-			
ROCK - VERY SOFT	2	-	-			
" - MODERATELY HARD	9	-	-			
" - HARD	12 (MIN)	-	-			

PRESSURES UNDER FOUNDATIONS.



COMBINED FOUNDATIONS.

- (1) METHOD APPLICABLE TO ANY NUMBER OF LOADS. (ALSO APPLICABLE WHEN $e = 0$ I.E. WHEN $P_1 = P_2 = P_x$)



$$e = \frac{\sum WZ}{\sum W} - \frac{L}{2}$$

CALCULATE P_1 AND P_2 AS ABOVE.

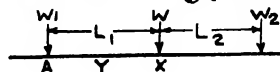
$$P_x = \left(\frac{L - a}{L} \right) (P_1 - P_2) + P_2$$

$$\text{B.M. AT } XX = \sum WX - \frac{a^2}{6} (2P_1 + P_x)$$

- (2) APPROXIMATE METHOD FOR MORE THAN FIVE LOADS.

B.M. AT X = $\frac{WL_1L_2}{6(L_1 + L_2)}$

B.M. AT Y = $\frac{MA + Mx}{3}$



EXAMPLES OF USE OF TABLE No. 18.

(a) To design a base for a 15-in. square column carrying 60 tons, (i) with ground pressure not exceeding 4 tons per sq. ft., (ii) with ground pressure not exceeding $1\frac{1}{2}$ tons per sq. ft.

(i) Adopt type (a):

$B \nabla \sqrt{14.4 \times 60} = 29.4$ in.; say, reinforced concrete footing 30 in. square

$d \nabla 4.67 \times 60 \left(\frac{1}{15} - \frac{15}{30^2} \right) = 14$ in.; reinforced concrete footing 15 in. deep.

Check: $B \nabla 15 + (2 \times 15) = 45$ in. Satisfactory.

$A \nabla \sqrt{\frac{144 \times 60}{4}} = 46.5$, say, mass concrete base 48 in. square.

$D \nabla 7 \times 60 \left(\frac{1}{30} - \frac{30}{48^2} \right) = 8.4$ in.; mass concrete base 18 in. thick (min.).

Check: $A \nabla 30 + (2 \times 18) = 66$ in. Satisfactory.

(ii) Adopt type (b):

$A \nabla 12 \sqrt{\frac{\text{say } 62}{1.5}} = 77$ in., say, 6-ft. 6-in. square.

$R = \frac{15}{78} = 0.192$.

For punching shear: $D \nabla \frac{4.67 \times 60}{78} \left(\frac{1}{0.192} - 0.192 \right) = 18$ in.

B.M. on x.x. = $\frac{60 \times 78}{24} (2 + 0.192)(1 - 0.192)^2 = 281$ in. tons.

NOTES.

Independent Bases, Types (b) and (c).

The resistance moment and area of steel required for these types of bases and similar types where the base is uniform in thickness, can be determined from the following formulæ.

If d_1 = maximum effective depth (inches).

d_2 = minimum effective depth (inches).

Q = resistance moment factor (see Table No. 27).

R.M. = $Qd_1^2 \left[C + \frac{H}{8} \right]$.

where $H = (A - C + 2d_1) \left(1 + \frac{d_1}{d_2} \right)^2$.

For bases of uniform thickness, R.M. = $Qd_1^2 \left(\frac{A + C}{2} + d_1 \right)$.

For all bases, the total area of the reinforcement is found from

$$A_T = \frac{2240 \text{ (B.M. on x.x.)}}{a_1 d_1 t}$$

where t = allowable tensile stress.

a_1 = lever arm factor (see Table No. 27).

$d_3 = d_1$ if 80 per cent. of the reinforcement is placed in a band of width equal to $C + 2d_1$ placed symmetrically under the column, or if $Qd_1^2 C$ is not less than the applied B.M. If all the reinforcement is uniformly spaced throughout the width of the base, then

$$d_3 = \frac{A + C + 2d_1}{2A} \left[d_2 + (d_1 - d_2) \left(\frac{A + C}{2A} \right) \right].$$

or for bases of uniform thickness $d_3 = \frac{A + C + 2d_1}{2A d_1}$.

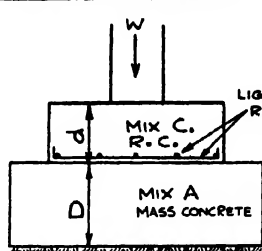
When $A \nabla C + 2d_1$ bending moments can be neglected and only nominal reinforcement provided.

The foregoing expressions can be readily modified to apply to a rectangular base.

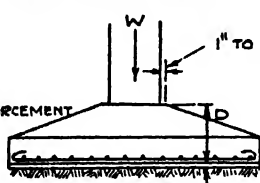
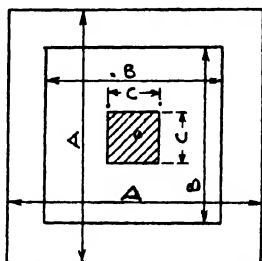
FOUNDATIONS .

TABLE N°16

INDEPENDENT BASES WITH CONCENTRIC LOADS .

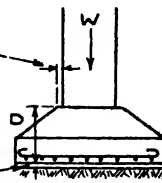
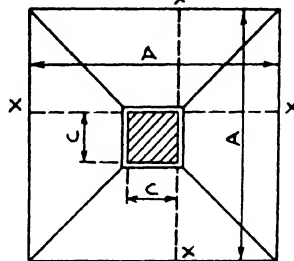


(a)



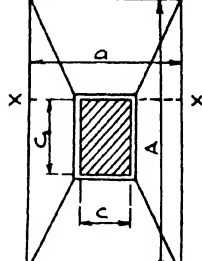
(b) SQUARE BASE

$$R = \frac{C}{A}$$



(c) RECTANGULAR BASE

$$R = \frac{C}{A} \quad r = \frac{C}{a}$$



ALL DIMENSIONS IN INCHES.

W = LOAD FROM COLUMN IN TONS.

W_b = WEIGHT OF BASE IN TONS.

P = SAFE BEARING PRESSURE ON GROUND IN TONS PER SQ. FT.

(a)

$$B \nless 14.4 W \nless C + 2d$$

$$A \nless \sqrt{\frac{144 W}{P}} \nless B + 2d$$

$$d \nless 4.67 W \left(\frac{1}{C} - \frac{C}{B^2} \right)$$

$$D \nless 7 W \left(\frac{1}{B} - \frac{B}{A^2} \right)$$

(b)

$$A \nless 12 \sqrt{\frac{W + W_b}{P}}$$

B.M. ON X.X. IN INCH-TNS:-

$$= \frac{W A}{24} (2 + R) (1 - R)^2$$

$$D \nless \frac{4.67 W}{A} \left(\frac{1}{R} - R \right)$$

(c)

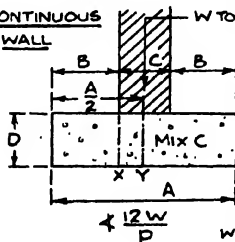
$$A d \nless \frac{144 (W + W_b)}{P}$$

B.M. ON X.X. IN INCH-TNS:-

$$= \frac{W A}{24} (2 + r) (1 - R)^2$$

$$D \nless 9.33 \left(\frac{1 - R r}{C_1 + C} \right)$$

CONTINUOUS WALL



W TONS PER LINEAL FOOT.

$$D \nless \frac{1.56 W B}{A}$$

(OR AS REQUIRED FOR BENDING MOMENT)

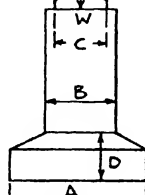
B.M. AT X = $\frac{W B^2}{2A}$

B.M. PER FOOT AT Y:-

$$= \frac{W B^2}{2A} + \frac{W B C}{4A}$$

WHEN B < D, B.M. = ZERO.

MASS CONCRETE PIERS. MIX A.



(SQUARE)

$$C \nless \sqrt{14.4 W}$$

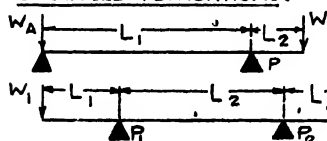
$$B \nless C$$

$$A \nless 12 \sqrt{\frac{W T}{P}} \nless B + 2D$$

WHERE W_T = W + WT. OF PIER

$$D \nless 7 W_T \left(\frac{1}{B} - \frac{B}{A^2} \right)$$

BALANCED FOUNDATIONS.



(MOMENTS AND REACTIONS DUE TO DEAD WT. OF BEAM TO BE ALLOWED FOR IN ADDITION.)

MIN. ANCHORAGE WT. = W_A = $\frac{W L_2}{L_1}$; $P = W \left(1 + \frac{L_2}{L_1} \right)$

$$P_1 = \frac{W_1 (L_2 + L_1) - W_2 L_3}{L_2}$$

$$P_2 = \frac{W_2 (L_2 + L_3) - W_1 L_1}{L_2}$$

NOTE:- BASES TO TAKE P, ETC., DESIGNED FOR CONCENTRIC LOAD.

EXAMPLES OF USE OF TABLE No. 19.

(a) To find the safe load on a pile 40 ft. long, 14 in. square, driven down to a gravel bed. Driving record as follows: 2 tons single acting steam hammer dropping 42 in.; 12 blows per final inch of penetration; weight of helmet, dolly, and stationary part of hammer = $\frac{1}{2}$ ton; dolly in good condition at end of driving; weight of pile = $\frac{40 \times 196}{2,240} = 3\frac{1}{2}$ tons; $P = 3\frac{1}{2} + \frac{1}{2} = 4$ tons; $\frac{P}{w} = \frac{4}{2} = 2$:

(i) By Dutch Formula:

From Table No. 19 with $\frac{P}{w} = 2$ and $w = 2$, safe load may be from 40 to 60 tons for $Hn = 360$. Adopting lower value with dolly in good condition, $Hn = 42 \times 12 = 504$, and allowing for 10 per cent. increase due to type of hammer, safe load = $40 \times \frac{504}{360} \times 1.1 = 49$ tons.

(ii) By Hiley Formula:

Effective drop = $H_1 = 0.90 \times 42 = 38$ in.

For $\frac{P}{w} = 2$, $e = 0.37$.

For medium driving, $c = 0.295$ for $L = 40$ ft.
Settlement load

$$= \frac{2 \times 38 \times 0.37 \times 12}{1 + 0.295 \times 12} + 2 + 4 = 73.5 + 6 = 79.5 \text{ tons}$$

(Check: driving pressure = $\frac{79.5 \times 2,240}{14^2} = 910$ lb.

Easy	driving	=	500	lb	per	sq.	in.
Medium	"	=	1,000	"	"	"	"
Hard	"	=	1,500	"	"	"	"
Very hard	"	=	2,000	"	"	"	"

Safe load = say $\frac{79.5}{2} = 40$ tons.

Hence by two different formulae the safe load seems to be between 40 and 50 tons.

(b) To find the safe load on a 15-in square pile driven 40 ft. into firm mud without finding any harder strata:

By Rhode formula: $k_1 = 0.0001$; $k_2 = 0.021$
 $B = D = 15$ in.

Safe load = $40[(0.0001 \times 40 \times 30) + 0.021] = 5.64$ tons, say, 6 tons exclusive of weight of pile.

(c) To find safe load on same pile as in (b) if bearing on wet clay:

k_3 (applicable to material driven through) = 0.015

k_4 (applicable to material founded upon) = 0.023

$W = (40 \times 0.015 \times 30) + (0.023 \times 225) = 23$ tons including weight of pile.

NOTES.

In all pile loading calculations it must be remembered that a formula can only pretend to give comparative values that must be combined with the results of tests and experience when assessing the safe load to which a given pile, driven under certain conditions, can be subjected.

PILES.

TABLE No 19.

DUTCH FORMULA.

$$\text{SAFE LOAD} = W = \frac{W^2 H n}{C(P+W)} \quad C = 4 \text{ TO } 6$$

W = WEIGHT OF HAMMER IN TONS.

H = DROP OF HAMMER IN INCHES.

P = WEIGHT OF PILE, HELMET, ETC.

n = NUMBER OF BLOWS PER INCH PENETRATION.

VALUES OF W FOR n = 10 AND H = 36" (TONS) MAX^m W: C = 4. MIN^m W: C = 6.

W	1/2 TON.		3/4 TON.		1 TON.		1 1/2 TON.		2 TON.		3 TON.		4 TON.	
P/W	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.	MIN.	MAX.
1/2	-	-	-	-	40	60	60	90	80	120	120	180	160	240
1	-	-	23	34	30	45	45	68	60	90	90	135	120	180
1 1/2	12	13	18	27	24	36	36	54	48	72	72	108	96	144
2	10	15	15	23	20	30	30	45	40	60	60	90	80	120
2 1/2	9	13	13	20	17	26	26	39	34	51	51	77	68	102
3	8	12	12	17	15	23	23	34	30	45	45	68	60	90
3 1/2	7	10	10	15	13	20	20	30	26	40	40	60	52	80
4	6	9	9	14	12	18	18	27	24	36	36	54	48	72

USE MAXIMUM VALUE IF NO HELMET IS USED OR IF DOLLY IS DESTROYED AT END OF DRIVING.

USE MINIMUM VALUE IF DOLLY AND PACKING ARE IN GOOD CONDITION AT END OF DRIVING.

FOR OTHER VALUES OF n AND H: $W = \frac{\text{TABULATED VALUES} \times H n}{360}$

VALUES OF W CAN BE INCREASED BY 10% IF SINGLE ACTING STEAM HAMMER IS EMPLOYED.

HILEY FORMULA. (MODIFIED)

$$\text{SETTLEMENT LOAD} = W_M = \frac{W H_1 e n}{1 + c n} + W + P$$

SAFE LOAD = $\frac{W_M}{3}$ TO $\frac{W_M}{1.5}$ (FOR SIGNIFICATION OF W, n, AND P SEE ABOVE)
(LOWER FACTOR OF SAFETY ASSOCIATED WITH HARDER DRIVING)

VALUES OF $\frac{\text{EFFECTIVE DROP (H}_1\text{)}}{\text{ACTUAL DROP}}$:- FREELY FALLING DROP HAMMER = 1.00
SINGLE ACTING STEAM HAMMER = .90
WINCH OPERATED DROP HAMMER = .80

VALUES OF P/W	1/2	1	1 1/2	2	2 1/2	3	4	5	6	7	8
EFFICIENCY OF BLOW = e	.69	.53	.44	.37	.33	.30	.25	.21	.19	.17	.15
VALUES OF C = TEMPORARY ELASTIC COMPRESSION FACTOR.	LENGTH OF PILE	10'-0"	20'-0"	30'-0"	40'-0"	50'-0"	60'-0"	TABULATED VALUES ARE MINIMUM AND ASSUME DOLLY AND PACKING IN GOOD CONDITION, AND GRAVEL SOIL. FOR MEDIUM DRIVING ADD :- FOR CLAY SOIL .05" FOR SOFT AND PEATY SOIL .20"			
	500 LBS./IN ² EASY DRIVING	.125	.128	.155	.170	.185	.200				
	1000 LBS./IN ² MEDIUM DRIVING	.205	.235	.265	.295	.325	.355				
	1500 LBS./IN ² HARD DRIVING	.285	.325	.370	.415	.460	.505				
	2000 LBS./IN ² VERY HARD DRIVING	.335	.395	.455	.515	.575	.635				

FRICTION PILES. (RECTANGULAR SECTION: B' x D')

(i) CONSISTENTLY SOFT MATERIAL :-

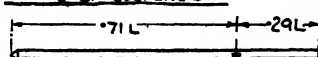
$$W = L_e [k_1 L_e (B+D) + k_2] \quad (\text{RHODE MODIFIED})$$

(ii) THROUGH SOFT MATERIAL AND BEARING ON SOMEWHAT FIRMER STRATA :-

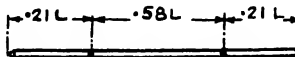
$$W = L_e k_3 (B+D) + k_4 B D \quad (\text{ADAPTED FROM A. HUNTER})$$

L_e = EMBEDDED LENGTH OF PILE IN FEET.

POINTS OF SUSPENSION (TO GIVE MINIMUM MOMENTS)



SINGLE POINT
LIFTING.



DOUBLE POINT
LIFTING

VALUES OF FACTORS K

TYPE OF GROUND	k ₁	k ₂	k ₃	k ₄
SOFT SILT	-	-	.012	-
FIRMER SILT, STIFF MUD, OR LOAM	.0001	.021	.015	.007
WET CLAY	.0006	.043	.016	.023
DRY CLAY OR SAND	.0014	.091	.033	.042
COARSE SAND OR GRAVEL	.0022	.143	.043	.114

EXAMPLES OF USE OF TABLE No. 20.

(a) To design a retaining wall (Type *a*) for the following conditions : $h = 10$ ft. ; $H = 12$ ft. $P_0 = 2,000$ lb. per ft. run of wall. $z = 3$ ft. 6 in. $P = 2,150$ " " " $y = 5$ ft. 3 in.

Weight of earth = 100 lb. per cu. ft. ; surcharge = 224 lb. per sq. ft.

B.M. at A.A. = $2,000 \times 3$ ft. 6 in. = 7,000 ft. lb. = 84,000 in. lb.

From Table No. 31, stresses 16,000 and 600, a suitable section for the wall stem would be 10 in. Therefore stem of wall at *AA* and base of wall at edge of haunches would be 10 in. thick tapering down to 6 in.

Assume $L = 6$ ft. and toe projects 6 in. in front of wall face.To determine W and x (by moments about toe of wall) :

Wall stem : 11 ft. 2 in. \times 96 av.	= 1,070 lb. \times 1 ft.	= 1,070 ft. lb.
Wall base : 6 ft. \times 96 av.	= 576 " \times 2 ft. 6 in.	= 1,440 "
Haunches : say	= 70 " \times 1 ft.	= 70 "
Earth : 4 ft. 8 in. by 11 ft. 4 in. \times 100 =	5,300 " \times 3 ft. 8 in.	= 19,400 "
Surcharge : 4 ft 8 in. \times 224	1,045 " \times 3 ft. 8 in.	= 3,840 "
	$W = 8,061$ lb.	$Wx = 25,820$ ft. lb.

$$x = \frac{25,820}{8,061} = 3.2 \text{ ft.}$$

$$\text{Stability : } -x < \frac{2,150 \times 5.25}{8,061} = 2.1 \text{ ft. Satisfactory.}$$

$$\text{Sliding : } -W < 3.75 \times 2,150 = 8,060 \text{ lb. Satisfactory.}$$

$$\text{Ground Pressure : } -e = \frac{2,150 \times 5 \text{ ft. } 3 \text{ in.}}{8,061} + \frac{6}{2} = 3.2 = 1.20$$

$$6e = 1.2 \times 6 = 7.2 \text{ which is greater than } L = 6 \text{ ft.}$$

Per appropriate formula given on Table No. 17

$$p = \frac{4 \times 8,061}{3 \times 1 (6 - 2 \times 1.2)}$$

$$\text{Max. press.} = 1.32 \text{ ton per sq. ft.}$$

(b) To design a sheet pile wall 12 ft high, freely cantilevering, with angle of repose of material = 35 deg. :

Here $H = 12$ ft. $\theta = 35$ deg.Hence $k_1 = 1.4$. $k_2 = 1.0$ Driven length of pile = $1.0 \times 12 = 12$ ft.Total length of pile = $12 + 12 = 24$ ft.Span of sheeting for purpose of calculating bending moment = $L = 1.4 \times 12 = 16.8$ ft.

The pressures in front of and behind the wall, and the point of application of these pressures can be computed as indicated on Table No. 5. Having calculated the moments, suitable sections can be determined from Chapter XI or Table No. 31.

(c) To determine the moments and forces in the walls of a cylindrical water tank 20 ft. in diameter, 30 ft. deep, with walls 10 in. thick at bottom :

Here $D = 20$ ft. ; $H = 30$ ft. ; $d = 0.83$ ft. ; $w = 62.4$ lb. per cu. ft.

$$\text{With } \frac{H}{D} = \frac{30}{20} = 1.5 ; \frac{H}{d} = \frac{30}{0.83} = 36 ;$$

$$F = 0.003 ; K = 0.20.$$

$$\text{B.M. (vertical) at } A : \text{B.M.} = 0.003 \times 62.4 \times 30^3 = 5,050 \text{ ft. lb.}$$

$$\text{Position of point of maximum ring tension : } L = 0.20 \times 30 = 6 \text{ ft. from bottom.}$$

$$\text{Max. ring tension} = 0.5 \times 62.4 \times 30 \times 20(1 - 0.20) = 15,000 \text{ lb. per ft. height of wall.}$$

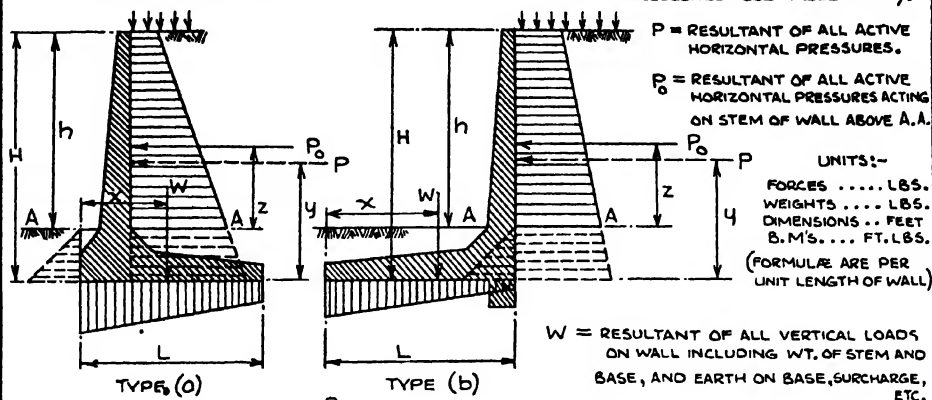
Horizontal steel (and concrete thickness) would be determined in accordance with formulae given in Chapter VII, and vertical steel at bottom in accordance with methods given in Chapter XI.

RETAINING WALLS & TANK WALLS

TABLE No 20

CANTILEVER RETAINING WALLS.

(FOR CALCULATION OF EARTH PRESSURES SEE TABLE No 5).



UNITS:-
 FORCES LBS.
 WEIGHTS LBS.
 DIMENSIONS .. FEET
 B.M.'S.... FT.LBS.
 (FORMULAE ARE PER UNIT LENGTH OF WALL)

STABILITY : $X \nless 1.5 \frac{Py}{W}$

SLIDING : APPLICABLE TO DESIGNS OMITTING RIB SHOWN IN (b).

$W \nless 3.75P$

GROUND PRESSURE : e = ECCENTRICITY OF COMBINED P AND $W = \frac{Py}{W} + \frac{L}{2} - X$
 SUBSTITUTE e IN APPROPRIATE FORMULA GIVEN ON TABLE No 17.

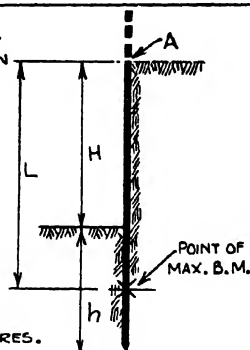
BENDING MOMENT : AT A.A. = $P_0 z$

SHEET PILE WALLS

SPAN OF SHEETING FOR MOMENT CALCULATION
 $= L = K_1 H.$

θ	CONDITION AT A					
	FREE		HINGED		FIXED	
	K_1	K_2	K_1	K_2	K_1	K_2
20°	2.0	2.0	1.23	1.23	1.11	1.11
30°	1.5	1.3	1.08	.88	1.07	.87
35°	1.4	1.0	1.07	.67	1.06	.66
45°	1.3	.8	1.06	.56	1.05	.55

MINIMUM EMBEDDED LENGTH OF SHEETING
 $= h = K_2 H$

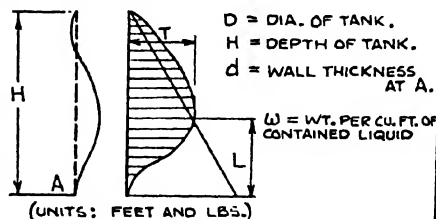


θ = ANGLE OF REPOSE OF EARTH IN FRONT OF SHEETING.

TABULATED VALUES ASSUME APPROX. TRIANGULARLY DISTRIBUTED PRESSURES.

RESTRAINT AT BASE OF WALLS OF CYLINDRICAL TANKS.

FACTORS		F				K			
H/d		10	20	30	40	10	20	30	40
VALUES OF I/D	1	.075	.047	.036	.028	-	-	-	-
	2	.046	.028	.022	.015	-	.50	.45	.40
	3	.032	.019	.014	.010	.55	.43	.38	.33
	4	.024	.014	.010	.007	.50	.34	.35	.30
	5	.020	.012	.009	.006	.45	.37	.32	.27
	1.0	.012	.006	.005	.003	.37	.28	.24	.21
	2.0	.006	.003	.002	.002	.30	.22	.19	.16
	4.0	.004	.002	.002	.001	.27	.20	.17	.14



(UNITS: FEET AND LBS.)
 B.M. AT A: $M = F W H^3$
 MAX. RING TENSION: $T = .5 W H \cdot D (1 - K)$
 POSITION OF MAX. RING TENSION: $L = K H.$

EXAMPLE OF USE OF TABLE No. 21.

(a) A single rectangular cell measuring in the clear 12 ft. 9 in. by 10 ft. 6 in. without cross walls or ties, and subject to a uniform horizontal pressure of 200 lb. per sq. ft. at a given depth; to find the maximum moments and horizontal direct tensions at the given depth, assuming the walls span horizontally:

Assuming the walls are 6 in. thick, the effective spans of the walls will

$$\text{be } 13 \text{ ft. } 3 \text{ in. and } 11 \text{ ft.}; \text{ hence } \frac{B}{D} = \frac{13.25}{11.0} = 1.2.$$

$k = 9.7$ (approx.) from Table.

$$M_1 \text{ -- B.M. at corners } = \frac{PD^2}{k} = \frac{200 \times 11^2}{9.7} = 2,500 \text{ ft. lb.}$$

$$\text{Free moment in } D = \frac{200 \times 11^2}{8} = 3,025 \text{ ft. lb.}$$

$$\text{Less corner moment} = 2,500 \text{ ,,}$$

$$\text{Positive B.M. at mid-span of } D = 525 \text{ ,,}$$

$$\text{Free moment in } B = \frac{200 \times 13.25^2}{8} = 4,390 \text{ ft. lb.}$$

$$\text{Less corner moment} = 2,500 \text{ ,,}$$

$$\text{Positive B.M. at mid-span of } B = 1,890 \text{ ,,}$$

Direct tension in short side

$$= \frac{PB}{2} = 0.5 \times 200 \times 12.75 = 1,275 \text{ lb. per ft.}$$

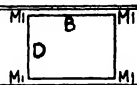
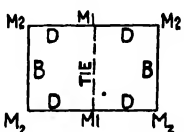
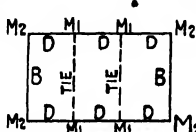
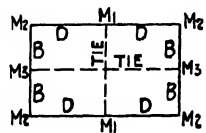
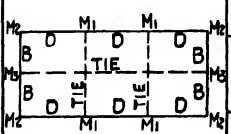
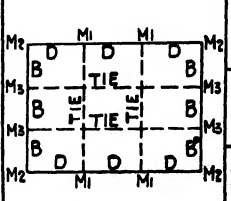
Direct tension in long side

$$= \frac{PD}{2} = 0.5 \times 200 \times 10.5 = 1,050 \text{ lb. per ft.}$$

The appropriate moments and tensions would be combined as described in Chapter XIV.

NOTES.

MOMENTS IN BIN WALLS. TABLE №21
(WITH UNIFORM HORIZONTAL PRESSURE (P) AT ANY GIVEN DEPTH).

FORM OF BIN	BENDING MOMENTS. $M = \frac{PD^2}{K}$								REACTION FORMULAE.
	FORMULAE.	BENDING MOMENT COEFFICIENTS k							
		VALUES OF $\frac{B}{D}$							
		0.5	0.8	1.0	1.2	1.5	1.8	2.0	
	$M_1 = \frac{P(D^3 + B^3)}{12(D + B)}$	16	14.3	12	9.68	6.86	4.92	4	$R_B = \frac{PB}{2}$ $R_D = \frac{PD}{2}$
	$M_1 = \frac{P(D^3 + 3D^2B - B^3)}{12(D + 2B)}$ $M_2 = \frac{P(D^3 + 2B^3)}{12(D + 2B)}$	10.1	10.8	12	14.2	22.6	97	MINUS 60	$R_B = \frac{PB}{2}$ $R_D = \frac{PD}{2} + \frac{M_2 - M_1}{D}$
	$M_1 = \frac{P(3D^3 + 6D^2B - B^3)}{12(3D + 5B)}$ $M_2 = \frac{P(3D^3 + 5B^3)}{12(3D + 5B)}$	11.2	11.5	12	12.75	14.7	18	22.3	$R_B = \frac{PB}{2}$ $R_D \text{ (END SPAN)} = \frac{PD}{2} + \frac{M_2 - M_1}{D}$ $R_D \text{ (CENTRE SPAN)} = \frac{PD}{2}$
	$M_1 = \frac{P(D+B)(2D-B)}{24}$ $M_2 = \frac{P(D^3 + B^3)}{12(D+B)}$ $M_3 = \frac{P(D+B)(2B-D)}{24}$	10.68	11.1	12	13.6	19.2	42.9	-	$R_B = \frac{PB}{2} + \frac{M_2 - M_3}{B}$ $R_D = \frac{PD}{2} + \frac{M_2 - M_1}{D}$
	$M_1 = \frac{P(6D^3 + 6D^2B - B^3)}{12(6D + 5B)}$ $M_2 = \frac{P(6D^3 + 5B^3)}{12(6D + 5B)}$ $M_3 = \frac{P(5B^3 + 9DB^2 - 3D^3)}{12(6D + 5B)}$	11.5	11.66	12	12.55	13.95	16.4	19.2	$R_B = \frac{PB}{2} + \frac{M_2 - M_3}{B}$ $R_D \text{ (END SPAN)} = \frac{PD}{2} + \frac{M_2 - M_1}{D}$ $R_D \text{ (CENTRE SPAN)} = \frac{PD}{2}$
	$M_1 = \frac{P(6D^3 + 6D^2B - B^3)}{60(D + B)}$ $M_2 = \frac{P(D^3 + B^3)}{12(D + B)}$ $M_3 = \frac{P(6B^3 + 6DB^2 - D^3)}{60(D + B)}$	11.4	11.61	12	12.6	14.11	16.71	20	$R_B \text{ (END SPAN)} = \frac{PB}{2} + \frac{M_2 - M_3}{B}$ $R_B \text{ (CENTRE SPAN)} = \frac{PB}{2}$ $R_D \text{ (END SPAN)} = \frac{PD}{2} + \frac{M_2 - M_1}{D}$ $R_D \text{ (CENTRE SPAN)} = \frac{PD}{2}$

NOTE :-

IN ADDITION TO BINS, THESE VALUES APPLY TO CONTINUOUS SPANS, WHERE INDEFINITE CONTINUITY, IN SIMILAR SEQUENCE, OCCURS.

EXAMPLES OF USE OF TABLE No. 22.

(a) For an example of the design of hopper bottoms see the typical examples given in the pages following Table No. 40.

(b) To determine the principal forces on the bottom of a "balanced" cylindrical tank (i.e. Intze type), given the following data:

$D = 40$ ft.; $d = 25$ ft.; $\phi = 48$ deg.; $\theta = 40$ deg.
 $W_1 = 582,000$ lb. $W_2 = 639,000$ lb. $W_3 = 296,000$ lb.

From Table No. 10; $\operatorname{cosec} \theta = \frac{1}{\sin \theta} = 1.55$; $\cot \theta = 1.192$

$$\cot \phi = 0.900.$$

Vertical shear along periphery of domed portion $= F_1 = \frac{582,000}{3.14 \times 25} = 7,400$
 lb. per ft. run.

Thrust at periphery of domed portion $= T_1 = 7,400 \times 1.55 = 11,450$ lb.
 per ft. run.

The values of F_1 and T_1 determine the thickness of the dome at the springing (see Chapter VII).

Outward horizontal thrust on ring beam at B from dome $= P_1 = 7,400 \times 1.192 = 8,820$ lb. per ft.

Shear per foot along inner periphery of conical portion

$$F_2 = \frac{639,000 + 296,000}{3.14 \times 25} = 11,900 \text{ lb. per ft.}$$

Inward thrust on ring beam at B from conical section $= P_2 = 11,900 \times 0.900 = 10,700$ lb. per ft.

Resultant circumferential force in ring beam at $B = \frac{d}{2}(P_1 - P_2) = 0.5 \times 25 (8,820 - 10,700) =$ thrust of 23,500 lb.

(If P_1 exceed P_2 , this circumferential force would be tensile; the ideal case is for P_1 and P_2 to be equal, thereby producing zero force in B —see note below.)

Shear per foot along outer periphery of conical portion $= F_3 = \frac{296,000}{3.14 \times 40} = 2,350$ lb. per ft.

Outward thrust on ring beam at top of conical portion $= P_3 = 2,350 \times 0.900 = 2,120$ lb. per ft.

Ring tension in beam at top of conical section $= 0.5 \times 40 \times 2,120 = 42,400$ lb.

Vertical walls must be reinforced for ring tension due to the horizontal pressure of water; ring tension $= \frac{62.4 D h}{2}$ lb. where h = head of water at section considered.

Conical portion must be reinforced to take a similar ring tension, and this steel can either be distributed throughout the height of the conical section, or be concentrated in the ring beams at the top and bottom of the section.

NOTES.

In large diameter Intze type tanks, the width of ring beam b may be considerable, in which case the weight of water immediately above the beam should not be considered as contributing to the forces on the dome and conical portion. With a wide beam, for the dome calculations

W_1 = weight of contents over the net area of the dome.

d = internal diameter of ring beam.

and for the conical portion

W_2 = weight of contents over the net area of the cone.

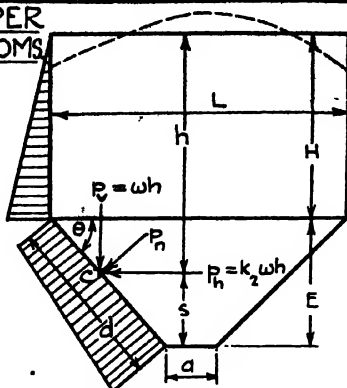
d = external diameter of ring beam.

If this adjustment were made for a ring beam of reasonable width in Example (b) above, P_1 would balance P_2 .

BOTTOMS OF BUNKERS & TANKS.

TABLE N°22.

HOPPER BOTTOMS



W = WT. PER CU. FT. OF FILLING.

W_s = WT. OF SLAB PER SQ. FT.

INTENSITY OF PRESSURE NORMAL TO SLAB :-

$$P_n = wh (k_2 \sin^2 \theta + \cos^2 \theta) + W_s \cos \theta$$

(SEE TABLE N°5 FOR VALUES OF k_2)

$$W_1 = w \left[\frac{s}{3} (ab + lD_1 + \sqrt{ab \cdot lD_1}) + lD_1 h \right] + W_s$$

$$W = w \left[\frac{E}{3} (ab + LB + \sqrt{ab \cdot LB}) + LBH \right] + W_s$$

W_1 = WT. OF BOTTOM BELOW LEVEL C.

W_2 = WT. OF COMPLETE BOTTOM.

DETERMINATION OF HORIZONTAL STEEL AT MIDSPAN

AND CORNERS :-

$$B.M. = .375 P_n D^2 \text{ IN. LBS/FT.}$$

$$\text{DIRECT TENSION} = .5 P_n \sin \theta \text{ LBS/FT.}$$

DETERMINATION OF LONGITUDINAL STEEL :-

AT CENTRE OF SLOPE :

$$B.M. = .375 P_n D^2 \text{ IN. LBS/FT.}$$

$$\text{DIRECT TENSION} = \frac{W_1}{2 \sin \theta (L + D)}$$

AT TOP OF SLOPE :

$$B.M. = .375 P_n D^2 \text{ IN. LBS/FT.}$$

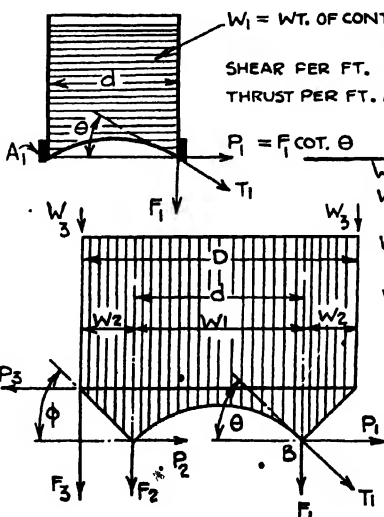
$$\text{DIRECT TENSION} = \frac{W}{2 \sin \theta (L + B)}$$

VERTICAL HANGING-UP FORCE AT BASE OF WALLS

$$= \frac{W}{2(L + B)} \text{ LBS/FT.}$$

DOMICAL BOTTOMS OF CYLINDRICAL TANKS.

(UNITS:-
FEET & LBS.)



W_1 = WT. OF CONTENTS ABOVE DOME INCLUDING WT. OF DOME.

$$\text{SHEAR PER FT.} = F_1 = \frac{W_1}{3 \cdot 14 d}$$

$$\text{THRUST PER FT. AT PERIPHERY OF DOME} = T_1 = F_1 \operatorname{cosec} \theta.$$

$$P_1 = F_1 \cot \theta \quad \text{RING TENSION IN BEAM AT } A_1 = .16 W_1 \cot \theta.$$

W_1 AS ABOVE.

W_2 = WT. OF CONTENTS IN PORTION ABOVE CONE INCLUDING WT. OF CONE.

W_3 = WT. OF WALLS ETC. AND ALL LOADS FROM ROOF, ETC. CARRIED BY WALLS.

VALUES OF F_1 , T_1 AND P_1 AS ABOVE.

$$F_2 = \frac{W_2 + W_3}{3 \cdot 14 d} \quad F_3 = \frac{W_3}{3 \cdot 14 D}$$

$$P_2 = F_2 \cot \phi. \quad P_3 = F_3 \cot \phi.$$

$$\text{RING TENSION IN BEAM AT B} = .5 d (P_1 - P_2)$$

(F_2 , F_3 , P_2 & P_3 EXPRESSED IN LBS. PER FOOT.)

(IDEAL CASE : $P_1 = P_2$).

NOTES.

CONCRETE.

TABLE NO. 23.

DESIGNATION			MIX A	MIX B	MIX C	MIX D	MIX E	MIX F	-
PROPORTIONS.	APPROXIMATE VOLUMETRIC PROPORTIONS.		1.3.6	1.2½.5	1.2.4	1.1⅓.3½	1.1½.3	1.1.2	-
	CEMENT.		112	112	112	112	112	112	LBS.
	FINE AGGREGATE.		3.75	3.10	2.5	2.07	1.86	1.26	CU. FT.
	COARSE AGGREGATE.		7.50	6.20	5.0	4.14	3.72	2.50	CU. FT.
	WATER. (SEE ALSO SLUMP TEST VALUES).		7	6¼	5¾	5½	5	4½	GALLS.
WORKING STRESSES (NORMAL HARDENING CEMENT).	COMPRESSION.	STANDARD.	400	500	700	750	800	875	LBS./IN. ²
		L.C.C.	-	-	600	650	675	750	LBS./IN. ²
	SHEAR.		40	50	60	63	65	70	LBS./IN. ²
	PUNCHING SHEAR.		80	100	120	125	130	140	LBS./IN. ²
	DIRECT TENSION. (REINFORCED).		-	-	180	190	195	210	LBS./IN. ²
ELASTIC MODULUS = E _c .	ORDINARY CALCULATIONS.	E _c	-	-	2 x 10 ⁶	2.2 x 10 ⁶	2.4 x 10 ⁶	3 x 10 ⁶	LBS./IN. ²
		m	-	-	15	13.6	12½	10	-
	DEFORMATION CALCULATIONS.	E _c	-	-	3 x 10 ⁶	3.35 x 10 ⁶	3.75 x 10 ⁶	5 x 10 ⁶	LBS./IN. ²
		m	-	-	10	9	8	6	-
COEFFICIENT OF LINEAR EXPANSION.			.0000054	.0000050	.0000066	.0000067	.0000068	-	PER °F
WATER:- ADD TO QUANTITIES GIVEN IF MATERIALS ARE ABSORBENT; DEDUCT IF MATERIALS ARE WET. STRESSES:- IF CEMENT IS MEASURED BY WEIGHT INCREASE STRESSES BY 20% FOR RAPID HARDENING CEMENT AND INCREASE BY 50% FOR ALUMINOUS CEMENT- FOR MODIFICATION TO STANDARD STRESSES SEE CHAP. IX.									
MIXES AND SLUMP TEST VALUES FOR VARIOUS USES.									
DESCRIPTION.	MIX.	MAX SLUMP.	DESCRIPTION.	MIX.	MAX SLUMP.	DESCRIPTION.	MIX	MAX SLUMP.	
GENERAL R.C. WORK			MASS CONCRETE:-			ROADS: FOUNDS.	B or C	1"	
BEAMS.	C	4" to 6"	FOUNDATIONS.	A	1" to 2"	BOTTOM LAYER.	C	½" to 1½"	
SLABS.	C	4" to 6"	PIERS (WITH PLUMS)	A	1" to 4"	SURFACING.	E	½" to 1½"	
WALLS.	C	5"	LIGHTLY REINFORCED	B	2" to 4"	SINGLE LAYER.	D or E	1" to 2"	
BUNKERS SILOS, ETC.	C	5"	WALLS.	A or B	3" to 5"	NON-SURFACED.			
RENDERED TANKS.	C	5"	BLINDING.	A	-	ROOF SLABS.	D	4" to 5"	
NON-RENDERED TANKS	E	6"	PILES.	D or E	3" to 5"	GROUND SLABS.	C	4" to 6"	
STEPS.	C	3" to 4"	BRIDGE WORK.	D	4" to 6"	COLUMNS: LOW LOAD	C	4" to 5"	
R.C. FOUNDS.	C	1½" to 3"	ARCHES.	D or E	3" to 5"	MED. LOAD.	E	3½" to 4½"	
CONCRETE DEPOSITED UNDER WATER.	D or E	0 to 2"	" LONG SPANS.	E or F	3" to 4"	HIGH LOAD.	F	3" to 4"	
PERCENTAGE VARIATION OF STRENGTH WITH AGE.						HORIZONTAL PRESSURE OF WET CONCRETE.			
AGE (ALL MIXES).	2 DAYS	4 DAYS	7 DAYS	14 DAYS	21 DAYS	1 MON	2 MONS	3 MONS	1 YEAR
NORMAL HARDENING PORTLAND CEMENT.	22	38	45	60	68	74	93	100	115
RAPID HARDENING PORTLAND CEMENT.	49	62	77	87	92	94	97	100	-
ALUMINOUS CEMENT.	90	95	-	-	-	-	-	100	-
						HEAD PRESSURE			
						UP TO 5'	140 LBS. PER SQ. FT.		
						5' TO 10'	120 " " " "		
						10' TO 20'	100 " " " "		
						OVER 20'	75 " " " "		
						DECREASE PRESSURES FOR DRY MIXES AND NARROW WIDTHS.			

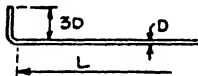
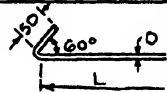
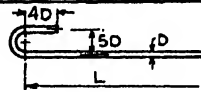
NOTES.

REINFORCEMENT — ADHESION

TABLE N^o24.

MINIMUM LAP LENGTHS. (TENSION)

(SEE ALSO CHAP. 8)

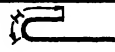

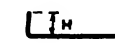
STRESS IN LBS. PER SQ. IN.	VALUES OF N.			L = N x DIA. OF BAR.		
	GENERAL USE	GENERAL USE	L.C.C.	GENERAL USE	GENERAL USE	L.C.C.
UP TO 10,000	40	22	42	18.75	12.5	25
11,000	40	24	40	20.75	14	27.5
12,000	40	26.25	50	22.5	15	30
13,000	40	28.5	54	24.5	16	32.5
14,000	40	30.5	58	26.25	17.5	35
15,000	40	32.75	62	28	19	37.5
16,000	40	35	67	30	20	40
17,000	42.5	37.5	(71)	32.5	22.5	(42.5)
18,000	45	40	(75)	35	25	(45)
19,000	47.5	42.5	(79)	37.5	27.5	(47.5)
20,000	50	45	(83)	40	30	(50)
22,500	56.25	51.25	(94)	46.25	36.25	(56.25)
25,000	62.5	57.5	(104)	42.5	42.5	(62.5)
TYPE OF ANCHORAGE	STRAIGHT					

VALUES IN BRACKETS REFER TO STRESSES NOT ALLOWED BY CURRENT L.C.C. REGULATIONS.

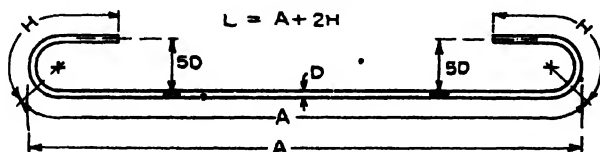
MINIMUM LAP LENGTH FOR COMPRESSION: $N = 24$; WHEN STRESS $> 9,600$: $N = \frac{\text{STRESS}}{400}$

STANDARD BENDING DIMENSIONS.

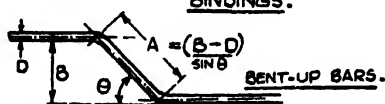
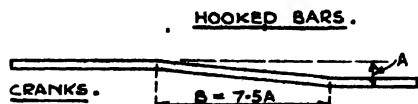
ONLY DIMENSIONS A, B,
H & L NEED BE ENTERED
ON BENDING SCHEDULE.

DIA. OF BAR	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"
	3"	3"	3"	4 1/2"	4 1/2"	4 1/2"	6"	7 1/2"	7 1/2"	9"	10 1/2"	12"	12"	13 1/2"
	1 1/2"	1 1/2"	3"	3"	3"	3"	3"	4 1/2"	4 1/2"	6"	6"	6"	7 1/2"	7 1/2"
	1 1/2"	1 1/2"	1 1/2"	1 1/2"	1 1/2"	1 1/2"	2"	3"	3"	3"	4 1/2"	4 1/2"	4 1/2"	4 1/2"

L = TOTAL LENGTH OF BAR.



BINDINGS.



EXAMPLES OF USE OF TABLE No. 25.

(a) To select suitable reinforcement for a beam requiring 7.42 sq. in. of steel.
Width of beam = 10 in. :

$$\text{Maximum number of bars in a single layer} = \frac{10}{2\frac{1}{2}} = 4.$$

From Table, four 1½-in. bars = 7.07 sq. in. which is insufficient ; hence most suitable number of bars may be 6 ; that is, 4 in bottom layer and 2 in second layer :

Six 1¼-in. bars = 7.36 sq. in. which should be satisfactory.

If the area must equal or exceed 7.42 sq. in., use

Three 1½-in. bars in bottom layer = 5.30 sq. in.

Three 1-in. bars in top layer = 2.36 „

$$\hline 7.66 \text{ „}$$

(b) To select suitable reinforcement for a square column requiring 3.06 sq. in.

Use four 1-in. bars = 3.14 sq. in.

or eight ¾-in. bars = 3.53 sq. in.

The 1-in. bars would be more economical, but size of column may necessitate 8 bars.

(c) To select suitable reinforcement for a 6-in. slab requiring 0.4 sq. in. of steel per foot width.

With maximum spacing : — 5-in. bars at 9-in. centres = 0.409 sq. in.

With minimum size bar : — 3-in. bars at 3-in. centres = 0.442 „

Most economical and practicable would be an intermediate diameter and spacing, say, ½-in. bars at 6-in. centres = 0.393 sq. in.

NOTES.

REINFORCEMENT — AREAS (INS²)

TABLE NO. 25

DIA.	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	
AREAS FOR GIVEN NUMBER OF BARS.	1	0.028	0.049	0.077	0.110	0.150	0.196	0.307	0.442	0.601	0.785	0.994	1.227	1.484	1.767
	2	0.055	0.098	0.153	0.221	0.301	0.393	0.614	0.884	1.203	1.571	1.988	2.45	2.97	3.53
	3	0.082	0.147	0.230	0.331	0.451	0.589	0.920	1.325	1.804	2.36	2.98	3.68	4.45	5.30
	4	0.110	0.196	0.307	0.442	0.601	0.785	1.227	1.767	2.41	3.14	3.98	4.91	5.94	7.07
	5	0.138	0.245	0.384	0.552	0.752	0.982	1.534	2.21	3.01	3.93	4.97	6.14	7.42	8.84
	6	0.165	0.295	0.460	0.663	0.902	1.178	1.841	2.65	3.61	4.71	5.96	7.36	8.91	10.60
	7	0.193	0.344	0.537	0.773	1.052	1.374	2.15	3.09	4.21	5.50	6.96	8.59	10.39	12.37
	8	0.221	0.393	0.614	0.884	1.203	1.571	2.45	3.53	4.81	6.28	7.95	9.82	11.88	14.14
	9	0.248	0.442	0.690	0.994	1.353	1.767	2.76	3.98	5.41	7.07	8.95	11.04	13.36	15.90
	10	0.276	0.491	0.767	1.104	1.503	1.963	3.07	4.42	6.01	7.85	9.94	12.27	14.85	17.67
	11	0.304	0.540	0.844	1.215	1.654	2.16	3.37	4.86	6.61	8.64	10.93	13.50	16.33	19.44
	12	0.331	0.589	0.920	1.325	1.804	2.36	3.68	5.30	7.22	9.42	11.93	14.73	17.82	21.21
	13	0.359	0.638	0.997	1.436	1.954	2.55	3.99	5.74	7.82	10.21	12.92	15.95	19.30	22.97
	14	0.387	0.687	1.074	1.547	2.10	2.75	4.30	6.19	8.42	11.00	13.92	17.18	20.79	24.74
	15	0.414	0.736	1.151	1.657	2.25	2.95	4.60	6.63	9.02	11.78	14.91	18.41	22.27	26.51
	16	0.442	0.785	1.227	1.768	2.41	3.14	4.91	7.07	9.62	12.57	15.90	19.64	23.76	28.27
	17	0.469	0.835	1.304	1.878	2.56	3.34	5.22	7.51	10.22	13.35	16.90	20.86	25.24	30.04
	18	0.497	0.884	1.381	1.989	2.71	3.53	5.52	7.95	10.82	14.14	17.89	22.09	26.73	31.81
	19	0.525	0.933	1.457	2.10	2.86	3.73	5.83	8.39	11.43	14.92	18.89	23.32	28.21	33.57
	20	0.552	0.982	1.534	2.21	3.01	3.93	6.14	8.84	12.03	15.71	19.88	24.54	29.70	35.34
AREAS PER FT. WIDTH FOR VARIOUS SPACINGS.	3"	0.110	0.196	0.307	0.442	0.601	0.785	1.227	1.767	2.405	3.142	GENERAL DATA			
	3 1/2"	0.095	0.168	0.263	0.379	0.515	0.673	1.052	1.515	2.06	2.69				
	4"	0.083	0.147	0.230	0.331	0.451	0.589	0.920	1.325	1.804	2.356	DIA.	PERI-METER	WT. PER FOOT	METRIC SIZE
	4 1/2"	0.074	0.131	0.205	0.295	0.401	0.524	0.818	1.178	1.604	2.09	INS.	INS.	LBS.	M.M.
	5"	0.066	0.118	0.184	0.265	0.361	0.471	0.736	1.060	1.443	1.885	3/16"	.589	.094	4.76
	5 1/2"	0.060	0.107	0.167	0.241	0.328	0.428	0.669	0.964	1.312	1.714	1/4"	.785	.167	6.35
	6"	0.055	0.098	0.153	0.221	0.301	0.393	0.614	0.884	1.203	1.571	5/16"	.98	.261	7.94
	6 1/2"	0.051	0.091	0.142	0.204	0.278	0.363	0.566	0.816	1.110	1.450	3/8"	1.18	.375	9.52
	7"	0.047	0.084	0.131	0.189	0.258	0.337	0.526	0.757	1.031	1.346	7/16"	1.37	.511	11.1
	7 1/2"	0.044	0.079	0.123	0.177	0.241	0.314	0.491	0.707	0.962	1.257	1/2"	1.57	.667	12.7
	8"	0.041	0.074	0.115	0.166	0.225	0.295	0.461	0.663	0.902	1.178	5/8"	1.96	1.043	15.9
	8 1/2"	0.039	0.069	0.108	0.156	0.212	0.277	0.433	0.624	0.849	1.109	3/4"	2.36	1.502	19.1
	9"	0.037	0.065	0.102	0.147	0.200	0.262	0.409	0.589	0.802	1.047	7/8"	2.75	2.044	22.2
	9 1/2"	0.035	0.062	0.097	0.140	0.190	0.248	0.388	0.558	0.760	0.992	1"	3.14	2.670	25.4
	10"	0.033	0.059	0.092	0.133	0.180	0.236	0.368	0.530	0.722	0.942	1 1/8"	3.54	3.379	28.6
	10 1/2"	0.032	0.056	0.088	0.126	0.172	0.224	0.351	0.505	0.687	0.898	1 1/4"	3.92	4.173	31.8
	11"	0.030	0.054	0.084	0.120	0.164	0.214	0.335	0.482	0.656	0.857	1 3/8"	4.31	5.049	34.9
12"	0.028	0.049	0.077	0.110	0.150	0.196	0.307	0.442	0.601	0.785	1 1/2"	4.71	6.008	38.1	
15"	0.022	0.039	0.061	0.088	0.120	0.157	0.245	0.353	0.481	0.628	SEE ALSO TABLE NO. 24 - ADHESION. TABLE NO. 26 - WEIGHTS.				
18"	0.018	0.033	0.051	0.074	0.100	0.131	0.205	0.295	0.401	0.524					
24"	0.014	0.025	0.038	0.055	0.075	0.098	0.153	0.221	0.301	0.393					

EXAMPLES OF USE OF TABLE No. 26.

(a) To find the weight of 13 ft. of 1-in. bar : By direct reading from Table, weight = 34.71 lb.

(b) To find the weight of 5,673 ft. of $\frac{1}{2}$ -in. bar : $= 5,673 \times 0.667 = 3,800$ lb.

(c) To find the number of tons of steel in 10,000 ft. of $\frac{3}{4}$ -in. bar : $= \frac{10,000}{1,491} = 6.7$ tons.

(d) To estimate the weight of reinforcement in a slab having $\frac{1}{2}$ -in. bars at 6 in. centres transversely and $\frac{5}{8}$ -in. bars at 15-in. centres longitudinally (bottom steel only).

$\frac{1}{2}$ -in. at 6 in. $= 12.02$ lb. per sq. yd.

$\frac{5}{8}$ -in. at 15 in. $= 1.88$ „ „ „

13.90

Add 10 per cent. for laps, hooks, etc. $= 13.90 + 1.39 = 15.29$ lb. per sq. yd.

Add $2\frac{1}{2}$ per cent. for rolling margin $= 0.38$ „ „ „

Total $= 15.67$ „ „ „

NOTES.

The amount of No. 14 or 16 gauge soft-iron binding wire required in the assembly of reinforcement is about 5 lb per ton of bars.

REINFORCEMENT - WEIGHTS (LBS)

TABLE Nº 26

DIA.	3/16"	1/4"	5/16"	3/8"	7/16"	1/2"	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	
LBS. PER FT.	0.094	0.167	0.261	0.375	0.511	0.667	1.043	1.502	2.044	2.670	3.379	4.173	5.049	6.008	
FT. PER TON	23830	13418	8682	5973	4384	3358	2148	1491	1096	839	663	537	444	373	
WT. PER GIVEN NUMBER & SIZES.	1	0.094	0.167	0.261	0.376	0.51	0.67	1.04	1.50	2.04	2.67	3.38	4.17	5.05	6.01
	2	0.188	0.334	0.522	0.752	1.02	1.34	2.09	3.00	4.09	5.34	6.76	8.34	10.10	12.02
	3	0.282	0.501	0.783	1.128	1.53	2.00	3.13	4.51	6.13	8.01	10.14	12.52	15.16	18.02
	4	0.376	0.668	1.044	1.504	2.04	2.67	4.17	6.01	8.18	10.68	13.52	16.69	20.20	24.03
	5	0.470	0.835	1.305	1.880	2.55	3.34	5.22	7.51	10.22	13.55	16.90	20.86	25.26	30.04
	6	0.564	1.002	1.566	2.266	3.06	4.01	6.26	9.01	12.26	16.02	20.28	25.03	30.29	36.05
	7	0.658	1.169	1.827	2.632	3.57	4.68	7.30	10.51	14.31	18.69	23.66	29.20	35.34	42.06
	8	0.752	1.336	2.088	3.008	4.08	5.34	8.34	12.02	16.35	21.36	27.04	33.38	40.89	48.06
	9	0.846	1.503	2.349	3.384	4.59	6.01	9.39	13.52	18.40	24.03	30.42	37.55	45.44	54.07
	10	0.940	1.670	2.610	3.760	5.11	6.68	10.43	15.02	20.44	26.70	33.80	41.72	50.49	60.08
	11	1.034	1.837	2.871	4.136	5.62	7.35	11.47	16.52	22.48	29.37	37.18	45.89	55.54	66.09
	12	1.128	2.004	3.132	4.512	6.13	8.02	12.52	18.02	24.53	32.04	40.56	50.06	60.59	72.10
	13	1.222	2.171	3.393	4.888	6.64	8.68	13.56	19.53	26.57	34.71	43.94	54.24	65.64	78.10
	14	1.316	2.338	3.654	5.264	7.15	9.35	14.60	21.03	28.62	37.38	47.32	58.41	70.68	84.11
	15	1.410	2.505	3.915	5.640	7.67	10.02	15.65	22.53	30.66	40.05	50.70	62.58	75.74	90.12
	16	1.504	2.672	4.176	6.016	8.18	10.69	16.69	24.03	32.70	42.72	54.08	66.75	80.78	96.13
	17	1.598	2.839	4.437	6.392	8.69	11.36	17.73	25.53	34.75	45.39	57.46	70.92	85.83	102.14
	18	1.692	3.006	4.698	6.768	9.20	12.02	18.77	27.04	36.79	48.06	60.84	75.10	90.86	108.14
WT. PER 50 YARD - BARS AT VARIOUS CENTRES.	2 1/2"	4.06	7.23	11.27	16.24	22.08	28.86	45.06	64.88	88.50	115.34	146.02	180.23	218.12	260.00
	3"	3.39	6.02	9.41	13.53	18.40	24.04	37.60	54.07	75.70	96.12	121.68	150.19	181.60	216.28
	3 1/2"	2.90	5.15	8.07	11.60	15.72	20.61	32.20	46.40	63.60	82.39	104.30	128.78	155.80	185.39
	4"	2.54	4.51	7.07	10.15	13.80	18.03	28.16	40.55	55.40	72.09	91.26	112.64	136.32	162.22
	4 1/2"	2.26	4.01	6.27	9.02	12.26	16.03	25.03	36.10	49.20	64.08	81.20	100.13	121.21	144.20
	5"	2.03	3.61	5.64	8.12	11.01	14.42	22.52	32.50	44.20	57.67	73.20	90.09	109.05	129.77
	5 1/2"	1.85	3.28	5.13	7.38	10.01	13.12	20.50	29.51	40.25	52.59	66.50	81.99	99.25	117.20
	6"	1.69	3.01	4.71	6.76	9.200	12.02	18.77	27.02	36.84	48.10	60.84	75.09	90.87	108.14
	6 1/2"	1.56	2.78	4.34	6.25	8.49	11.09	17.52	24.95	34.67	44.40	56.25	69.48	83.89	99.82
	7"	1.45	2.58	4.05	5.80	7.85	10.31	16.09	23.17	31.68	41.20	52.20	64.37	77.89	92.89
	7 1/2"	1.35	2.41	3.77	5.41	7.36	9.62	15.02	21.63	29.50	38.50	48.80	60.08	72.71	86.80
	8"	1.27	2.26	3.52	5.07	6.90	9.02	14.08	20.31	27.70	36.10	45.63	56.30	68.16	81.25
	8 1/2"	1.19	2.12	3.32	4.78	6.50	8.49	13.25	19.06	26.07	33.97	43.05	53.10	64.15	76.60
	9"	1.13	2.00	3.14	4.51	6.14	8.02	12.51	18.02	24.53	32.08	40.60	50.06	60.56	72.30
	9 1/2"	1.07	1.90	2.97	4.27	5.79	7.59	11.86	17.05	23.28	30.38	38.60	47.42	57.39	68.50
	10"	1.02	1.80	2.82	4.06	5.51	7.22	11.26	16.22	22.08	28.80	36.58	45.06	54.52	64.88
	11"	0.92	1.64	2.57	3.69	5.02	6.55	10.24	14.74	20.07	26.22	33.30	40.96	49.37	58.90
	12"	0.85	1.50	2.36	3.38	4.60	6.02	9.39	13.52	18.41	24.00	30.42	37.54	45.44	54.07
	15"	0.68	1.20	1.88	2.71	3.68	4.81	7.54	10.82	14.72	19.20	24.38	30.10	36.35	43.26
	18"	0.56	1.00	1.57	2.26	3.06	4.01	6.26	9.01	12.26	16.02	20.28	25.04	30.29	36.05

EXAMPLES OF USE OF TABLES NOS. 27, 28, AND 29.

(See also page facing Table No. 28.)

(a) To design a rectangular beam section to take 500,000 in. lb. with maximum stresses of 17,000 and 700 lb. per sq. in.:

$$(i) \text{ From Table No. 27: } Q = 116.7; \text{ hence } bd^2 = \frac{500,000}{116.7} = 4,280.$$

For a section with a total depth of 21 in. and 12 in. wide,

$$bd^2 = 12 \times 19.5^2 = 4,560$$

$$r = \frac{17,000}{700} = 24.3; \text{ from the Table, } a_1 = 0.87.$$

$$\text{Hence } A_T = 0.87 \times \frac{500,000}{19.5 \times 17,000} = 1.73 \text{ sq. in.}$$

= three $\frac{7}{8}$ -in. bars (Table No. 25).

(ii) Alternative method: Assume effective depth = 19 in., for which, from Table No. 29, M. of R. of beam 1 in. wide without compression steel 42,100 in. lb.

$$b = \frac{500,000}{42,100} = 11.9 \text{ in., say, 12 in. wide.}$$

$$A_T = 11.9 \times 0.150 = 1.78 \text{ sq. in.} = \text{three } \frac{7}{8}\text{-in. bars.}$$

$$\text{Total depth of beam} = 19 + 1.5 = 20.5 \text{ in., say, 21 in.}$$

Therefore a section 21 in. by 12 in. with three $\frac{7}{8}$ -in. bars is satisfactory.

(b) To find the maximum stresses in a rectangular section 15 in. overall depth (effective depth = 13½ in.) and 9 in. wide, reinforced in tension with two 1-in. bars, and subject to a B.M. of 200,000 in. lb. $A_T = 1.57$ (Table No. 25).

$$p = \frac{1.57 \times 100}{9 \times 13.5} = 1.29$$

From Table No. 27: $a_1 = 0.85$ and $r = 18$

$$t = \frac{200,000}{0.85 \times 13.5 \times 1.57} = 11,100 \text{ lb. per sq. in.}$$

$$c = \frac{11,100}{18} = 617 \text{ lb. per sq. in.}$$

(c) To find the resistance moment of the section specified in (b) if stresses are not to exceed either 18,000 or 700 lb. per sq. in.

$$p \text{ (as above)} = 1.29, a_1 = 0.85, n_1 = 0.46.$$

$$\text{R.M. (steel)} = 1.57 \times 18,000 \times 0.85 \times 13.5 = 324,000 \text{ in. lb.}$$

$$\text{R.M. (concrete)} = 9 \times 0.46 \times \frac{700}{2} \times 0.85 \times 13.5^2 = 224,000 \text{ in. lb.}$$

Hence the compressive stress controls, and maximum R.M. = 224,000 in. lb.

NOTES.

EXAMPLES OF USE OF TABLES Nos. 27, 28 AND 29.

(See also page facing *Table No. 27.*)

(a) To find the amount of compressive and tensile reinforcement that should be provided in a 12-in. square section subject to a bending moment of 150,000 in. lb.; maximum stresses not to exceed 16,000 and 600 lb. per sq. in.:

From *Table No. 27*: $Q = 95$.

R.M. of concrete = $95 \times 12 \times 10.5^2 = 126,500$ in. lb.

$R_s = 150,000 - 126,500 = 23,500$ in. lb.

$r = \frac{16,000}{600} = 26.7$; from *Table No. 27*: $a_1 = 0.88$; $n_1 = 0.36$.

Assume $f = 1\frac{1}{2}$ in and $m = 15$

$a_s = 10.5 - 1.5 = 9$ in $n_1 = 0.36 \times 10.5 = 3.78$ in.

$f_c = \frac{3.78}{1.5} \times 14 \times 600 = 5,060$ lb per sq. in.

$A_c = \frac{23,500}{5,060} = 4.64$ sq. in., say, two $\frac{5}{8}$ -in. bars.

$a_r = \frac{150,000}{(0.515 \times 5,060) + (0.5 \times 12 \times 3.78 \times 600)} = 9.2$ in.

$A_r = \frac{150,000}{9.2 \times 16,000} = 1.02$ sq. in., say, two $\frac{7}{8}$ -in. bars.

(b) Section in example (a) subject to a moment of 200,000 in. lb.:

$R_s = 200,000 - 126,500 = 73,500$ in. lb.

$A_c = \frac{73,500}{5,060} = 14.5$ sq. in.

Approximate $a_r = 9.1$ in

$A_r = \frac{200,000}{16,000} = 12.5$ sq. in.

Since A_c is greater than A_r , the "steel-beam theory" is applicable, and

$$A_r = A_c = \frac{200,000}{16,000 \times 0.5} = 12.5 \text{ sq. in.},$$

say, two 1-in. bars top and bottom.

(c) To select the shallowest rectangular section 10 in. wide to resist a B.M. of 500,000 in. lb. with normal stresses of 16,000 and 600, but if $A_c = A_r$, maximum concrete stress may be 800 lb. per sq. in. (that is, "steel-beam theory" not allowed).

R.M. per inch width = $\frac{500,000}{10} = 50,000$ in. lb.

With $A_c = A_r$ and stresses 16,000 and 800, from *Table No. 28*, effective depth = 13 in (R.M. = 52,000); total depth, say, 14 in., $A_c = A_r = 10 \times 0.290 = 2.90$ sq. in., say, three $1\frac{1}{8}$ -in. bars

(d) To select a rectangular section to take 400,000 in. lb. without compression steel, with maximum stresses 18,000 and 675 lb per sq. in.

Stress ratio = 26.7; therefore use the tabulated values given on *Table No. 28* for 16,000 and 600;

$$k_1 = \frac{18,000}{16,000} = 1.125.$$

Therefore select a section from *Table No. 28* to give a R.M. = $\frac{400,000}{1.125}$

= 355,000 in. lb.

With effective depth = 18 in., R.M. per inch = 30,800 in. lb.

Hence $b = \frac{355,000}{30,800} = 11.5$ in.

$A_r = 11.5 \times 0.121 = 1.4$ sq. in.

Therefore a suitable section would be 12 in. wide, 20 in. overall depth with three $\frac{3}{4}$ -in. bars in bottom.

RESISTANCE MOMENTS - RECT^r BEAMS.

TABLE NO. 28.

MAXIMUM STRESS IN TENSION STEEL = 16,000 LBS. PER SQ. IN.

EFFECTIVE DEPTH INS.	MAX. C = 600 LBS. PER IN. ²				MAX. C = 800 LBS. PER IN. ²			
	A _c = 0		A _c = A _t		A _c = 0		A _c = A _t	
	R. M.	A _t	R. M.	A _t	R. M.	A _t	R. M.	A _t
10	9,540	•068	15,000	•100	14,700	•107	28,000	•200
11	11,500	•074	18,500	•115	17,800	•117	36,000	•230
12	13,700	•081	21,500	•128	21,100	•128	43,000	•260
13	16,100	•088	25,500	•140	24,800	•139	52,000	•290
14	18,700	•095	30,000	•150	28,700	•150	60,000	•320
15	21,400	•101	35,000	•165	33,000	•160	70,000	•335
16	24,400	•108	40,000	•175	37,600	•171	80,000	•350
17	27,500	•115	45,000	•185	42,400	•182	90,000	•370
18	30,800	•121	50,000	•198	47,500	•192	100,000	•390
19	34,500	•128	55,000	•210	53,000	•203	110,000	•410
20	38,100	•135	61,000	•223	58,800	•214	122,000	•430
21	42,000	•142	67,000	•235	65,000	•225	134,000	•450
22	46,000	•148	75,000	•245	71,000	•235	148,000	•475
23	50,000	•155	84,000	•258	77,500	•246	162,000	•500
24	56,000	•162	92,500	•270	86,000	•257	178,000	•525
25	59,500	•169	102,000	•280	92,000	•267	196,000	•550
26	64,000	•175	110,000	•290	99,000	•278	215,000	•580
27	69,000	•182	120,000	•300	107,000	•288	235,000	•610
28	74,500	•189	130,000	•310	115,000	•300	255,000	•640
29	80,000	•196	138,000	•325	124,000	•310	275,000	•665
30	85,500	•205	148,000	•340	132,000	•320	293,000	•700
31	92,000	•209	158,000	•350	141,000	•331	320,000	•730
32	98,000	•216	168,000	•365	150,000	•342	343,000	•760
33	104,000	•222	179,000	•375	168,000	•352	364,000	•790
34	110,000	•229	189,000	•390	170,000	•363	383,000	•815
35	117,000	•236	200,000	•400	180,000	•374	415,000	•840
36	123,000	•243	215,000	•417	190,000	•385	440,000	•875

R. M. = RESISTANCE MOMENT IN INCH LBS. PER ONE INCH WIDTH OF SECTION.

A_t = AREA OF TENSILE REINFORCEMENT IN IN.² PER ONE INCH WIDTH.

A_c = AREA OF COMPRESSIVE REINFORCEMENT IN IN.² (VALUES OF R. MS.

WITH A_c = A_t ARE APPROXIMATE ONLY, AS EXACT VALUES DEPEND UPON SIZE AND ARRANGEMENT OF BARS.)

m = 15.

FOR NOTES ON THE USE OF THIS TABLE FOR OTHER STRESSES SEE TABLE NO. 27.

NOTES.

(For examples of use of this Table, see pages facing *Tables* Nos. 27 and 28.)

RESISTANCE MOMENTS-RECT² BEAMS. TABLE N^o 29.

MAXIMUM STRESS IN TENSION STEEL = 17,000 LBS. PER SQ. IN.

EFFECTIVE DEPTH INS.	MAX. C = 700 LBS./IN. ²		MAX. C = 850 LBS./IN. ²			
	A _c = 0		A _c = 0		A _c = A _t	
	R. M.	A _t .	R. M.	A _t .	R. M.	A _t .
10	11,700	• 079	15,600	• 114	29,800	• 212
11	14,100	• 087	18,900	• 124	38,300	• 245
12	16,800	• 095	22,500	• 136	35,600	• 276
13	19,800	• 103	26,500	• 148	55,300	• 308
14	22,900	• 110	30,500	• 159	64,000	• 340
15	26,300	• 118	35,000	• 170	74,200	• 356
16	30,000	• 126	40,000	• 181	85,000	• 372
17	33,800	• 134	45,000	• 193	95,500	• 394
18	37,900	• 142	50,500	• 204	106,300	• 415
19	42,100	• 150	56,400	• 216	117,000	• 435
20	46,700	• 158	60,500	• 227	130,000	• 470
21	51,600	• 166	69,100	• 239	142,000	• 480
22	56,500	• 174	75,200	• 250	157,000	• 505
23	61,800	• 182	82,500	• 262	172,000	• 530
24	68,500	• 190	91,500	• 273	189,000	• 560
25	73,000	• 198	98,000	• 283	208,000	• 585
26	79,000	• 205	105,000	• 296	228,000	• 620
27	85,000	• 213	114,000	• 306	250,000	• 650
28	91,500	• 221	122,000	• 319	271,000	• 680
29	98,500	• 229	132,000	• 330	282,000	• 710
30	105,000	• 237	140,000	• 340	312,000	• 745
31	113,000	• 244	150,000	• 352	340,000	• 775
32	120,000	• 253	160,000	• 363	365,000	• 810
33	128,000	• 260	170,000	• 373	386,000	• 840
34	135,000	• 268	180,000	• 385	408,000	• 870
35	144,000	• 276	191,000	• 396	441,000	• 895
36	152,000	• 284	202,000	• 410	467,000	• 930

R. M. = RESISTANCE MOMENT IN INCH LBS PER ONE INCH WIDTH OF SECTION.

A_t = AREA OF TENSILE REINFORCEMENT IN INS.² PER ONE INCH WIDTH.

A_c = AREA OF COMPRESSIVE REINFORCEMENT IN INS.² (VALUES OF

R. M. S. WITH A_c = A_t ARE APPROXIMATE ONLY, AS EXACT VALUES DEPEND UPON SIZE AND ARRANGEMENT OF BARS.)

m = 15

FOR NOTES ON USE OF THIS TABLE FOR OTHER STRESSES SEE TABLE N^o 27.

EXAMPLES OF USE OF TABLE No. 30.

To design a slab to take a bending moment of 2,000 ft. lb. with stresses not exceeding 16,000 lb. per sq. in. on steel, and 700 lb. per sq. in. on concrete.

(a) Maximum concrete and steel stresses:

$$\text{Effective depth required} = 0.0912 \sqrt{2,000} = 4.07 \text{ in.}$$

$$\text{Total slab thickness} = 5 \text{ in.}$$

$$\text{Area of steel required} = 0.0095 \sqrt{2,000} = 0.420 \text{ sq. in.}$$

(b) Specified total slab thickness = 6 in.; effective depth = 5.25 in.

$$k_1 = \frac{5.25}{\sqrt{2,000}} = 0.118.$$

With $t = 16,000$, corresponding value of $k_2 = 0.007$.

$$\text{Area of steel required} = 0.007 \sqrt{2,000} = 0.312 \text{ sq. in.}$$

(c) Specified total slab thickness = $4\frac{1}{2}$ in.; effective depth = 3.75 in.

$$k_1 = \frac{3.75}{\sqrt{2,000}} = 0.084.$$

With $c = 700$ corresponding value of $k_2 = 0.014$ approx.

$$\text{Area of steel required} = 0.014 \sqrt{2,000} = 0.625 \text{ sq. in.}$$

(See Table No. 31 for further suitable sections with compression reinforcement.)

SLAB DESIGN FACTORS

TABLE N°30

t	12,000		14,000		16,000		17,000		18,000		20,000		22,400	
C	k ₂	k ₁	k ₂	k ₁	k ₂	k ₁	k ₂	k ₁	k ₂	k ₁	k ₂	k ₁	k ₂	k ₁
100	•0024	•432	•0019	•462	•0016	•490	•0014	•503	•0013	•517	•0011	•542	•0010	•570
150	•0035	•299	•0028	•318	•0023	•336	•0021	•344	•0020	•353	•0017	•369	•0014	•338
200	•0046	•231	•0037	•245	•0031	•259	•0028	•265	•0026	•271	•0022	•283	•0019	•297
250	•0057	•191	•0046	•202	•0038	•212	•0035	•217	•0032	•222	•0027	•231	•0023	•242
300	•0067	•164	•0054	•173	•0045	•181	•0041	•185	•0038	•189	•0033	•196	•0028	•205
350	•0077	•145	•0062	•151	•0052	•159	•0047	•162	•0044	•165	•0038	•172	•0032	•179
400	•0087	•130	•0070	•136	•0058	•142	•0053	•145	•0049	•148	•0042	•153	•0036	•160
450	•0096	•118	•0078	•124	•0065	•129	•0059	•131	•0055	•134	•0047	•139	•0040	•144
500	•0105	•109	•0085	•114	•0071	•118	•0065	•121	•0060	•123	•0052	•127	•0044	•132
525	•0110	•105	•0089	•1097	•0074	•114	•0068	•116	•0063	•118	•0054	•122	•0046	•127
550	•0114	•102	•0093	•1058	•0077	•1098	•0071	•112	•0066	•114	•0057	•118	•0048	•122
575	•0118	•098	•0096	•1022	•0080	•1061	•0074	•1079	•0068	•1097	•0040	•113	•0050	•117
600	•0123	•095	•0100	•0990	•0083	•1026	•0077	•1043	•0071	•1061	•0061	•110	•0052	•113
625	•0127	•092	•0103	•0960	•0086	•0994	•0079	•1011	•0073	•1027	•0063	•106	•0054	•110
650	•0131	•090	•0107	•0932	•0089	•0965	•0082	•0980	•0076	•0996	•0066	•103	•0056	•106
675	•0135	•087	•0110	•0906	•0092	•0937	•0085	•0952	•0078	•0967	•0068	•0996	•0058	•103
700	•0139	•085	•0113	•0882	•0095	•0912	•0087	•0926	•0081	•0941	•0070	•0968	•0060	•1001
725	•0143	•083	•0117	•0859	•0098	•0888	•0090	•0901	•0083	•0915	•0072	•0942	•0062	•0973
750	•0147	•081	•0120	•0838	•0101	•0866	•0093	•0879	•0086	•0892	•0074	•0917	•0064	•0947
775	•0151	•079	•0123	•0819	•0103	•0845	•0095	•0857	•0088	•0870	•0077	•0894	•0066	•0923
800	•0155	•078	•0127	•0800	•0106	•0825	•0098	•0837	•0091	•0849	•0079	•0873	•0067	•0900
825	•0159	•076	•0130	•0783	•0109	•0807	•0100	•0818	•0093	•0830	•0081	•0852	•0069	•0879
850	•0163	•074	•0133	•0766	•0112	•0789	•0103	•0800	•0095	•0811	•0083	•0833	•0071	•0859
875	•0166	•073	•0136	•0750	•0114	•0773	•0105	•0783	•0098	•0794	•0085	•0815	•0073	•0840
900	•0170	•071	•0139	•0736	•0117	•0757	•0108	•0767	•0100	•0778	•0087	•0798	•0075	•0822
925	•0174	•070	•0142	•0722	•0120	•0742	•0110	•0752	•0102	•0762	•0089	•0782	•0076	•0805
950	•0177	•069	•0146	•0708	•0122	•0728	•0113	•0738	•0105	•0748	•0091	•0766	•0078	•0789
975	•0181	•068	•0149	•0696	•0125	•0715	•0115	•0724	•0107	•0734	•0093	•0752	•0080	•0773
1000	•0185	•067	•0152	•068	•0127	•070	•0118	•071	•0109	•072	•0095	•074	•0082	•076

NOTES:-

M° = BENDING MOMENT IN FT. LBS. PER FOOT WIDTH.

t = MAX. ALLOWABLE TENSILE STRESS IN LBS. PER IN.²

C = MAX. ALLOWABLE COMPRESSIVE STRESS IN LBS. PER IN.²

$$\text{EFFECTIVE DEPTH} = k_1 \sqrt{M} \quad \text{INCHES.}$$

$$\text{AREA OF TENSILE STEEL} = k_2 \sqrt{M} \quad \text{SQ. IN.}$$

EXAMPLES OF USE OF TABLE No. 31.

To design a slab to take a bending moment of 2,000 ft. lb. (= 24,000 in. lb.) per ft. width. Maximum stresses: $t = 16,000$, and $c = 700$ lb. per sq. in. (Compare with examples for *Table No. 30.*)

(a) With no compression steel:

Resistance moment of a 5-in. slab with 0.422 sq. in. of steel per ft.
= 23,800 in. lb.

(b) With $A_c = 0.5A_t$:

Resistance moment of a $4\frac{1}{2}$ -in. slab with 0.461 sq. in. of steel per ft. in bottom and 0.230 sq. in. of steel per ft. in top = 23,720 in. lb.

(c) With $A_c = A_t$:

By interpolation: $4\frac{1}{4}$ -in. slab with 0.500 sq. in. of steel per foot width in top and bottom is suitable.

NOTES.

RESISTANCE MOMENTS - SLABS

TABLE N^o 31

MAXIMUM STRESS IN TENSION STEEL = 16,000 LBS. PER SQ. IN.

CONCRETE STRESS	SLAB THICK- NESS	EFFECTIVE DEPTH, INS.	$A_c = 0$		$A_c = .5 A_t$		$A_c = A_t$	
			R. M.	A_t	R. M.	A_t	R. M.	A_t
MAXIMUM $C = 600$ LBS./IN. ²	3"	2.34"	6,260	.190	6,580	.202	6,940	.215
	3½"	2.81	9,070	.229	9,780	.249	10,580	.275
	4"	3.31	12,500	.268	13,910	.301	15,730	.345
	4½"	3.75	16,050	.304	17,410	.332	19,010	.366
	5"	4.25	20,600	.345	23,980	.397	27,650	.466
	5½"	4.56	23,700	.370	27,350	.416	29,920	.476
	6"	5.06	29,200	.410	33,150	.470	38,430	.548
	6½"	5.56	35,300	.450	40,810	.523	48,200	.624
	7"	5.88	39,500	.477	44,560	.542	51,300	.631
	7½"	6.38	46,400	.519	53,070	.595	62,200	.704
	8"	6.88	53,900	.557	62,450	.649	74,500	.780
	9"	7.69	67,200	.621	77,350	.721	91,300	.858
	10"	8.69	86,000	.703	100,900	.830	122,200	1.01
	11"	9.50	103,000	.767	120,100	.904	144,100	1.09
	12"	10.50	126,000	.850	149,000	1.010	182,600	1.24
MAXIMUM $C = 700$ LBS./IN. ²	3"	2.31	7,730	.240	8,248	.260	8,860	.284
	3½"	2.75	10,900	.286	11,850	.316	12,900	.328
	4"	3.25	15,250	.336	17,205	.385	19,820	.450
	4½"	3.75	20,300	.389	23,720	.461	28,670	.564
	5"	4.06	23,800	.422	26,900	.484	31,060	.567
	5½"	4.56	29,900	.475	33,750	.568	39,050	.672
	6"	4.88	34,300	.507	38,750	.581	44,750	.681
	6½"	5.38	41,800	.559	48,230	.654	57,300	.788
	7"	5.58	50,000	.613	57,910	.718	69,000	.864
	7½"	6.19	55,300	.643	63,660	.749	75,200	.899
	8"	6.69	64,600	.695	75,530	.822	91,200	1.008
	9"	7.69	85,100	.798	102,250	.965	128,600	1.225
	10"	8.50	104,000	.882	124,200	1.061	154,700	1.335
	11"	9.50	130,200	.987	158,900	1.21	203,800	1.56
	12"	10.50	159,500	1.090	197,700	1.355	264,500	1.865

R.M. = RESISTANCE MOMENT IN INCH LBS. PER FOOT WIDTH OF SLAB.

A_t = AREA OF TENSION STEEL IN SQ. INS. PER FOOT WIDTH OF SLAB.

A_c = AREA OF COMPRESSION STEEL.

FOR NOTES ON THE USE OF THIS TABLE FOR OTHER STRESSES SEE TABLE N^o 27.

EXAMPLES OF USE OF TABLE No. 32.

(a) To find the moment of resistance in compression and the corresponding amount of tensile reinforcement for a tee beam, the rib of which is 10 in. wide and extends 16 in. below the soffit of a 6-in. slab. Span of beam = 18 ft. and distance between adjacent beams = 8 ft. Maximum stresses not to exceed 17,000 and 700 lb per sq. in.

Effective breadth :

$$(1) \text{ Available slab } = 96 \text{ in.}$$

$$(2) \frac{\text{Span}}{3} = 72 \text{ in.}$$

$$(3) 12d_s + b = (12 \times 6) + 10 = 82 \text{ in.}$$

Hence maximum $b_s = 72$ in ; effective depth, say, 20.5 in.

$$\text{From Table No. 27, } n_1 \left(\text{for } r = \frac{17,000}{700} \right) = 0.39 \times 20.5 = 8 \text{ in.}$$

and neutral axis falls below slab.

$$a_1 = 20.5 - 3 = 17.5 \text{ in.}$$

R.M. in compression

$$= \frac{700 \times 72 \times 17.5 \times 6}{2 \times 8} = (2 \times 8 - 6) = 3,300,000 \text{ in. lb.}$$

$$A_r = \frac{3,300,000}{17,000 \times 17.5} = 11.10 \text{ sq. in.}$$

(b) To find the moment of resistance in compression and the corresponding amount of tensile reinforcement for an ell beam, the rib of which is 6 in. wide and extends 10 in. below a 7-in. slab. Span of beam = 14 ft. and available slab width exceeds 3 ft. Maximum stresses : 16,000 and 600 lb. per sq. in.

Effective breadth : (1) Available slab = 36 in. minimum.

$$(2) \frac{\text{Span}}{3} = 56 \text{ in.}$$

$$(3) 4d_s + b = 4 \times 7 + 6 = 34 \text{ in.}$$

Hence $b_s = 34$ in ; effective depth, say, 15.5 in.

$$\text{From Table No. 27, } n_1 \left(\text{for } r = \frac{16,000}{600} \right) = 0.36 \times 15.5 = 5.6 \text{ in.,}$$

thus the neutral axis falls within the slab.

$$a_2 = 15.5 - \frac{5.6}{3} = 13.6 \text{ in.}$$

$$\text{R.M. in compression} = 0.5 \times 34 \times 5.6 \times 13.6 \times 600 = 770,000 \text{ in. lb.}$$

$$A_r = \frac{770,000}{16,000 \times 13.6} = 3.54 \text{ sq. in.}$$

(c) To find the minimum distance from the supports of an end-span beam, loaded with uniformly distributed load, that two bars can be bent up to assist in shear resistance ; the total number of equal diameter bars in the bottom at mid-span is six ; span of beam = 20 ft.

For second bar of six, $k_2 = 0.30$ and $k_3 = 0.18$. Hence distance from inner support $0.30 \times 20 = 6$ ft. and distance from outer support $= 0.18 \times 20 = 3$ ft. 7 in.

BEAM DATA

TABLE No. 32.

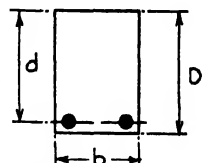
RECTANGULAR BEAMS.

PROPORTIONS :- MIN. $d = f \times \text{SPAN}$.

TENSILE STRESS :- 12,000 14,000 15,000 16,000 18,000

VALUE OF f :- .0375 .044 .047 .05 .056

$D = b$ TO $3b$



FLANGED BEAMS

$$b = \frac{dr}{3}$$

MAX. VALUE OF $b_s =$ LEAST OF FOLLOWING :-

(1) DISTANCE BETWEEN CENTRES OF ADJACENT BEAMS.

(2) ONE-THIRD SPAN OF BEAM

(3) TEE BEAM: $12d_s + b$

ELL BEAM: $4d_s + b$

LEVER ARM :

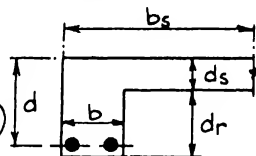
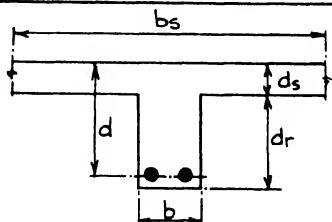
NEUTRAL AXIS BELOW SLAB: $a_1 = d - .5d_s$

NEUTRAL AXIS WITHIN SLAB: $a_2 = d - \frac{n}{3}$

RESISTANCE MOMENT (COMPRESSION)

NEUTRAL AXIS BELOW SLAB: $R.M. = \frac{cb_s a_1 d_s (2n - d_s)}{2n}$

NEUTRAL AXIS WITHIN SLAB: $R.M. = .5b_s n a_2 c$.

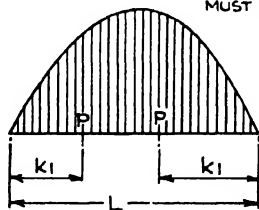


SECTIONS AT WHICH BOTTOM BARS CAN BE STOPPED (OR BENT UP).

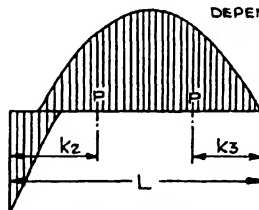
UNIFORMLY DISTRIBUTED LOAD.

MAX. DISTANCE FROM SUPPORT TO $P = kL$ WHERE $P =$ POINT OF STOPPING OR BENDING UP. IF BAR IS NOT BENT UP, SUFFICIENT ANCHORAGE LENGTH

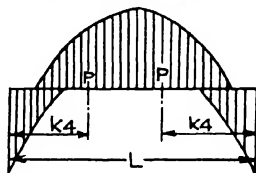
MUST BE PROVIDED ON SUPPORT SIDE OF P ; ANCHORAGE LENGTH WILL DEPEND ON RATE OF CHANGE OF B.M.



FREELY SUPPORTED SPAN



END SPAN



INTERIOR SPAN

NO OF BARS AT MIDSPAN	k_1						k_2						k_3						k_4					
	ORDER OF						STOPPING OFF OR						BENDING UP BARS											
	1ST	2ND	3RD	4TH	5TH	6TH	1ST	2ND	3RD	4TH	5TH	6TH	1ST	2ND	3RD	4TH	5TH	6TH	1ST	2ND	3RD	4TH	5TH	6TH
1	0	-	-	-	-	-	.11	-	-	-	-	-	0	-	-	-	-	-	.09	-	-	-	-	-
2	.15	0	-	-	-	-	.24	.11	-	-	-	-	.13	0	-	-	-	-	.21	.09	-	-	-	-
3	.21	.09	0	-	-	-	.30	.19	.11	-	-	-	.18	.08	0	-	-	-	.27	.16	.09	-	-	-
4	.25	.15	.07	0	-	-	.33	.24	.17	.11	-	-	.22	.13	.05	0	-	-	.30	.21	.15	.09	-	-
5	.27	.19	.12	.05	0	-	.35	.28	.21	.16	.11	-	.24	.15	.09	.04	0	-	.31	.24	.18	.13	.09	-
6	.30	.21	.15	.04	.04	0	.36	.30	.24	.19	.14	.11	.25	.18	.13	.08	.03	0	.33	.27	.21	.16	.12	.09
7	.31	.23	.17	.12	.08	.04	.39	.32	.26	.22	.18	.14	.29	.20	.15	.11	.07	.03	.34	.29	.23	.19	.15	.12
8	.32	.25	.19	.15	.10	.07	.40	.33	.27	.24	.20	.17	.30	.22	.17	.13	.09	.05	.35	.30	.25	.21	.18	.15

EXAMPLES OF USE OF TABLE No. 33.

(a) To design the shear reinforcement for the specified sections to take a shear force of 30,000 lb. Allowable stresses: bent-up bars: 16,000 lb. per sq. in.; binders: 14,000 lb. per sq. in.; maximum unit effective shear stress on concrete (Mix C) 60 lb. per sq. in. Lever arm of all sections = $0.87 \times$ effective depth.

(i) Effective depth = 40 in.; breadth = 15 in.

$$S = \frac{30,000}{0.87 \times 40 \times 15} = 57.4 \text{ lb. per sq. in.}$$

Therefore no shear reinforcement is required.

(ii) Effective depth = 30 in.; breadth = 12 in.

$$S = \frac{30,000}{0.87 \times 30 \times 12} = 96 \text{ lb. per sq. in., which is less than 120 and more than 60.}$$

From Table No. 33, $r = 0.58$.

Shear to be taken on steel = $0.58 \times 30,000 = 17,400 \text{ lb.}$

If taken on binders alone: $V = \frac{17,400}{0.87 \times 30} = 667$, which is given by $\frac{1}{2}$ -in. single binders at 8-in. centres or by $\frac{1}{2}$ -in. double binders at 16-in. centres, which are rather heavy for the given section.

If taken on bent-up bars alone (arranged so as to give double shear), single shear value required = $\frac{17,400}{2} = 8,700 \text{ lb.}$ which is given by one 1-in. bar bent up at 45 deg.

If taken on binding and bent-up bars, one $\frac{3}{4}$ -in. bar bent-up at 30 deg. and $\frac{3}{8}$ -in. double binding at 12-in. centres would be sufficient (see example (b)).

(iii) Effective depth = 20 in.; breadth = 10 in.

$$S = \frac{30,000}{0.87 \times 20 \times 10} = 173 \text{ lb. per sq. in.}$$

Therefore the whole shear would be taken on reinforcement.

Three $\frac{3}{4}$ -in. bars bent-up at 45 deg. at 16,000 lb. per sq. in. arranged in double shear = $3 \times 2 \times 4,998 = 30,000 \text{ lb.}$

Provide nominal binding; say $\frac{3}{8}$ -in. single at 18-in. centres.

(b) To calculate the resistance of the shear reinforcement in the section specified in (ii) if reinforced with one $\frac{3}{4}$ -in. bar bent-up at 30 deg. (in single shear) and $\frac{3}{8}$ -in. double binders at 12-in. centres:

Value of one $\frac{3}{4}$ -in. bar bent-up at 30 deg. in single shear at

$$16,000 \text{ lb. per sq. in.} = 0.707 \times 4,998 = 3,530 \text{ lb.}$$

$\frac{3}{8}$ -in. double binders at 12-in. centres at 14,000 lb. per sq. in.

$$\text{take } 2 \times 0.87 \times 30 \times 257 = 13,400 \text{ ,,}$$

$$\text{Total} = 16,930 \text{ ,,}$$

From the calculation in (ii) above, the shear resistance required from the steel is 17,400 lb. Hence the resistance of 16,930 lb. provided might be just sufficient, since the unit stresses are not excessive.

SHEAR REINFORCEMENT.

TABLE N^o 33.

PROPORTION OF TOTAL SHEAR TAKEN ON STEEL : $r = \frac{\text{UNIT SHEAR}}{\text{SAFE SHEAR}} - 1$																	
UNIT SHEAR	60	65	70	75	80	85	90	95	100	105	110	115	120	125	130	135	140
MIX C	NIL	.08	.17	.25	.33	.42	.50	.58	.67	.75	.83	.92	1.00	1.00	1.00	1.00	1.00
MIX D	NIL	.03	.11	.19	.27	.35	.43	.51	.58	.67	.75	.83	.91	1.00	1.00	1.00	1.00
MIX E	NIL	NIL	.07	.15	.23	.31	.38	.46	.54	.62	.69	.77	.85	.92	1.00	1.00	1.00
MIX F	NIL	NIL	NIL	.07	.14	.21	.29	.35	.43	.50	.57	.65	.71	.78	.86	.93	1.00

SHEAR VALUE OF SINGLE BINDERS (TWO ARMS) : $F = V(\text{LEVER ARM})$.

DIA.	STRESS IN LBS./IN. ²	VALUES OF V FOR VARIOUS SPACINGS.															
		2"	3"	4"	4½"	5"	6"	7"	7½"	8"	9"	10"	11"	12"	15"	18"	24"
¼"	10,000	440	326	246	216	146	164	140	130	122	110	98	88	80	66	54	40
	12,000	586	390	292	260	234	196	168	156	148	130	118	106	98	78	66	48
	14,000	684	456	342	306	274	250	196	184	172	154	138	124	117	92	76	56
	15,000	736	440	368	328	294	246	210	196	184	164	148	134	122	98	82	62
	16,000	782	522	392	348	314	262	224	208	195	174	156	142	130	104	86	66
5/16"	10,000	768	510	382	340	306	255	214	204	191	170	153	139	128	102	85	64
	12,000	920	618	460	408	368	306	262	172	250	204	184	167	153	122	102	77
	14,000	1032	716	535	476	428	357	306	285	268	238	214	194	178	143	119	90
	15,000	1160	765	575	510	460	382	328	306	287	255	230	208	191	253	127	96
	16,000	1225	817	615	545	490	408	350	327	306	273	245	223	204	163	136	102
3/8"	10,000	1100	735	550	490	440	367	314	293	275	244	220	200	183	147	122	92
	12,000	1320	880	660	587	529	440	378	352	330	294	264	240	220	176	147	110
	14,000	1540	1030	770	685	617	513	441	411	385	342	308	280	257	205	171	128
	15,000	1650	1100	825	735	660	548	470	439	411	366	330	300	273	219	183	137
	16,000	1760	1175	880	783	705	587	503	470	440	391	352	320	293	235	196	147
½"	10,000	1960	1310	984	878	786	655	561	524	492	436	395	357	328	262	218	163
	12,000	2350	1572	1180	1048	944	787	674	629	590	524	472	429	393	315	262	196
	14,000	2740	1834	1375	1221	1100	917	785	734	688	611	550	500	458	367	306	228
	15,000	2940	1950	1470	1300	1170	980	840	785	735	655	587	534	489	391	326	245
	16,000	3130	2096	1571	1395	1258	1048	898	839	787	700	629	572	524	419	349	261

SHEAR VALUE OF DIAGONAL BARS AT 45°

WHEN BARS ARE INCLINED AT 30° TO HORIZONTAL .707 OF THE VALUES SHOULD BE TAKEN

DIAM.	STRESS IN LBS. PER SQ. INCH.							
	8,000	10,000	12,000	14,000	15,000	16,000	17,000	18,000
5/8"	1735	2169	2603	3036	3230	3470	3688	3904
¾"	2499	3123	3748	4373	4680	4998	5310	5623
7/8"	3401	4251	5101	5951	6380	6802	7227	7652
1"	4442	5553	6663	7774	8330	8884	9440	9995
1 1/8"	5622	7028	8433	9838	10550	11244	11947	12649
1 1/4"	6941	8676	10411	12147	13016	13882	14748	15617
1 3/8"	8398	10496	12598	14697	15750	16747	17847	18897
1 1/2"	9992	12493	14992	17491	18744	19994	21243	22493

NOTE:- SEE CHAP. XI FOR NOTES AND FORMULAE.

SEE TABLE N^o 32 FOR POINTS AT WHICH TENSION BARS CAN BE BENT UP.

EXAMPLES OF USE OF TABLE No. 34.

(a) To determine the maximum safe load on a column, 18 in. square, 28 ft. in height, end conditions approximating to both ends hinged; reinforced with four 1½-in. longitudinal bars, and ⅝-in. double binders at 6-in. centres; concrete is Mix E:

(i) On total section:

$$\text{Binders: } p = \frac{400 \times 0.077 \times 2}{6 \times 18 (\text{overall})} = 0.57 \text{ per cent. (more than 0.2 per}$$

cent. minimum).

Slenderness: $k = 1.4$

$$R = \frac{1.4 \times 28 \times 12}{18} = 26.1$$

$$\frac{c_1}{c} = 0.53 \text{ or } 0.56 \text{ (by alternative "general use" formulae).}$$

Stress: From Table No. 23 or 35, safe stress for Mix E: 800 lb. per sq. in., $m = 12.5$; $c_s = 0.53 \times 800 = 423$ lb. per sq. in.

From Table No. 34: $A_c = 3.98$ sq. in.

$$\text{Safe load } W = [18^2 + (12.5 - 1) \times 3.98] 423 = 157,000 \text{ lb.} = 70 \text{ tons}$$

(ii) On core section:

$$\text{Binders: } p = \frac{400 \times 0.077 \times 2}{6 \times 15 (\text{core})} = 0.685 \text{ per cent.}$$

$$\frac{c_1}{c} = 1 + \frac{0.685}{10} = 1.07.$$

Slenderness, stress, etc., as above

$$c_s = 1.07 \times 0.53 \times 800 = 453 \text{ lb per sq in.}$$

$$\text{Safe load} = [15^2 + (12.5 - 1) \times 3.98] 453 = 123,000 \text{ lb.} = 55 \text{ tons.}$$

NOTES.

COLUMNS

TABLE No 34.

COLUMN REINFORCEMENT - LIMITING VALUES.

	MAXIMUM	MINIMUM
LONGITUDINAL BARS :	0.8% TOTAL AREA	4% CORE AREA
INDEPENDENT BINDERS :	0.2% TOTAL VOLUME	5% CORE VOLUME
HELICAL BINDERS :	0.5% CORE VOLUME	3.13% CORE VOLUME
COVER OF CONCRETE (MINIMUM):	1" FOR COLUMNS LESS THAN 12"; 1 1/2" FOR COLS. 12" AND OVER.	

PROPERTIES OF LONGITUDINAL BARS

DIAMETER:	5/8"	3/4"	7/8"	1"	1 1/8"	1 1/4"	1 3/8"	1 1/2"	
AREA OF 4 NO BARS. IN ² :	1.23	1.77	2.41	3.14	3.98	4.91	5.94	7.07	PRO RATA FOR OTHER NUMBERS OF BARS. = 24 DIAM. = 16 DIAMS.
WEIGHT OF 4 NO BARS. LBS/FT.	4.17	6.01	8.18	10.7	13.5	16.7	20.2	24.0	
MINIMUM LAP LENGTH:	1'-3"	1'-6"	1'-9"	2'-0"	2'-3"	2'-6"	2'-9"	3'-0"	
MAX. SPACING OF BINDERS:	10"	12"	14"	16"	18"	20"	22"	24"	

STRESS-INCREASE FACTORS $\frac{C_1}{C}$ FOR MORE THAN MINIMUM BINDERS.

VOLUME OF BINDERS EXPRESSED AS % OF CORE VOLUME (P)	INDEPENDENT RECTANGULAR BINDERS						HELICAL BINDERS				C = NORMAL SAFE STRESS FORMULAE FOR GENERAL USE :- INDEPENDENT :- $\frac{C_1}{C} = 1 + \frac{P}{10}$ HELICAL :- $\frac{C_1}{C} = 1 + 32P$ FORMULA PER L.C.C. :- $\frac{C_1}{C} = 1 + 0P$ AT = CROSS SECTION AREA OF BINDER
	GENERAL USE	L.C.C.					GENERAL USE	L.C.C.			
		SPACING OF BINDERS						SPACING OF BINDERS			
		*2D OR LESS a = 16	*3D a = 12	*4D a = 08	*5D a = 04	*6D OR MORE a = 0		*2D a = 32	*3D a = 24	*4D a = 16	
0.5	1.05	1.08	1.06	1.04	1.02	1.00	1.16	1.16	1.12	1.08	$\frac{C_1}{C} = 1 + 32P$
1.0	1.10	1.16	1.12	1.08	1.04	MAX. -	1.32	1.32	1.24	1.16	
1.5	1.15	1.24	1.18	1.12	1.06	-	1.48	1.48	1.36	1.24	
2.0	1.20	1.32	1.24	1.16	1.08	-	1.64	1.50	1.48	1.32	
2.5	1.25	1.33	1.30	1.20	1.10	-	1.80	MAX. -	1.50	1.40	
3.0	1.30	MAX. -	1.33	1.24	1.12	-	1.96	-	MAX. -	1.48	$\frac{C_1}{C} = 1 + 0P$ AT = CROSS SECTION AREA OF BINDER
3.13	1.31	-	MAX. -	1.25	1.13	-	2.0 MAX. -	-	-	1.50 MAX. -	
3.5	1.35	-	-	1.28	1.14						
4.0	1.40	-	-	1.32	1.16						
5.0	1.50 MAX.	-	-	1.33	1.20						

$$P = \frac{100 \text{ (VOL. OF BINDERS)}}{\text{VOL. OF CORE}} \left(= \frac{400 \text{ AT}}{\text{FOR } \frac{S}{D} \text{ COLS.}} \right)$$

D = LEAST WIDTH OF CORE; S = SPACING.

$$P = \frac{100 (\text{VOL. OF BINDERS})}{\text{VOL. OF CORE}} = \frac{400 A_t}{S D} \quad (\text{FOR SQ COLS.})$$

D = LEAST WIDTH OF CORE; S = SPACING.

SLENDERNES FACTORS. $\frac{C_2}{C}$ = STRESS REDUCTION FACTOR.

RECTANGULAR COLS.	GENERAL USE	$R = \frac{K(\text{ACTUAL HT.})}{\text{OVERALL WIDTH}}$	12	15	20	25	30	35	40	42	50	60	VALUES OF K.	
		$\frac{C_2}{C} = (1.4 - \frac{R}{30})$	1.00	.90	.73	.57	.40	.24	.07	NIL	-	-	END CONDITIONS	K
		$\frac{C_2}{C} = \frac{1}{1 + .0012 R^2}$.85	.79	.68	.57	.48	.40	.34	.32	.25	.19	BOTH FIXED	.7
		$\frac{C_2}{C}$	1.00	.80	.60	.40	.20	NIL	-	-	-	-	FIXED AND HINGED	1.0
ANY SECTION	GENERAL USE	$R = \frac{K(\text{ACTUAL HT.})}{\text{CORE WIDTH}}$	10.7	12.8	15	17.1	19.3	21.4	-	-	-	-	BOTH HINGED	1.4
		$\frac{C_2}{C}$	1.00	.80	.60	.40	.20	NIL	-	-	-	-	FIXED AND FREE	2.8
		$\frac{C_2}{C}$	1.00	.80	.60	.40	.20	NIL	-	-	-	-	RADI OF GYRATION	
		$R_1 = \frac{K(\text{ACTUAL HT.})}{\text{RAD. OF GYRATION}}$	32.1	40	50	64.2	70	80	100	120	140	160	SQUARE OR RECTANGLE:	.288 D
ANY SECTION	L-C-C	$\frac{C_2}{C} = \frac{1}{1 + .0008 R_1^2}$.91	.86	.80	.71	.67	.61	.50	.41	.34	.28	CIRCLE:	.50 D
		$\frac{C_2}{C}$	1.00	.75	.44	NIL	-	-	-	-	-	-	OCTAGON:	.257 D
		$\frac{C_2}{C}$	1.00	.75	.44	NIL	-	-	-	-	-	-	ANNULAR:	$.5 \sqrt{R^2 + r^2}$
		$\frac{C_2}{C}$	1.00	.75	.44	NIL	-	-	-	-	-	-		

SAFE LOADS (SEE ALSO TABLE No 35)

FOR VALUES OF M SEE TABLE No 23.

$$W = [A + (1n-1) A_c] C_x$$

A = EFFECTIVE CONCRETE AREA.

A_c = AREA OF LONGITUDINAL REINFORCEMENT.

C_x = SAFE STRESS ALLOWING FOR % OF BINDERS, SLENDERNES, MIX, ETC

EXAMPLES OF USE OF TABLE No. 35.

(a) To select a design for a column to carry 200 tons subject to the following conditions: (i) to occupy the minimum of floor space; (ii) square column constructed in concrete Mix C:

- (i) Smallest square column = 19 in. by 19 in. = 361 sq. in. of floor space.
Smallest circular column = 18½ in. overall diameter = 269 sq. in. of floor space.

Therefore an 18½-in. circular (or octagonal) column constructed in Mix E concrete, with 4 per cent. of longitudinal steel and 3.13 per cent. of helical binders, would be satisfactory. (Note that owing to the decrease in m , with high percentages of longitudinal steel there is little advantage in using Mix F in place of Mix E.)

- (ii) Referring to Table No. 35:

24-in. square column with $A_c = 0.8$ per cent. of total area and binders = 0.2 per cent. total volume.

$A_c = 0.8$ per cent. $\times 24^2 = 4.61 =$ Eight $\frac{7}{8}$ -in. bars.

Binders: Volume required = 0.2 per cent. $\times 24^2 \times 12 = 13.8$ cb. in. per ft. run of column.

Area per ft. = $\frac{13.8}{4 \times 21} = 0.165$ sq. in. = (From Table No. 25) $\frac{5}{16}$ -in.

binders at 11-in. centres double system.

This design takes the full area of the concrete into account. If, from the point of view of protection from fire or other damage, the cover has to be discounted as a load-bearing part of the column, one of the following designs should be adopted: 28-in. square column. $A_c = 0.8$ per cent. (core); Binders = 0.5 per cent. (core).

$A_c = 0.8 \times 25^2 = 5.0$ sq. in. — Eight $\frac{7}{8}$ -in. bars.

Binders: by rearranging formula given on Table No. 34: $\frac{A_T}{s} = \frac{pD}{400}$

$$= \frac{0.5 \times 25}{400} = 0.0313.$$

For double system of $\frac{5}{16}$ -in. binders at 7-in. centres, $\frac{A_T}{s} = \frac{2 \times 0.11}{7} = 0.0314.$

With a 23-in. square column; $A_c = 0.8$ per cent. (core); Binders = 5.0 per cent. (core).

With a 20-in. square column; $A_c = 4$ per cent. (core); Binders = 5.0 per cent. (core).

The exact nature of the requisite amount of reinforcement for the columns would be determined as above, but generally such designs would be less economical than the first alternative.

NOTES.

COLUMNS - SAFE LOADS (TONS)

TABLE No. 35.

CONCRETE MIX & NORMAL WORKING STRESS. SEE TABLE No. 23			MIX C. 700 LBS. PER SQ. IN. $m = 15$				MIX E. 800 LBS. PER SQ. IN. $m = 12.5$				MIX F. 875 LBS. PER SQ. IN. $m = 10$			
LONGITUDINAL STEEL			0.8%	0.8%	0.8%	4.0%	0.8%	0.8%	0.8%	4.0%	0.8%	0.8%	0.8%	4.0%
			TOTAL	CORE	CORE	CORE	TOTAL	CORE	CORE	CORE	TOTAL	CORE	CORE	CORE
BINDERS			0.2%	0.5%	MAX.		0.2%	0.5%	MAX.		0.2%	0.5%	MAX.	
COL. TYPE	OVERALL SIZE	CORE WIDTH	TOTAL	CORE			TOTAL	CORE			TOTAL	CORE		
SQUARE COLUMNS (MAX. VOL. OF BINDERS = 5% OF CORE)	9"	7"	28	18	25	36	31	20	29	38	34	22	31	39
	10"	8"	35	23	33	47	39	26	37	50	42	28	40	51
	11"	9"	42	30	42	59	47	33	47	64	51	36	51	64
	12"	9"	50	30	42	59	56	33	47	64	60	36	51	64
	13"	10"	59	37	52	73	66	41	58	79	71	44	63	80
	14"	11"	68	44	63	88	76	49	71	95	87	53	76	97
	15"	12"	78	53	75	105	88	59	84	113	94	63	91	114
	16"	13"	89	62	88	123	100	69	99	132	107	74	107	134
	17"	14"	100	72	102	143	112	80	115	154	121	86	124	156
	18"	15"	112	82	117	165	126	92	132	177	136	99	142	179
	19"	16"	125	94	133	187	141	105	150	200	151	112	161	203
	20"	17"	138	105	150	210	156	118	169	226	168	127	182	229
	21"	18"	153	118	169	236	172	132	189	254	185	142	204	257
	22"	19"	168	132	188	264	188	148	212	284	203	159	228	287
	24"	21"	204	161	230	322	229	181	259	346	246	194	278	351
	27"	24"	252	177	305	429	283	240	343	460	305	258	370	465
	30"	27"	311	266	379	531	350	298	426	571	378	320	458	577
	33"	30"	379	329	468	659	425	369	526	706	457	395	566	715
	36"	33"	405	399	567	799	505	446	638	855	545	480	685	868
COLUMNS WITH HELICALLY BOUND CORES. (MAX. VOL. OF BINDERS = 3-13% OF CORE)	12"	9"	-	26	44	62	-	29	49	68	-	31	53	67
	13"	10"	-	32	54	76	-	35	60	83	-	38	66	83
	14"	11"	-	38	68	93	-	43	74	101	-	46	80	100
	15"	12"	-	46	81	110	-	51	87	120	-	55	95	120
	16"	13"	-	53	95	129	-	60	102	141	-	64	111	141
	17"	14"	-	62	107	150	-	70	119	163	-	75	129	163
	18"	15"	-	71	122	172	-	80	136	187	-	86	148	187
	19"	16"	-	84	140	196	-	91	155	214	-	98	168	213
	20"	17"	-	91	157	221	-	103	175	241	-	110	189	242
	21"	18"	-	102	176	248	-	115	196	270	-	123	212	270
	22"	19"	-	114	197	277	-	129	219	301	-	136	237	301
	24"	21"	-	139	240	338	-	157	267	369	-	168	290	367
	27"	24"	-	182	319	449	-	209	355	489	-	223	385	490
	30"	27"	-	230	396	556	-	259	440	608	-	276	478	607
	33"	30"	-	284	490	689	-	320	545	750	-	342	590	750
	36"	33"	-	345	595	835	-	390	660	910	-	415	715	910

IF OTHER NORMAL WORKING STRESSES ARE ADOPTED, SAFE LOADS WOULD BE ADJUSTED PRO RATA.
LOADS TABULATED ABOVE ARE NOT IN ACCORDANCE WITH L.C.C. REGULATIONS.
SEE TABLE No. 34 FOR CONDITIONS ON WHICH ABOVE LOADS ARE COMPUTED.

EXAMPLES OF USE OF TABLE No. 36.

A rectangular section is 18 in. deep and 12 in. wide and is reinforced with two 1-in. bars in the top and three 1-in. bars in the bottom. The centres of the bars are $1\frac{1}{2}$ in. from the top and bottom faces respectively. To find the stresses produced in the section under the conditions of direct thrust and moment given below, assuming a modular ratio of 15.

For all cases $D = 18$ in., $b = 12$ in., $d = 16.5$ in., $f_1 = \frac{16.5 - 1\frac{1}{2}}{16.5} = 0.909$.

$A_T = 2.36$ sq. in.; $A_G = 1.57$ sq. in.
(a) $M = 200,000$ in. lb. $N = 100,000$ lb.

$$c = \frac{200,000}{100,000} = 2 \text{ in., that is, } < 0.167D (= 3 \text{ in.}).$$

$$A_1 = 14 \times 3.93 = 55 \text{ sq. in. } A_2 = 12 \times 18 = 216 \text{ sq. in.}$$

$$Z = \frac{2 \times 55}{18} (9 - 16.5 + 15)^2 + (0.167 \times 216 \times 18) = 988 \text{ in.}$$

$$c = \frac{100,000}{55 + 216} \pm \frac{200,000}{988} = 571 \text{ and } 165 \text{ lb. per sq. in.}$$

(b) $M = 200,000$ in. lb. $N = 50,000$ lb.

$$e = \frac{200,000}{50,000} = 4 \text{ in., that is, } > 0.167D (= 3 \text{ in.}) \text{ and } < 0.5d (8.25 \text{ in.}).$$

A_1 , A_2 , and Z as for (a).

$$c_{\text{in}} = \frac{50,000}{55 + 216} - \frac{200,000}{988} = 184 - 203 = -19 \text{ lb. per sq. in.}$$

This is less than one-sixth of the allowable maximum stress (say $\frac{600}{6}$) and therefore maximum compression stress = $184 + 203 = 387$ lb. per sq. in.

(c) $M = 300,000$ in. lb. $N = 15,000$ lb.

$$e = \frac{300,000}{15,000} = 20 \text{ in., that is, } > 0.5d (8.25 \text{ in.}) \text{ and } < 1.5d (= 25 \text{ in.}).$$

$$k = \frac{20 \times 9}{16.5} + 1 = 1.666. \text{ Assume } n_1 = 0.55.$$

$$j = 0.55 \times 12 \times 16.5 \times 0.5 = 55.$$

From Table No. 36: $G = 0.224$, $H = 11.5$ approx.

$$c = \frac{15,000 \times 1.666}{(0.224 \times 12 \times 16.5) + (11.5 \times 1.57 \times 0.909)} = 412 \text{ lb. per sq. in.}$$

$$t = \frac{412(55 + 11.5 \times 1.57) - 15,000}{2.36} = 6,440 \text{ lb. per sq. in.}$$

Corresponding value of $n_1 = \frac{1}{1 + \frac{6,440}{412 \times 15}} = 0.49$, compared with the

assumed value 0.55. Re-working with an intermediate value of 0.51,

$$j = 12 \times 16.5 \times 0.5 \times 0.51 = 50.5.$$

$$G = 0.5 \times 0.51 \left(1 - \frac{0.51}{3}\right) = 0.212.$$

$$H = \frac{14}{0.51} (0.51 + 0.909 - 1) = 11.5.$$

$$c = \frac{15,000 \times 1.666}{(0.212 \times 12 \times 16.5) + (11.5 \times 1.57 \times 0.909)} = 426 \text{ lb. per sq. in.}$$

$$t = \frac{426(50.5 + 11.5 \times 1.57) - 15,000}{2.36} = 6,020 \text{ lb. per sq. in.}$$

Corresponding value of $n_1 = \frac{1}{1 + \frac{6,020}{426 \times 15}} = 0.514$; this is close enough to

second trial value.

(d) $M = 10,000$ in. lb. $N =$ direct pull of 2,000 lb.

$$e = \frac{10,000}{2,000} = 5 \text{ in. or } < 0.5f_1d (= 7.5 \text{ in.}).$$

$$\text{Tensile stress in } A_T = \frac{2,000}{2.36} (7.5 + 5) = 10,600 \text{ lb. per sq. in.}$$

$$\text{Tensile stress in } A_G = \frac{2,000}{1.57} (7.5 - 5) = 3,180 \text{ lb. per sq. in.}$$

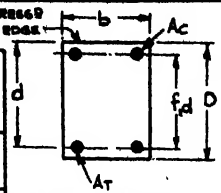
BENDING & DIRECT FORCE.

TABLE NO. 36.

RECTANGULAR SECTIONS.

$$e = \frac{\text{MOMENT}}{\text{DIRECT FORCE}} = \frac{M}{N}$$

COMPRESSED EDGE



$$e < .467d$$

$$A_1 = (m-1)(A_c + A_t); \quad z = \frac{2A_1(.5d - d + f_d)^2 + .167A_2 D}{D}$$

$$A_2 = bD$$

$$C_{MAX} \& C_{MIN} = \frac{N}{A_1 + A_2} \pm \frac{M}{Z}$$

$e > .467d$
 COMPUTE C (MIN.) AS ABOVE; IF NEGATIVE AND $> \frac{\text{ALLOWABLE } C}{6}$
 RECALCULATE BY METHOD GIVEN BELOW.

EVALUATE $F = \frac{e - .5d}{d} + 1$; ASSUME n_1 ; FIND G, H , AND $J = \frac{bdn_1^2}{2}$
 SUBSTITUTE IN $C = \frac{NF}{Gbd + HA_c f_i}$ $t = \frac{C(J + HA_c H) - N}{A_t}$

CHECK ASSUMED VALUE OF n_1 FROM $n_1 = \frac{1}{1 + t/mc}$ AND REWORK IF MORE THAN 10% ERROR.

VALUES OF $t/c, H, \& C. (H = 1/n_1(m-1)(n_1 + f_i - 1))$

VALUE OF n_1	.20	.25	.30	.35	.40	.45	.50	.55	.60	.65	.70	.75
$G = 5n_1(1 - n_1/3)$.043	.114	.135	.155	.173	.191	.208	.224	.240	.254	.268	.281
$m = 15$	t/c	60	45	35	28	22.5	18.3	15	13.7	10	8	6.5
	VALUES OF f_i	.80	-	2.8	4.6	6.0	7.0	7.7	8.4	9.0	9.4	9.9
		.85	3.5	4.6	7.0	8.0	8.8	9.4	10.2	10.5	10.7	11.0
		.90	7.0	8.4	9.4	10.1	10.5	10.9	11.2	11.5	11.8	12.0
		.95	10.5	11.2	11.6	12.0	12.3	12.4	12.6	12.7	12.9	13.0
$m = 12.5$	t/c	50	37.5	29	23	18.8	15.2	12.5	10.2	8.3	6.8	5.4
	VALUES OF f_i	.80	-	2.3	3.8	4.9	5.7	6.3	6.9	7.4	7.7	8.0
		.85	2.9	4.6	5.7	6.6	7.2	7.7	8.0	8.4	8.7	8.8
		.90	5.7	6.9	7.7	8.3	8.6	9.0	9.2	9.5	9.6	9.9
		.95	7.8	9.2	9.5	9.9	10.1	10.2	10.4	10.4	10.6	10.7
$m = 10$	t/c	40	30	23.3	18.5	15	12.2	10	8.2	6.7	5.4	4.3
	VALUES OF f_i	.80	-	1.8	3.0	3.9	4.5	5.0	5.4	5.8	6.1	6.3
		.85	2.3	3.6	4.5	5.1	5.6	6.1	6.3	6.6	6.8	6.9
		.90	4.5	5.4	6.1	6.5	6.8	7.0	7.2	7.4	7.5	7.7
		.95	6.8	7.2	7.5	7.7	7.9	8.0	8.1	8.2	8.3	8.4
$m = 8$	t/c	32	24	18.6	14.8	12	9.8	8	6.5	5.4	4.3	3.4
	VALUES OF f_i	.80	-	1.4	2.3	3.0	3.5	3.9	4.2	4.5	4.7	4.9
		.85	1.8	2.8	3.5	4.0	4.4	4.7	4.9	5.1	5.3	5.4
		.90	3.5	4.2	4.7	5.1	5.3	5.5	5.6	5.8	5.8	6.0
		.95	5.8	5.6	5.8	6.0	6.2	6.2	6.3	6.4	6.5	6.5

$e > .5d$

$e > .5d$ CALCULATE STRESSES C_1 AND t_1 DUE TO M ONLY AND DETERMINE $n = n_1 d$ FOR THESE STRESSES
 EVALUATE $C_2 = \frac{N}{bn + A_t m + A_c(m-1)}$; THEN $C = C_1 + C_2$
 $t = t_1 - m C_2$

$e < .5f_d$ TENSILE STRESS IN $A_t = \frac{N}{A_t}(.5f_d + e)$; TENSILE STRESS IN $A_c = \frac{N}{A_c}(.5f_d - e)$.

$e > .5f_d$ EVALUATE $L = \frac{e + .5d}{d} - 1$; ASSUME n_1 AND FIND G, H , AND $J = .5bdn_1^2$
 SUBSTITUTE IN $C = \frac{NL}{bdG + HA_c f_i}$ $t = \frac{C(J + A_c H) + N}{A_t}$
 CHECK n_1 AS ABOVE

$e > .5d$ CALCULATE STRESSES C_1 AND t_1 DUE TO M ONLY AND DETERMINE $n = n_1 d$ FOR THESE STRESSES
 EVALUATE C_2 AS ABOVE; THEN $C = C_1 - C_2$ $t = t_1 + m C_2$

BENDING AND DIRECT COMPRESSION

BENDING AND DIRECT TENSION

EXAMPLES OF USE OF TABLE No. 37.

- (a) To convert 17 ft. 8
- $\frac{1}{8}$
- in. into metric units :

$$17 \text{ ft. } 8 \text{ in.} = 5.385 \text{ metres.}$$

$$\frac{1}{8} \text{ in.} = 0.0079 \text{ ,,}$$

$$17 \text{ ft. } 8 \frac{1}{8} \text{ in.} = 5.393 \text{ ,,}$$

- (b) To convert 4.067 metres into English units :

$$4.067$$

$$4.064 = 13 \text{ ft. } 4 \text{ in.}$$

$$0.003 = \frac{1}{8} \text{ (At foot of Table : } \frac{1}{8} \text{-in.} = 3.18 \text{ mm.)}$$

$$13 \text{ ft. } 4 \frac{1}{8} \text{ in.}$$

- (c) To express
- $\frac{1}{2}$
- in. bars at 8-in. centres in metric units :

$$\frac{1}{2} \text{ in.} = 12.7 \text{ mm. } 8 \text{ in.} = 0.203 \text{ m.}$$

Nearest practical values : 12-mm. bars at 20-cm. centres.

- (d) To express 600 lb. per sq. in. in metric units :

$$= \frac{600}{14.25} = 42 \text{ kg. per sq. cm. approximately.}$$

- (e) To convert 6 lb. per sq. yd. to metric units :

$$6 \text{ lb.} = \frac{6}{2.21} = 2.71 \text{ kg.}$$

$$1 \text{ sq. yd.} = \frac{1}{1.196} = 0.836 \text{ sq. m.}$$

$$\therefore 6 \text{ lb. per sq. yd.} = \frac{2.71}{0.836} = 3.24 \text{ kg. per sq. m.}$$

NOTES.

EXAMPLES OF USE OF TABLE No. 38.

(a) To express 10 ft. $5\frac{5}{8}$ in. as a decimal of a foot :

$$\begin{array}{r} 10 \text{ ft.} = 10.00000 \\ 5\frac{5}{8} \text{ in.} = 0.42969 \end{array}$$

(b) To convert 0.6732 ft. to inches (nearest $\frac{1}{32}$ in.) :

$$\begin{array}{r} 0.6732 \text{ ft.} \\ 0.67187 \text{ ft.} = 8\frac{1}{16} \text{ in.} \end{array}$$

Difference = 0.00133 ft. which is slightly more than $\frac{1}{84}$ in. Therefore 0.6732 ft. $\approx 8\frac{1}{16}$ in.

NOTES.

DECIMAL EQUIVALENTS

TABLE N°38.

INS.	0	1	2	3	4	5	6	7	8	9	10	11
	-	•08333	•16666	•25	•33333	•41666	•5	•58333	•66666	•75	•83333	•91666
$\frac{1}{32}$	•00260	•08344	•16927	•2528	•33594	•41927	•5026	•58594	•66927	•7526	•83594	•91927
$\frac{1}{16}$	•00521	•08654	•17187	•25521	•33854	•42187	•50521	•58854	•67187	•75521	•83854	•92187
$\frac{3}{32}$	•00781	•09114	•17448	•25781	•34114	•42448	•50781	•59114	•67448	•75781	•84114	•92448
$\frac{1}{8}$	•01041	•09374	•17707	•26041	•34374	•42707	•51041	•59374	•67707	•76041	•84374	•92707
$\frac{5}{32}$	•01302	•09635	•17969	•26302	•34635	•42969	•51302	•59635	•67969	•76302	•84635	•92969
$\frac{3}{16}$	•01562	•09895	•18228	•26562	•34895	•43228	•51562	•59895	•68228	•76562	•84895	•93228
$\frac{7}{32}$	•01823	•10156	•18489	•26823	•35156	•43489	•51823	•60156	•68489	•76823	•85156	•93489
$\frac{1}{4}$	•02083	•10416	•1875	•27083	•35416	•4375	•52083	•60416	•6875	•77083	•85416	•9375
$\frac{9}{32}$	•02344	•10677	•1901	•27344	•35677	•4401	•52344	•60677	•6901	•77344	•85677	•9401
$\frac{5}{16}$	•02604	•10937	•1927	•27604	•35937	•4427	•52604	•60937	•6927	•77604	•85937	•9427
$\frac{11}{32}$	•02864	•11198	•19531	•27864	•36198	•44531	•52864	•61198	•69531	•77864	•86198	•94531
$\frac{3}{8}$	•03125	•11458	•19791	•28125	•36458	•44791	•53125	•61458	•69791	•78125	•86458	•94791
$\frac{13}{32}$	•03385	•11718	•20052	•28385	•36719	•45052	•53385	•61719	•70052	•78385	•86719	•95052
$\frac{7}{16}$	•03646	•11979	•20312	•28646	•36979	•45312	•53646	•61979	•70312	•78646	•86979	•95312
$\frac{15}{32}$	•03906	•12239	•20573	•28906	•37239	•45573	•53906	•62239	•70573	•78906	•87239	•95573
$\frac{1}{2}$	•04166	•125	•20832	•29166	•375	•45833	•54166	•625	•70832	•79166	•875	•95833
$\frac{17}{32}$	•04427	•1276	•21094	•29427	•3776	•46094	•54427	•6276	•71094	•79427	•8776	•96094
$\frac{9}{16}$	•04687	•1302	•21353	•29687	•3802	•46353	•54687	•6302	•71353	•79687	•8802	•96353
$\frac{19}{32}$	•04948	•13281	•21614	•29948	•38281	•46614	•54948	•63281	•71614	•79948	•88281	•96614
$\frac{5}{8}$	•05208	•13541	•21874	•30208	•38541	•46875	•55208	•63541	•71874	•80208	•88541	•96875
$\frac{21}{32}$	•05469	•13809	•22135	•30469	•38802	•47135	•55469	•63802	•72135	•80469	•88802	•97135
$\frac{11}{16}$	•05729	•14062	•22395	•30729	•39062	•47395	•55729	•64062	•72395	•80729	•89062	•97395
$\frac{23}{32}$	•05989	•14323	•22656	•30989	•39323	•47656	•55989	•64323	•72656	•80989	•89323	•97656
$\frac{3}{4}$	•0625	•14583	•22916	•3125	•39583	•47916	•5625	•64583	•72916	•8125	•89583	•97916
$\frac{25}{32}$	•0651	•14844	•23177	•3151	•39844	•48177	•5651	•64844	•73177	•8151	•89844	•98177
$\frac{13}{16}$	•06771	•15104	•23437	•31771	•40104	•48437	•56771	•65104	•73437	•81771	•90104	•98437
$\frac{27}{32}$	•07031	•15364	•23698	•32031	•40364	•48698	•57031	•65364	•73698	•82031	•90364	•98698
$\frac{7}{8}$	•07292	•15625	•23958	•32292	•40625	•48958	•57292	•65625	•73958	•82292	•90625	•98958
$\frac{29}{32}$	•07552	•15885	•24219	•32552	•40885	•49219	•57552	•65885	•74219	•82552	•90885	•99219
$\frac{15}{16}$	•07813	•16146	•24479	•32813	•41146	•49479	•57813	•66146	•74479	•82813	•91146	•99479
$\frac{31}{32}$	•08073	•16406	•24739	•33073	•41406	•49739	•58073	•66406	•74739	•83073	•91406	•99739
	0	1	2	3	4	5	6	7	8	9	10	11

DIVISIONS OF A FOOT EXPRESSED IN FEET

NOTES.

MISCELLANEOUS DATA.

TABLE N^o39.

QUANTITIES OF MATERIALS PER CU. YD. OF CONCRETE.

MIX.	A	C	D	E	F	-
APPROX. PROPORTIONS	1 · 3 · 6	1 · 2 · 4	1 · 1 ³ / ₅ · 3 ¹ / ₂	1 · 1 ¹ / ₂ · 3	1 · 1 · 2	BY VOL.
COARSE AGGREGATE	24	22 ² / ₃	22 ¹ / ₃	22	21 ¹ / ₃	CU. FT.
FINE AGGREGATE	12	11 ¹ / ₃	11 ¹ / ₆	11	10 ² / ₃	CU. FT.
CEMENT	360	510	585	660	960	LBS.

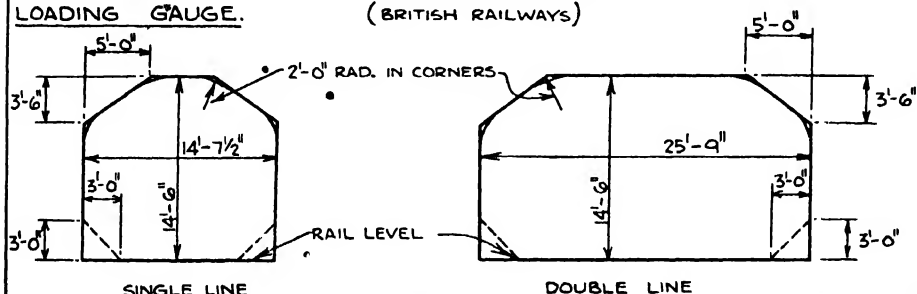
THIS TABLE SHOULD BE READ IN CONJUNCTION WITH TABLE N^o 23.

QUANTITIES ARE APPROXIMATE ONLY BUT ARE APPLICABLE TO BALLAST AND SANDS OF NORMAL GRADING. AN ADDITIONAL 5% SHOULD BE ADDED TO BOTH COARSE AND FINE AGGREGATE QUANTITIES IF CRUSHED STONE IS USED.

FOR WASTE, ETC., ADD ABOUT 10% TO AGGREGATE AND 5% TO CEMENT.

LOADING GAUGE.

(BRITISH RAILWAYS)



PROPERTIES OF SECTIONS.

SECTION	AREA	MODULUS	MOMENT OF INERTIA
	DB	XX. $\frac{BD^2}{6}$ YY. $\frac{BD^2}{3}$ ZZ. $\frac{B^2D^2}{6(B^2+D^2)}$	XX. $\frac{BD^3}{12}$ YY. $\frac{BD^3}{3}$ ZZ. $\frac{B^3D^3}{6(B^2+D^2)}$
	$Bd + b(D-d)$	$\frac{M. OF I}{N} = \frac{bD^2 + d^2(B-b)}{2[Bd + b(D-d)]}$	$\frac{1}{3}[BN^3 + bn^3 - h^3(B-b)]$
	D^2	$.118D^3$	$\frac{D^4}{12}$
	$.828D^2$	XX. $.109D^3$ YY. $.1016D^3$	XX. $.055D^4$ YY. $.055D^4$
	$.7854D^2$	$.0982D^3$	$.0491D^4$

ESSENTIAL TRIGONOMETRICAL FORMULÆ.

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$\sin (\theta + \phi) = \sin \theta \cos \phi + \cos \theta \sin \phi$$

$$\sin (\theta - \phi) = \sin \theta \cos \phi - \cos \theta \sin \phi$$

$$\cos (\theta + \phi) = \cos \theta \cos \phi - \sin \theta \sin \phi$$

$$\cos (\theta - \phi) = \cos \theta \cos \phi + \sin \theta \sin \phi$$

$$\tan (\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi}$$

$$\tan (\theta - \phi) = \frac{\tan \theta - \tan \phi}{1 + \tan \theta \tan \phi}$$

Applicable to any triangle ABC in which $AB = c$; $BC = a$; $AC = b$.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\text{Area} = \frac{bc \sin A}{2} = \frac{ac \sin B}{2} = \frac{ab \sin C}{2} = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$

$$\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}}$$

NOTES.

TRIGONOMETRICAL RATIOS.

TABLE No 40.

ANGLE DEGREES	SINE	TANGENT	COTANGENT	COSINE	-
1	•0175	•0175	57.2900	•9998	89
2	•0349	•0349	28.6363	•9994	88
3	•0523	•0524	19.0811	•9986	87
4	•0698	•0699	14.3007	•9976	86
5	•0872	•0875	11.4301	•9962	85
6	•1045	•1051	9.5144	•9945	84
7	•1219	•1226	8.1443	•9925	83
8	•1392	•1405	7.1154	•9903	82
9	•1564	•1584	6.3138	•9877	81
10	•1736	•1763	5.6713	•9848	80
11	•1908	•1944	5.1446	•9816	79
12	•2079	•2126	4.7046	•9781	78
13	•2250	•2309	4.3315	•9744	77
14	•2419	•2493	4.0108	•9703	76
15	•2588	•2679	3.7321	•9659	75
16	•2756	•2867	3.4874	•9613	74
17	•2924	•3057	3.2769	•9563	73
18	•3090	•3249	3.0777	•9511	72
19	•3256	•3443	2.9042	•9455	71
20	•3420	•3640	2.7475	•9397	70
21	•3584	•3839	2.6051	•9336	69
22	•3746	•4040	2.4751	•9272	68
23	•3907	•4245	2.3539	•9205	67
24	•4067	•4452	2.2460	•9135	66
25	•4226	•4663	2.1445	•9063	65
26	•4384	•4877	2.0503	•8988	64
27	•4540	•5095	1.9626	•8910	63
28	•4695	•5317	1.8807	•8829	62
29	•4848	•5543	1.8040	•8746	61
30	•5000	•5774	1.7321	•8660	60
31	•5150	•6009	1.6643	•8572	59
32	•5299	•6249	1.6003	•8480	58
33	•5446	•6494	1.5399	•8387	57
34	•5592	•6745	1.4826	•8290	56
35	•5736	•7002	1.4281	•8192	55
36	•5878	•7265	1.3764	•8090	54
37	•6018	•7536	1.3270	•7986	53
38	•6157	•7813	1.2799	•7880	52
39	•6293	•8098	1.2349	•7771	51
40	•6428	•8391	1.1918	•7660	50
41	•6561	•8693	1.1504	•7547	49
42	•6691	•9004	1.1106	•7431	48
43	•6820	•9325	1.0724	•7314	47
44	•6947	•9657	1.0355	•7193	46
45	•7071	1.0000	1.0000	•7071	45
-	COSINE	COTANGENT	TANGENT	SINE	

ADDITIONAL EXAMPLES.

The following examples have been compiled to indicate the use of a series of the foregoing Tables in the design of a number of complete structural parts, together with examples of taking-off and approximately pricing bills of quantities.

- (i) TYPICAL DESIGN OF FLOOR PANEL—BEAM AND SLAB CONSTRUCTION (page 258).
- (ii) TYPICAL DESIGN OF FLOOR PANEL—FLAT SLAB CONSTRUCTION.
 - (a) WITH DROPPED PANELS (page 261).
 - (b) WITHOUT DROPPED PANELS (page 264).
- (iii) TYPICAL DESIGN OF HOPPER BOTTOM (page 265).
- (iv) TYPICAL EXAMPLE OF QUANTITIES AND COST ESTIMATING. (THE QUANTITIES FOR DESIGN (i) ARE TAKEN-OFF AND PRICED IN COMPARISON WITH THOSE FOR (iia) and (iib)) (page 268).

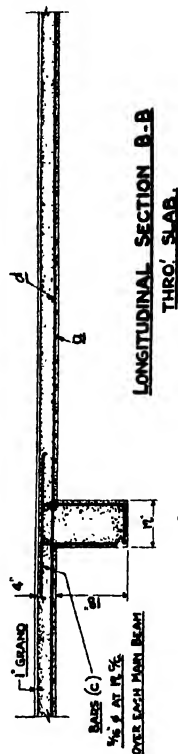


BENDING SCHEDULE

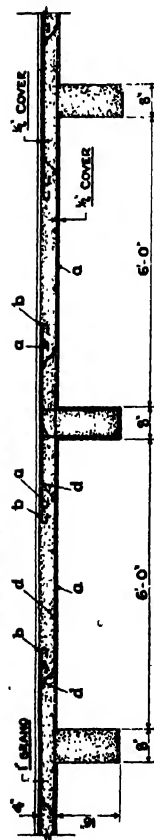
REF.	NO OF BARS (PER PANEL)	DIA.	TOTAL LENGTH	BENDING	REMARKS.
a.	108	3/4"	9'-6"		MAIN TRANSVERSE BARS
b.	51	3/4"	4'-9"		CLAMPS IN TOP OVER SECONDARY BARS
c.	18	5/8"	3'-3"		PUTT OVER MAIN BARS
d.	18	5/8"	20'-6"	STRAIGHT	DISTRIBUTION BARS

- BARS (d)
5/16" DISTRIBUTION BARS

NOTE: BARS SHOWN DOTTED IN PLAN ARE IN TOP OF SLAB



LONGITUDINAL SECTION B-B
THRO' SLAB



TRANSVERSE SECTION A-A. THRO' SLAB.

FIG. 48c- TYPICAL DETAILS OF
BEAM AND SLAB CONSTRUCTION;

(i) TYPICAL DESIGN OF FLOOR PANEL—BEAM AND SLAB CONSTRUCTION.

(See *Figs. 48(a)* and *48(b)* for details and general arrangement)

Warehouse floor in panels 20 ft. square; 1-in. granolithic finish not to be included in resistance section.

Adopt concrete Mix C (see *Table No. 23*) and working stresses thus:

Concrete in compression 700 lb. per sq. in.

Steel in tension 17,000 " " "

Binders 14,000 " " "

Bent-up bars 16,000 " " "

With 17,000 and 700 (from *Table No. 27* with stress ratio = 24.3), lever arm = $a_1 = 0.87$; neutral axis depth ratio = $n_1 = 0.39$.Adopt live load 224 lb. per sq. ft. (< 200 lb. per *Table No. 1*).SLAB. Dead load: 1-in. granolithic (*Table No. 1*) 12 lb. per sq. ft.

4-in. slab 4 × 12 48 " " "

Total dead load 60 " " "

Live load 224 " " "

Total load: 284 " " "

Ratio of live to dead load $\frac{224}{60} = 3.73$ With this ratio compare B.M. coefficientsgiven on *Tables Nos. 10* and *14*; adopting coefficients on *Table No. 14*:Support B.M. $0.083 \times 284 \times 6.67^2 = 1,045$ ft. lb.Midspan B.M. $0.067 \times 284 \times 6.67^2 = 833$ "Slab thickness (see *Table No. 30*):Effective depth = $d = 0.0926 \sqrt{1,045} = 2.99$ in.A 4-in. slab provides $d = 3.25$ in.Midspan: $A_T = \frac{833 \times 12}{17,000 \times 0.87 \times 3.25} = 0.208$ sq. in.Refer to *Table No. 25* and provide $\frac{3}{8}$ -in. bars at 6-in. centres.Support: $A_T = \frac{1,045 \times 12}{17,000 \times 0.87 \times 3.25} = 0.261$ sq. in. $\frac{3}{8}$ -in. bars at 6-in. centres = 0.221 "

Remainder 0.040 "

Remainder can be made up by inserting additional $\frac{3}{8}$ -in. clamps at 12-in. centres over supports.SECONDARY BEAMS Effective span = 20 ft.; clear span = 19 ft. Assume section = 16 in. by 8 in.; $d = 18.5$ in.Dead load: Slab $6.67 \times 60 = 400$ lb. per ft. run.

Beam rib, say = 130 " " "

Total dead load 530 " " "

Live load: $6.67 \times 224 = 1,500$ " " "

Total load 2,030 " " "

Ratio of live to dead load is approximately 3. Referring to *Table No. 10*, the beam can either be designed for a B.M. coefficient of $\frac{1}{12}$ at midspan and support, or for 0.073 at midspan and 0.104 at supports; since the secondary beams have main beams for their supports, adopt the former coefficients.

B.M. at midspan and support

 $= 0.083 \times 2,030 \times 19 \times 20 \times 12 = 772,000$ in. lb.

Maximum breadth of flange (Table No. 32):

- (i) 6.67 ft. = 80 in.
 (ii) $\frac{20}{3}$ = 80 in.
 (iii) $(12 \times 4) + 8 = 56$ in.; therefore $b_f = 56$ in.
 $n = 0.39 \times 18.5 = 7.2$ in., and the neutral axis is below the slab;
 $a = 16.5$ in.

$$\text{R.M. of compression at midspan} = \frac{700 \times 56 \times 16.5 \times 4}{2 \times 7.2} (14.4 - 4) = 1,870,000 \text{ in. lb., which is ample.}$$

$$\text{Midspan } A_r = \frac{772,000}{17,000 \times 16.5} = 2.75 \text{ sq. in.}$$

Two 1½-in. and two 1-in. bars = 2.773 sq. in.

$$\text{At support: } A_r = \frac{772,000}{17,000 \times 0.87 \times 17.5} = 2.98 \text{ sq. in.}$$

Four 1-in. bars = 3.14 sq. in.

R.M. of compression at support: From Table No. 29, R.M. of concrete only $8 \times 36,000$, say, 288,000 in. lb. which is insufficient. With an increased compressive stress of 850 lb per sq. in. at the support, and $A_c = A_r$, R.M. = $8 \times 100,000 = 800,000$ in. lb., which is satisfactory if $A_r = A_c = 8 \times 0.4$ (say) = 3.2 sq. in. (3.14 sq. in. have been provided). Alternatively, by "steel-beam theory,"

$$A_r = A_c = \frac{772,000}{17,000 \times 15} = 3.03 \text{ sq. in.}$$

Maximum shear force = $2,030 \times 19 \times 0.5 = 19,300$ lb.

$$s = \frac{19,700}{8 \times 15} = 164 \text{ lb per sq. in. Satisfactory.}$$

From Table No. 33, one 1-in. bar bent-up at 45 deg at 16,000 lb per sq. in. takes 8,900 in "single shear"; if the bars are arranged in "double shear" the value would be $2 \times 8,900 = 17,800$ lb. This is sufficient with nominal binding.

At end of second bent-up bar (3 ft. from support) shear force = $2,030 \times 7$

$$14,210 \text{ lb. and } s = \frac{14,210}{8 \times 15.5} = 108 \text{ lb. per sq. in.; therefore, from}$$

Table No. 33, shear to be taken on steel = $0.83 \times 14,210 = 11,800$ lb

If this shear is taken on binders, the required value of V is $\frac{11,800}{16.5} = 716$.

This is provided by ¾-in. single binders at 4-in. centres at 14,000 lb per sq. in. (Alternatively if one of the remaining top bars were bent down at 30 deg., the resistance provided would be $0.707 \times 8,900 \times 2 = 12,600$ lb. in double shear.)

MAIN BEAMS.—Effective span = 20 ft.; clear span = 18 ft. 9 in (15-in. column).

Assume section: 18 in. by 12 in. net.

Dead loads. Point loads = $2 \times 530 \times 19 = 20,200$ lb.

Uniformly distributed beam = 216 lb per ft. run.

Slab, etc. = 60 " " "

$$\text{Total} = 18\frac{1}{2} \times 276 = 5,180 \text{ lb.}$$

Live loads. Point loads = $2 \times 1,500 \times 19 = 57,000$ lb.

Uniformly distributed (= live load on beam width)
 = $18\frac{1}{2} \times 1 \times 336 = 6,300$ lb.

Considering the span as equivalent to the centre span of a five-span beam system, the moment coefficients can be taken from Table No. 8:

Midspan B.M.

Dead-load point loads:	$0.061 \times 20,200 \times 20 \times 12 =$	296,000
" distributed loads:	$0.046 \times 5,180 \times 20 \times 12 =$	57,000
Live-load point loads:	$0.115 \times 57,000 \times 20 \times 12 =$	1,570,000
" distributed load:	$0.086 \times 6,300 \times 20 \times 12 =$	130,000

$$\text{Total} = 2,053,000$$

s

Support B.M.:		in.-lb.
Dead-load point loads:	$0.106 \times 20,200 \times 20 \times 12 =$	513,000
„ distributed load:	$0.080 \times 5,180 \times 20 \times 12 =$	100,000
Live-load point loads:	$0.148 \times 57,000 \times 20 \times 12 =$	2,022,000
„ distributed load:	$0.111 \times 6,300 \times 20 \times 12 =$	168,000

Total = 2,803,000

Midspan section: Maximum breadth of flange = $(12 \times 4) + 12 = 60$ in.

$n = 0.39 \times 19.75 = 7.7$ in.; $a = 17.75$ in. approximately.

R.M. of compression.

Flange (excluding stem)

$$= \frac{700 \times 48 \times 17.75 \times 4}{2 \times 7.7} (15.4 - 4) = 1,765,000 \text{ in. lb.}$$

$$\text{Stem alone} = 116.7 \times 12 \times 19.75^2 = 546,000 \text{ „}$$

Total = 2,311,000 „

Thus the assumed section provides sufficient compressive resistance at midspan.

$$A_T = \frac{2,053,000}{17,000 \times 17.75} = 6.80 \text{ sq. in.}$$

Four $1\frac{1}{2}$ -in. bars ($= 7.07$ sq. in.) in one layer

Support Section. With c not exceeding 850 lb. per sq. in., $A_T = A_c$, and with $d = 18$ in., from Table No. 29, R.M. = $12 \times 106,300 = 1,276,000$ in. lb., which is insufficient. R.M. (per inch width) required

$$= \frac{2,803,000}{12} = 234,000 \text{ in. lb.}$$

This is given by a section with $d = 27$ in. or a total depth of 31 in. (allowing for tension steel in two layers). Hence a 9-in. haunch must be provided, and $A_T = A_c$.

$$a_s = 0.87 \times 27 = 23.5 \text{ in.}; a_s = 23.5 \text{ in.}$$

$$A_T = \frac{2,803,000}{17,000 \times 23.5} = 7.02 \text{ sq. in.}$$

= Four $1\frac{1}{2}$ -in. bars top and bottom.

Maximum shear force = one-half the total load = 44,340 lb. Two $1\frac{1}{2}$ -in. bars bent-up at 30 deg. (double shear) at 16,000 lb. take

$$0.707 \times 2 \times 2 \times 20,000 = 56,500 \text{ lb.,}$$

which is ample. This shear remains practically constant up to the point load; thus the same resistance is required throughout.

$$\text{Approximate shear at end of haunch} = \frac{44,340}{12 \times 17.75} = 208 \text{ lb. per sq. in.,}$$

which is allowable for main beams.

Maximum shear beyond point load = $\frac{1}{3}(5,180 + 6,300) = 1,900$ lb., which can be safely taken on the concrete.

The anchorage lengths of, and types of, hooks for the various bars are calculated from the data given on Table No. 24, and the points of bending-up or stopping-off are determined in accordance with Table No. 32. When the detailed arrangement of the bars has been finally determined, as in Figs. 48(a) and 48(b), the effective depths and lever arms of the various sections may be slightly different from the values assumed in the calculations, but these divergencies would make no difference to the steel areas provided.

(ii) TYPICAL DESIGN OF FLOOR PANEL—FLAT SLAB CONSTRUCTION

(a) WITH DROPPED PANELS (Fig. 49).

Floor panel 20 ft. square, with finish, loading, stresses, etc., as in Example (i).
See Table No. 14 for special formulæ, etc. Minimum size of dropped panel
 $= 0.4 \times 20 = 8.0$ ft.

For convenience, make panel 10 ft. square.

Minimum diameter of column capital $= 0.225 \times 20 = 4$ ft. 6 in.

$B = L = 20$ ft. $= 240$ in.

$S_B = S_L = 240 - 0.67 \times 54 = 204$ in.

Loading: Live $= 224$ lb. per sq. ft.

1-in. Granolithic $= 12$ " " "

Dead (say) $= 114$ " " "

$w = 350$ " " "

Minimum slab thickness to comply with ruling that depth of slab with dropped panels shall not be less than 6 in., nor less than $[0.00083(240 + 240)\sqrt{350}] + 1.0 = 8\frac{1}{2}$ in.; hence assumed dead load is reasonable.

$$\text{Total positive B.M.} = \frac{350 \times 240 \times 204^2}{3,600} = 972,000 \text{ in. lb.}$$

(a) 30 per cent. to be provided in centre half $= \frac{0.3 \times 972,000}{10} = 29,200$ in. lb. per ft.

(b) 35 per cent. to be provided in each outer quarter

$$= \frac{0.35 \times 972,000}{5} = 68,100 \text{ in. lb. per ft.}$$

$$\text{Total negative B.M.} = \frac{350 \times 240 \times 204^2}{2,160} = 1,625,000 \text{ in. lb.}$$

(c) 20 per cent. to be provided in centre half $= \frac{0.2 \times 1,625,000}{10} = 32,500$ in. lb. per ft.

(d) 40 per cent. to be provided in each outer quarter

$$= \frac{0.40 \times 1,625,000}{5} = 130,000 \text{ in. lb. per ft.}$$

Moments (a), (b), and (c) will control the thickness of the slab between the dropped panels; from Table No. 30, effective depth required $= 0.0926\sqrt{\frac{68,100}{12}} = 7$ in.

Slab $8\frac{1}{2}$ in. thick gives $d = 7.75$ in. for bottom layer of steel and 7.25 in. for second layer.

Moment (d) will control the thickness of the dropped panel:

$$d = 0.0926\sqrt{\frac{130,000}{12}} = 9.8 \text{ in.}$$

Making the dropped panel 11 in. thick gives $d = 10.06$ in. for top layer of bars and 9.44 in. for second layer of bars.

The slab thicknesses provided give a dead load approximately equal to that assumed.

• Calculation of steel areas.

In centre half,

$$\bullet \text{ in bottom at midspan, area} = \frac{29,200}{7.25 \times 0.87 \times 17,000} = 0.272 \text{ sq. in.}$$

Say, $\frac{1}{2}$ -in. bars at $7\frac{1}{2}$ -in. centres.

$$\bullet \text{ in top over column centre line, area} = \frac{32,500}{7.25 \times 0.87 \times 17,000} = 0.303 \text{ sq. in.}$$

Say, $\frac{1}{2}$ -in. bars at $7\frac{1}{2}$ -in. centres.

See Detail A-A for
Typical Dropped Panel

See Detail A-A for
Typical Dropped Panel

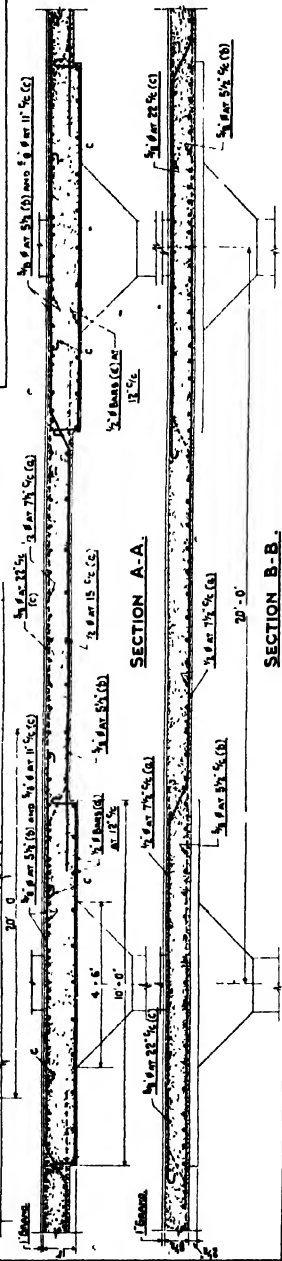
NOTES
All Bars to Have $\frac{1}{4}$ " Minimum Cover or Concrete
Base, (a) and (b) - Alternative Bars Accepted.
Reinforcing Bars to be Arranged so That
Points of Bending-Up and Bending-Down
Bars, Success Derived in Plan to be in Top of Slab

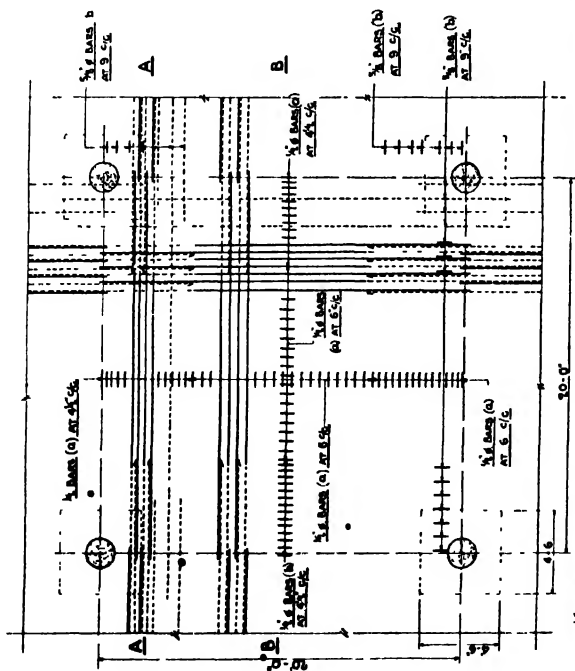
BENDING LIST (PER 20'-0" SQUARE PANEL)

(a)	30	$\frac{1}{4}$ "	26'-6"	15'-0"	10'-0"	24"
(b)	44	$\frac{5}{8}$ "	25'-0"	11'-8"	10'-0"	6"
(c)	11	$\frac{5}{8}$ "	26'-0"	25'-0"	9'-4"	4" ∇ 9"
(d)	22	$\frac{1}{4}$ "	17'-0"	25'-0"	9'-4"	4" ∇ 9"

(SEE TABLE NO 24)

FIG-49. TYPICAL DETAIL OF FLAT SLAB CONSTRUCTION WITH DROPPED PANEL





NOTES

- ALL BARS TO HAVE $\frac{1}{4}$ " MINIMUM COVER OF CONCRETE.
- BARS (a) - ALTERNATE BARS REVERSED.
- BEND-UP BARS TO BE SO ARRANGED THAT POINT OF BENDING-UP ARE STAGGERED BY 9"
- BARS SHOWN DOTTED IN PLAN TO BE IN TOP OF SLAB.

BENDING LIST (PER 20'-0" PANEL)

(a)	96 - $\frac{1}{4}$ " - 76.6'	15'-0"	9'-0"	4'-0"
(b)	14 - $\frac{1}{4}$ " - 76.0'	6'	25'-0"	6'

(SEE TABLE NO. 74)

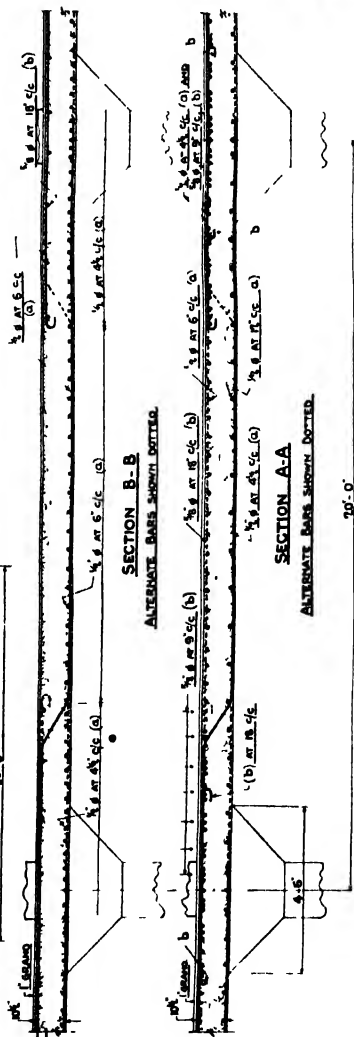


FIG. 50- TYPICAL DETAIL OF
FLAT SLAB CONSTRUCTION
WITHOUT DROPPED PANELS.

In outer quarters,

$$\text{in bottom at midspan, area} = \frac{68,100}{7.25 \times 0.87 \times 17,000} = 0.634 \text{ sq. in.}$$

Say, $\frac{1}{2}$ -in. bars at $5\frac{1}{2}$ -in. centres.

$$\text{in top over column centre line, area} = \frac{130,000}{9.44 \times 0.87 \times 17,000} = 0.931 \text{ sq. in.}$$

Say, $\frac{1}{2}$ -in. bars at $5\frac{1}{2}$ -in. centres and $\frac{1}{2}$ -in. bars at 11-in. centres.

(b) WITHOUT DROPPED PANELS (Fig. 50).

Assuming value of loading as in example with dropped panels, minimum slab thickness = $[0.001(240 + 240)\sqrt{350}] + 1.5 = 10\frac{1}{2}$ in. Providing this slab thickness, the actual loading will be:

Live load	= 224 lb. per sq. ft.
1-in. Granolithic	= 12 " " "
10½-in. slab	= 126 " " "
Total	= 362 " " "

By calculations similar to previous example, the following moments are obtained:

Total positive B.M. = 1,000,000 in. lb.

(a) 40 per cent. to be provided in centre half = 40,000 in. lb. per ft.

(b) 30 per cent. to be provided in each outer quarter = 60,000 in. lb. per ft.

Total negative B.M. = 1,680,000 in. lb.

(c) 30 per cent. to be provided in centre half = 50,400 in. lb. per ft.

(d) 35 per cent. to be provided in each outer quarter = 117,500 in. lb. per ft.

Moment (d) will control slab thickness:

$$\text{Effective depth required} = 0.0926 \sqrt{\frac{117,500}{12}} = 8.95 \text{ in.}$$

A 10½-in. slab gives an effective depth of 9.75 in. for the first layer of bars and 9.25 in. for the second layer.

Calculation of steel areas:

In centre half,

$$\text{in bottom at midspan} \frac{40,000}{9.25 \times 0.87 \times 17,000} = 0.292 \text{ sq. in. Use } \frac{1}{2}\text{-in. bars at 6-in. centres.}$$

$$\text{in top over column centre line} \frac{50,400}{9.25 \times 0.87 \times 17,000} = 0.368 \text{ sq. in.}$$

Use $\frac{1}{2}$ -in. bars at 6-in. centres.

In outer quarters,

$$\text{in bottom at midspan} \frac{60,000}{9.25 \times 0.87 \times 17,000} = 0.437 \text{ sq. in. Use } \frac{1}{2}\text{-in. bars at } 4\frac{1}{2}\text{-in. centres.}$$

$$\text{in top over column centre line} \frac{117,500}{9.25 \times 0.87 \times 17,000} = 0.857 \text{ sq. in.}$$

Use $\frac{1}{2}$ -in. bars at $4\frac{1}{2}$ -in. centres and $\frac{1}{2}$ -in. bars at 9-in. centres.

(iii) TYPICAL DESIGN OF HOPPER BOTTOM.

(See Fig. 51 for details and general arrangement. See Table No. 22 for formulæ.)

To design the hopper bottom of a coal bunker 15 ft. square, that is, $L = B = 15$ ft.; $a = b = 1$ ft. 6 in.; $H = 12$ ft.; $E = 7$ ft. 6 in.

Hence by calculation or scaling, $d = 10$ ft.

$$\text{and } \sin \theta = \frac{7.5}{10.0} = 0.75$$

$$\cos \theta = \frac{6.75}{10.0} = 0.675.$$

By drawing the inscribed circle on a normal plan of one side of the bottom :

$D = 8$ ft. and $D_1 = 9$ ft. 6 in.

$h = 15$ ft.; $S = 4$ ft. 6 in.; and $l = 9.5$ ft.

Assuming a 5-in. slab, $w_s = 60$ lb. per sq. ft.

From Table No. 5, for coal, $w = 50$ lb. per cu. ft.; $k_1 = 0.271$

$$p_n = 50 \times 15(0.271 \times 0.675^2 + 0.675^3) + (60 \times 0.675)$$

$= 495$ lb. per sq. ft. normal to slab.

$w_1 =$ wt. of bottom below $C = 8,500$ lb. approximately.

$w_2 =$ wt. of complete bottom $= 22,500$ lb. approximately.

$$W' = 50[2.5(1.5^2 + 1.5^2 + \sqrt{1.5^2 \times 1.5^2}) + (1.5^2 \times 12)] + 22,500$$

$$= 188,750 \text{ lb.}$$

$$W_1 = 50 \left[\frac{4.5}{3}(1.5^2 + 9.5^2 + \sqrt{1.5^2 \times 9.5^2}) + (9.5^2 \times 15) \right] + 8,500$$

$$= 93,650 \text{ lb.}$$

At critical sections B.M. $= 0.375 \times 495 \times 8^2 = 12,000$ in. lb. Neglecting effect of direct tension, effective depth required $= 0.0926 \sqrt{\frac{12,000}{12}}$, say, 3 in. with stresses not exceeding 700 and 17,000 lb. per sq. in. Thus 5 in., as assumed, is ample, but it is not practicable to make slab much thinner.

Horizontal reinforcement: $N = 0.5 \times 495 \times 0.75 \times 9.5 = 1,760$ lb. per ft.

Adopting the method given in Chapter XIV for combined stresses,

$$e = \frac{12,000}{1,760} = 6.8 \text{ in.}$$

$$e_s = 6.8 - \frac{5}{2} + 0.75 = 5.05 \text{ in.}$$

$$B_s = 1,760 \times 5.05 = 8,900 \text{ in. lb.}$$

$$A_T = \frac{8,900}{0.87 \times 4.25 \times 17,000} = 0.143 \text{ sq. in.}$$

$$A_{T1} = \frac{1,760}{17,000} = 0.103 \text{ „}$$

$$0.246 \text{ „}$$

Provide $\frac{1}{2}$ -in. bars at 9-in. centres.

Longitudinal reinforcement in bottom at centre of slope :

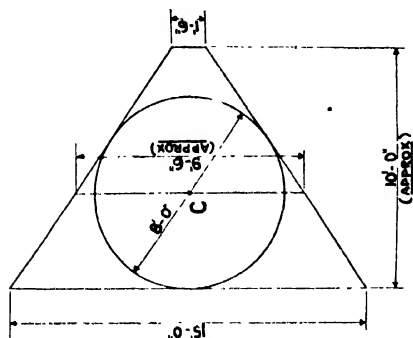
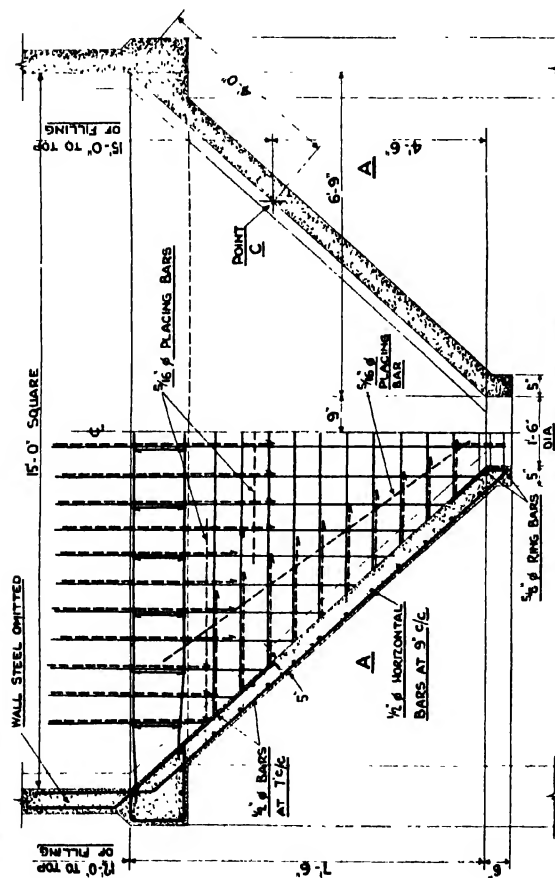
$$N = \frac{93,650}{2 \times 0.75 \times 2 \times 9.5} = 3,300 \text{ lb. per ft.}$$

$$e = \frac{12,000}{3,300} = 3.63 \text{ in.}$$

$$e_s = 3.63 - 2.5 + 0.75 = 1.88 \text{ in.}$$

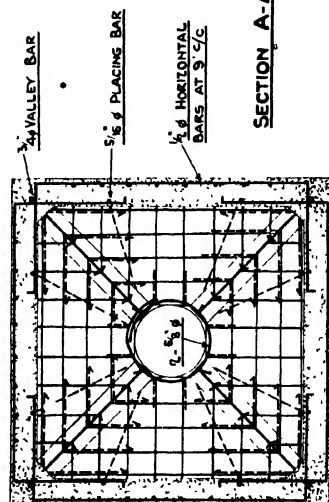
$$A_{T2} = \frac{3,300}{17,000} \left(\frac{1.88}{0.87 \times 4.25} + 1 \right) = 0.293 \text{ sq. in.}$$

Provide $\frac{1}{2}$ -in. bars at 7-in. centres.



NORMAL PLAN OF SIDE OF HOPPER:
 (SINCE BUNKER IS SQUARE ON PLAN AND
 OUTLET IS PLACED CENTRAL DETAIL OF ALL
 SIDES OF HOPPER ARE IDENTICAL)

NOTE: ALL BARS TO HAVE $\frac{1}{4}$ " MINIMUM COVER
 OF CONCRETE



SECTION A-A

**FIG. 51.- TYPICAL DETAIL OF
 HOPPER BOTTOM.**

Longitudinal reinforcement in top of slab at top of slope.

$$N = \frac{188,750}{2 \times 0.75 \times 2 \times 15} = 4,150 \text{ lb. per ft.}$$

$$e = \frac{12,000}{4,150} = 2.9 \text{ in.}$$

$$e_s = 2.9 - 2.5 + 0.75 = 1.15 \text{ in.}$$

$$A_r = \frac{4,150}{17,000} \left(0.87 \times 4.25 + 1 \right) = 0.320 \text{ sq in.}$$

Provide $\frac{1}{2}$ -in. bars at 7-in. centres.

(iv) TYPICAL EXAMPLE OF QUANTITIES AND COST ESTIMATING.

Quantities are taken-off for one 20-ft. square panel of floor of beam and slab construction, as detailed on *Fig. 48* (columns excluded).

	CONCRETE.	SHUTTERING.
Slab	20' 0" 20' 0" 4" 133.3	20' 0" 20' 0" 400
Secondary Beams.	2/19' 0" 1' 4" 8" 33.0	2/2/19' 0" 1' 4" 101
	18' 9" 8" 1' 4" 16.7	2/18' 9" 1' 4" 50
Main Beams.	18' 9" 1' 0" 1' 6" 28.1	2/18' 9" 1' 6" 56
	2/1/2' 3" 1' 0" 9" 1.7	2/2/1/2' 3" 6" 2
	Total 213.7 cb. ft.	Deduct 1' 3" 1' 3" 2
		Total 607 sq. ft.
		= 67½ sq. yd.
1-in. Granolithic:	20' 0" 20' 0" 400	

Deduct 1' 3"
1' 3" 2

398 sq. ft. = 44½ sq. yds.

REINFORCEMENT (see descriptive bending list of details on *Figs. 48(a)* and *48(b)*.)

Diameter	$\frac{5}{16}$ in.	$\frac{3}{8}$ in.	$\frac{1}{2}$ in.	1 in.	1½ in.	1½ in.
Slab	60	1,026				
	369	242				
Secondary beams (3 No.)		456	72	86	132	
				36		
				69		
Main beams (1 No.)	127		20			39
	10					26
	11					51
	12					39
						22
						18
Total length:	589	1,724	92	191	132	195 ft.
Weight per ft.:	0.261	0.376	0.668	2.67	3.38	6.01 lb. (see <i>Table No. 26</i>)
Total weight	154	648	62	510	446	1,171 lb.

Total weight of reinforcement (allowing for rolling margin):

Below $\frac{5}{16}$ in. = 864 lb. = 7½ cwt.
 $\frac{5}{16}$ in. and over = 2,127 lb. = 19 cwt.

COSTING: The foregoing quantities are transferred to the following schedule and priced out at rates based on inclusive prices given in current periodicals.

SCHEDULE OF QUANTITIES.

BEAM AND SLAB CONSTRUCTION.

Item No.	Description.	Quantities.	Rate.	Total.
				£ s. d.
1.	Mixing and placing concrete, mix C, in slabs and beams (rough screeding only).	cb. yd. 7.9	46s. 6d.	18 7 0
2.	Close-boarded shuttering to soffits of slabs, and sides and soffits of beams and haunches.	yd. sup. 67½	6s. 6d.	21 17 8
3.	Provide and lay 1-in. granolithic surfacing.	yd. sup. 44½	2s. 6d.	5 10 8
4.	Provide, bend, and fix reinforcement in bars less than ½ in. diameter including tying wire.	cwt. 7½	16s. 6d.	6 7 11
5.	Ditto ½ in. and over.	cwt. 19	15s.	14 5 0
Total estimated cost per panel 20 ft. square.				£66 8 3

Inclusive estimated cost = 30s. per sq. yd.

These inclusive rates for both the concrete and the shuttering can be derived as follows:

Concrete.

	s. d.
4-in. slab: 44½ sq. yd. at 4s. 6d.	199 2
Beams: 80 cb. ft. at 1s. 9d.	140 0
	<hr/> 339 2

s. d.

Average = $\frac{339s. 2d.}{7.9} = 43 \text{ 0 per cb. yd.}$

Add for hoisting, etc. = 3 6 „ „

Total 46 6 „ „

Shuttering—

	s. d.
Slab soffit: 340 sq. ft. at 6d.	= 170 0
Beam sides and soffits: 260 sq. ft. at 10d.	= 216 8
Extra for haunches: 5 lin. ft. at 3d.	= 1 3
Forming chamfers: 160 lin. ft. at 3d.	= 40 0

427 11

Average = $\frac{428}{67\frac{1}{2}}$, say, 6s. 6d. per sq. yd.

The following schedules are for the quantities taken off the alternative designs of flat slab construction illustrated in Figs. 49 and 50. Some of the price rates differ from those adopted for the beam and slab construction schedule, the modifications being due to the following considerations:

Concrete: rate reduced as placing in slabs only (without screeding) is cheaper than in beams.

Shuttering: rate reduced as there is less labour in making flat soffits than in forming beams and haunches; rate for flat slab without dropped panels is less than rate with dropped panels.

Reinforcement: rate for bars ½ in. and over increased as there are no bars over ½ in., hence more bars per cwt. to handle and bend; decrease in rate for bars under ½ in. because all are ½-in. diameter.

FLAT SLAB WITH DROPPED PANELS.

Item.	Description.	Quantity.	Rate.	Total.		
				£	s.	d.
1.	Concrete Mix C.	yd. cb. 11.1	44s.	24	8	5
2.	Shuttering.	yd. sup. 45½	5s. 6d.	12	8	11
3.	1-in. granolithic.	yd. sup. 44½	2s. 6d.	5	10	8
4.	Reinforcement: under ⅝ in.	cwt. 13	16s.	11	8	0
5.	Do. ⅝ in.	cwt. 7	15s. 6d.	5	8	6
6.	Extra for splayed column head. No. 1		40s.	2	0	0
Total estimated cost per 20-ft. square panel				£61	4	
Inclusive estimated cost = 27s. 6d. per sq. yd.						

FLAT SLAB WITHOUT DROPPED PANELS.

Item.	Description.	Quantity.	Rate	Total		
				£	s.	d.
1.	Concrete Mix C.	yd. cb. 12.9	44s.	28	8	7
2.	Shuttering.	yd. sup. 44½	5s.	11	1	3
3.	1-in. granolithic.	yd. sup. 44½	2s. 6d.	5	10	8
4.	Reinforcement: under ⅝ in.	cwt. 15½	16s.	12	8	0
5.	Do. ⅝ in.	cwt. 5	15s. 6d.	3	17	6
6.	Extra for splayed column head. No. 1.		40s.	2	0	0
Total estimated cost per 20-ft. square panel.				£63	6	0
Inclusive estimated cost = 28s. 9d. per sq. yd.						

These inclusive costs per unit area are commented upon at the conclusion of Chapter IV.

PART III .
DESCRIPTIVE BIBLIOGRAPHY

PART III

DESCRIPTIVE BIBLIOGRAPHY

THIS section has been compiled to direct the reader to some of the most suitable and accessible sources of information that may supplement the present volume, but cannot be considered in any way a complete bibliography of works dealing with reinforced concrete design. A few of the references may lead to authorities whose points of view, although contrary to those expressed in this handbook or opposed to accepted practice, are worthy of consideration. Where convenient, British texts are referred to in preference to foreign, since the former are usually more accessible.

PUBLISHERS' NOTE: Any of the books mentioned can be supplied by The Book Department of Concrete Publications, Ltd., 20 Dartmouth Street, London, S.W., on receipt of a remittance for the published price, plus 6d. per volume to cover postage to any address in the world on books costing more than 2s., or 3d. on books costing 2s. or less. Proceedings and papers of professional institutions are not supplied.

I.—Reinforced Concrete Engineering.

GENERAL AUTHORITIES.—(The references given in this section are to works that are frequently referred to in the text or in this bibliography, or will otherwise be of use and interest to the reinforced concrete engineer.)

"Regulations of the London County Council in respect to Buildings of Reinforced Concrete". (—L.C.C. Regulations)

"Bestimmungen des Deutschen Ausschusses für Eisenbetonbau" (1925) (—German Regulations).

"Building Science Abstracts" (= B.S. Abs.).—A monthly journal published by the Department of Scientific and Industrial Research, giving résumés of the subject matter in the latest technical periodicals and books (9d. net).

"Concrete and Constructional Engineering" (= "C. & C.E.")—A monthly journal devoted to the practice of concrete engineering (1s. 6d. net).

"Proceedings of the Institution of Civil Engineers" (= Proc. I.C.E.).

"British Standard Specifications" (= B.S.S.), issued by the British Standards Institution (various prices).

"Proceedings of 27th annual meeting of the American Society for Testing Materials."—Standard Specification for Concrete and Reinforced Concrete compiled by the Joint Committee of the Am. Soc. of Civil Engs.; Am. Soc. for Testing Materials; Am. Rly. Eng. Assn.; Am. Conc. Inst.; and Am. P.C. Assn. (= American Joint Committee).

"Illustrated Technical Dictionaries"; Vol. 8 (Reinforced Concrete).—A dictionary of technical terms in German, English, French, Russian, Italian, and Spanish.

"Joint Committee on Reinforced Concrete" (2nd Report, 1911), Royal Institute of British Architects (= R.I.B.A. Report) (4s.)

"Beton-Kalendar," Parts I and II; issued annually (7.50 R.M.).

"Handbüch für Eisenbetonbau."—A comprehensive multi-volume consideration of the theory and practice of plain and reinforced concrete.

ARCHITECTURE AND ECONOMY.

"Architectural Design in Concrete," by T. P. Bennett.—Illustrations of buildings in various parts of the world (30s.).

"Ideals of Engineering Architecture," by C. E. Fowler.—Useful photographs

of and comments on reinforced concrete bridges, power houses and towers, and remarks on ornamentation.

"Reinforced Concrete Construction" (Vol. III), by G. A. Hool.—Photographs and critical comments on bridge architecture (30s.).

"The Bridges of the Rhine," by Karl Möhringer.—Architecture and the adoption of various constructional materials to bridge construction.

"Practical Points in R.C. Design," by E. A. Scott (an unpublished paper read before the Institution of Civil Engineers, Glasgow Students' Section, 1927).—A comprehensive study of the various factors affecting economy in reinforced concrete design.

"Engineering Failures and their Lessons," by Edward Godfrey.—Interesting and controversial papers on weaknesses in the design and construction of reinforced concrete structures.

II.—Loads.

DEAD AND LIVE LOADS.

"L.C.C. Regulations" for live loads on buildings in London.

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott (16s.).—Comprehensive list of dead and live loadings, with a review of data adopted in other countries.

"Code of Practice for Structural Steel"—London Building Act (1930).

WIND PRESSURE.

"Transactions and Notes of the Concrete Institute," Vol. VI, Part II.—A paper read by Mr. G. Keevil (1915), gives a comprehensive survey of practice and experiments on wind pressure on structures.

"B.S.S. No. 153, Part 3."—Wind pressure on bridges.

"Arrol's Bridge Engineer's Handbook," by Adam Hunter.—Particulars of experiments made in connection with the Forth Bridge.

"Proc. I.C.E." Vol. 171.—Description of experiments made at the National Physical Laboratory.

"Code of Practice for Structural Steel"—London Building Act (1930).

"Kempe's Engineer's Year Book."—Intensity of wind pressure on brick chimneys (31s. 6d.).

MOVING LOADS.

"B.S.S. No. 153, Appendix No. 1."—Standard highway load.

Ministry of Transport Standard Loading for Highway Bridges.—For application of the "equivalent loading" to various types of structures, see article by G. H. Hargreaves, "C. & C.E.," Dec. 1931.

IMPACT.

"B.S.S. No. 153, Part 3"—Formula for highway and railway bridges.

"Bridges of the Rhine," by Karl Möhringer.—Particulars of impact factors allowed in various parts of the Cologne-Mülheim Suspension Bridge.

POINT LOADS.

"Reinforced Concrete Construction" (Vol. III), by G. A. Hool (30s.).—Gives an American method and an account of investigations on the dispersion of point loads on slabs.

"Journal of Institution of Municipal and County Engineers" (1926).—Dispersion of point loads on slabs is considered in an article by Sir E. Owen Williams. (This article was reproduced in "C. & C.E.," May 1926.)

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott.—Proposes an amended dispersion formula.

(See also the bibliography of M. Pigeaud's method given in Section IV.)

III. - Horizontal Pressures due to Contained Materials.**THEORETICAL PRESSURES.**

"Theory of Structures," by E. S. Andrews (13s. 6d. net).—General consideration of Rankine's Theory.

"Design of Walls, Bins and Grain Elevators," by M. S. Ketchum.—Gives derivation of the Coulomb Theory and development of the Rankine and Cain formulæ. (Note that factors given by Ketchum in application of the formulæ are primarily for steel structures.)

"Mining Structures," by M. S. Ketchum.—Gives a résumé of "Walls, Bins and Grain Elevators" with reference to the pressures on retaining walls and the walls of shallow bunkers.

"Earth Pressures, Walls and Bins," by W. Cain (14s.).

"Handbuch für Eisenbetonbau."—Gives formulæ for pressures and properties of various materials.

GRAPHICAL DETERMINATION.

"Structural Engineering," by J. Husband and W. Harby (16s. net).—For Rebhann's graphical solution of earth pressure problems.

"Stability of Retaining Walls" (paper read before the Association of Yorkshire Students of the Inst. of Civil Engs. by Professor Charnock).—Gives a proof of Rebhann's construction.

EXPERIMENTAL DATA.

"Some Earth Pressure Theories in Relation to Engineering Practice" (paper by Lt.-Col. J. Mitchell Moncrieff, before the Institution of Structural Engineers (see "Structural Engineer," 1928).

"Proc. I.C.E.," Vol. 199 (1915).—A. L. Bell's consideration of pressures due to clay and formulæ for use in design.

"Proc. I.C.E.," Vols. 203 and 209.—For papers by R. M. Crosthwaite on earth pressures, viz. "Experiments on Earth Pressures" and "Experiments on the horizontal Pressure of Sand."

FLEXIBLE WALLS.

"Proc. I.C.E.," Vol. 226.—Paper by R. N. Stroyer on horizontal earth pressures on flexible walls. The regulations of Danish Society of Engineers are given as an appendix to this Paper.

"Concrete and Constructional Engineering," October and December, 1929, Articles by R. N. Stroyer.

DEEP SILOS.

"Reinforced Concrete Design," by G. P. Manning (21s.).—For derivation of Janssen's formula.

"Design of Walls, Bins and Grain Elevators," by M. S. Ketchum.—Comparison of Airy and Janssen's formulæ.

"C. & C.E.," 1922.—A summary of factors affecting pressures in deep silos.

"Proc. I.C.E.," Vol. 131.—Airy's paper on pressures in deep silos.

"Kempe's Engineer's Yearbook."—Article by A. Drew on pressures in deep silos.

"Handbuch für Eisenbetonbau" (Vol. 14) (16.50 R.M.).—Gives details of design and construction of silos.

IV. - Bending Moments and Shearing Force.**THEORY.**

"Theory of Structures," by A. Morley (12s. 6d. net).—For derivation of simple bending moments and shear force; derivation of Clapeyron's Three Moment Theorem; relation between load, shear, moment, slope, and deflection and derivation of formulæ for general and particular cases.

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott (16s.).—Formulæ for numerous common loadings on cantilevers, free and fixed beams, for shear, moment,

and deflection. Mr. Scott also quotes M. Maurice Levy with regard to "Fixed Points."

"Strength of Materials," by A. Morley (12s. 6d. net).—Gives a method for determining the moments in a continuous beam with varying moment of inertia.

"Kontinuierliche Träger Tabellen," by G. Griot.—Data for constructing influence lines for beams continuous over equal spans or over a system of spans having symmetrical inequality.

"L.C.C. Regulations."—Moments in continuous beams

"German Regulations."—Moments in slabs continuous over supports.

"Engineering Record," Nov. 6, 1907.—A method of determining deflection of reinforced concrete beams (quoted in "Kemp's Engineer's Yearbook").

RECTANGULAR SLABS.

"R.I.B.A. Report," Appendices III, VII and VIII.—Consideration of the Grashof and Rankine and the French Government formulae, and Bach's theory of flat slabs supported on all edges and uniformly loaded.

"Reinforced Concrete Construction" (Vol. I), by G. A. Hool (17s. 6d.).—Derivation of Grashof and Rankine Rule.

"Manual of Reinforced Concrete," by C. F. Marsh and W. Dunn (21s.)—Gives curves of the Grashof and Rankine rule extended to various conditions of fixity at the edges of rectangular panels

"The Strength of Rectangular Slabs," by A. Ingerslev (paper read before the Concrete Institute, Dec. 1922)—Gives the author's proposed formula and a comparative chart of most of the acknowledged rules.

"Joint Committee Report"—For American formula.

"Annales des Ponts et Chaussées Français," Feb. 1921.—Publication of the method proposed by M. Pigeaud.

"Reinforced Concrete Bridges," by W. L. Scott (28s. net).—Gives M. Pigeaud's method (in English).

"C. & C.F.," March, April, and May, 1930—Articles by W. L. Scott on M. Pigeaud's method.

FLAT SLABS.

"Concrete Engineer's Handbook," by G. A. Hool and N. C. Johnson (30s.)—Gives most of the United States rulings.

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott (16s.).—Comparison of various United States and Continental rulings.

"German Regulations"—For accepted German practice.

"Standard Specification for Concrete and Reinforced Concrete" (published by the Canadian Engineering Standards Association).—Detailed regulations for flat-slab designs.

V.—Framed Structures.

THEORY: (i) SLOPE-DEFLECTION METHOD.

"Analysis of Statically Indeterminate Structures by the Slope Deflection Method," by Wilson, Richart, and Weiss (University of Illinois Bulletin No. 108).—Comprehensive mathematical treatment, establishing the fundamental formulæ and applying them to numerous cases. Bibliographical references are given in footnotes.

"Concrete Engineer's Handbook," by G. A. Hool and N. C. Johnson (30s.).—Gives a résumé of Wilson, Richart, and Weiss's work.

"Reinforced Concrete Reservoirs and Tanks," by W. S. Gray (10s. net).—Application of slope-deflection formulæ to practical problems.

THEORY: (ii) PRINCIPLE OF LEAST WORK.

"Elastic Stresses in Structures," by E. S. Andrews.—Gives the development of the formulæ from Castigliano's Theory, being a translation of Castigliano's "Théorème de l'équilibre des systèmes élastiques."

"Theory of Structures," by A. Morley (12s. 6d. net).—Application of fundamental principles to portal frames.

"Stresses in Portals and Similar Structures," by T. C. Brown ("The Engineer," Oct 15 et seq., 1920).

PRACTICAL FORMULÆ

"Rahmenformeln" (25s) and "Mehrstielgerahmen," by A. Kleinogel—Formulae for practically all types of frames and loadings

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott (16s).—Formulae and tabulated factors for portals and interior and exterior columns

"Reinforced Concrete Simply Explained," by Oscar Faber (5s). Factors for application to exterior columns

"American Joint Committee Report"—Gives United States formulae for calculating moments in columns (also given in "C. & C.E.," 1925)

"German Regulations" (German formula) —(Also given in "C. & C.E.," 1926).

"Guia 'El Constructor'."—Gives the Mendoza Regulations which include a method of allowing for bending in exterior columns

EARTHQUAKE-RESISTING STRUCTURES

"Earthquake Resisting Construction" (Bulletin No 16, Auckland (N.Z.) University College).—A review of earthquake proof design

"Earthquake-proof Structures"—Paper read by Mr H. D. Dewell before the International Congress of Metallic Structures, 1930

"Guia 'El Constructor'."—Gives Mendoza Regulations for earthquake-proof structures; summarized in article in "C. & C.E.," May, 1931.

"Report of Building Regulations Committee."—New Zealand (1931).

"Amendments (1924) to the Building Law of Tokyo, Japan," by Dr. Naito.—Gives requirements of construction for earthquake-proof structures in timber, brick, steel and reinforced concrete

GENERAL.

"Concrete, Plain and Reinforced," by Taylor, Thompson and Smulski.—Determination of moment of inertia of reinforced concrete sections Vol I, 40s; Vol II, 37s 6d

"Reinforced Concrete Bridges," by W. L. Scott (28s net).—Gives details of hinges used in concrete structures

"R.I.B.A. Report," Appendix V.—Moment of inertia of reinforced concrete sections

VI.—Foundations.

GENERAL.

"Foundations," by W. Simpson.—Deals solely with methods of and appliances for the examination and testing of ground; contains a good bibliography.

"Foundation Pressures" ("Building," March 1927).—A method of testing ground by driving test piles.

"Concrete Year Book" (3s 6d net).—Gives a table of standard splayed footings.

"Proc. of I. of C.E.," Vol 128.—Bending moments and shear forces in wall and pier foundations.

"Arrol's Bridge and Structural Engineer's Handbook," by A. Hunter.—Bearing values of various soils; caisson foundations; the section headed "Notes on Foundations" tabulates much useful data with regard to the foundations of a number of important structures.

"Reinforced Concrete Reservoirs and Tanks," by W. S. Gray (10s. net).—Design of foundations for structures in mining areas.

"Theory of Structures," by A. Morley (12s. 6d. net).—Derivation of "spewing" formula.

PILES.

"Handbuch für Eisenbetonbau" (Vol III) (22.50 R.M.).—Gives about sixteen pile-driving formulae with comparative notes; effect of shoe shapes.

"Engineering," March and June, 1925; Mr. A. Hiley's formula described.

"Structural Engineer," Vol. 8.--"Pile Driving Calculations," by A. Hiley; notes on driving forces and ground resistance

"Notes on Foundations and Erection" (paper read before the Institution of Structural Engineers, November 1929, by J. McCarthy).—Application of Hiley formula to an important building.

"Arrol's Bridge and Structural Engineer's Handbook," by A. Hunter.—Friction pile formula.

"Pile Formulae," by J. H. Nicholson ("Selected Engineering Papers" No. 4675, Inst. of C.E.)

"The Design of Piles," by G. B. R. Pimm ("Selected Engineering Papers," No. 4695, Inst. of C.E.)—Advantage of tapered piles

"Pile Driving and the Supporting Capacity of Piles," by R. Bennett (Selected Engineering Papers, No. 4803, Inst. of C.E.)

"Proc. of Society of Engineers" (1921) —Two papers were read describing investigations on the physical properties of clay in relation to pile driving, by A. S. E. Ackermann and Lt.-Col. H. R. Lordly (Cornell University).

"The Resistance of a Group of Piles," by H. W. Westergaard (paper read before Western Society of Engineers, Chicago) —Makes particular reference to inclined piles.

"Kemp's Engineer's Year Book" (3rs. 6d.) —Gives a selection of pile formulae

VII. —Retaining Walls and Containers.

"Marine Structures in Concrete," etc. —A series of articles by R. N. Stroyer, "C. & C.E." 1928, quotes data given by A. Freund

"Proc. of I. of C.E." (Vol. 41). —Description of experiments made by J. Sandeman on the resistance of piles subject to a horizontal pull

• "Arrol's Bridge and Structural Engineer's Handbook," by A. Hunter —The holding power of anchorages in earth

"Reinforced Concrete Reservoirs and Tanks," by W. S. Gray (10s. net).

"Handbuch für Eisenbetonbau" (Vol. V) —Moments in tank walls; method developed by Dr. Reissner

"C. & C.E.," June, 1929.—Article by H. Carpenter giving the application of Dr. Reissner's method

VIII.—Bridges, Buildings, and Other Structures.

ARCHES

"Reinforced Concrete Bridges," by W. L. Scott (28s. net) —Design of hinged and fixed arches

"Reinforced Concrete Construction" (Vol. III), by G. A. Hool (30s) —Analysis of arches of any profile.

"Handbuch für Eisenbetonbau" (Vol. 7) (38 RM) —Design and construction of arch bridges.

"Recent Developments in Arch Design," by W. L. Scott ("C. & C.E.," 1931) —Suggested modifications to the usual assumptions for application to long span arches

• "Reinforced Concrete Design," by G. P. Manning (21s) —Design of arches of all shapes (see also this author's articles in "C. & C.E.," 1930 & 1931)

"Der Bogen und das Brücken-gewölbe," by Dr. Strassner.—Method of arch design.

"Concrete, Plain and Reinforced" (Vol. I, 40s.; Vol. II, 37s. 6d.), by Taylor, Thompson and Smulski —Strassner's method of arch analysis

"Arched Bridges with Fixed Supports," by H. Carpenter ("C. & C.E.," 1930). —Application of Strassner's method

GIRDER BRIDGES

"Journal of Institution of Municipal and County Engineers" (1926) —Article by Sir Owen Williams on economical design of girder bridges (abstract in "C. & C.E.," May, 1926).

"Reinforced Concrete Bridges," by W. L. Scott (28s. net).

"Ideals of Engineering Architecture," by C. E. Fowler.—Bridge piers, with particular reference to the shapes of cutwaters.

CULVERTS.

"Rahmenformeln," by A. Kleinlogel (25s.).—Formule for culverts with constant and varying moments of inertia.

"Unsymmetrical Loading of Box Culverts," by G. H. Hargreaves ("C. & C.E.," Nov. 1931)

BUILDINGS.

"Reinforced Concrete Construction" (Vol. 2), by G. A. Hool (30s.).—Details of United States building practice.

"Mining Structures," by M. S. Ketchum.

"Concrete Year Book" (3s. 6d. net).—Hollow tile floors.

"Reinforced Concrete Design," by G. P. Manning (21s.).—Hollow tile floors.

ROADS.

"Concrete Year Book."—Description of British methods of concrete road construction

"Concrete and Constructional Engineering" (1928-31).—Articles by R. A. B. Smith.

"Concrete Road Construction in Argentina," by Chas. E. Reynolds ("C. & C.E.," Dec. 1930).—Design, construction and materials

"B.S.S. No. 76, 1930 and 1931."—Tars for road purposes.

GENERAL.

"Reinforced Concrete Reservoirs and Tanks," by W. S. Gray (10s. net).—Typical details of joints in tanks, walls, floors, etc.

"Reinforced Concrete Design," by G. P. Manning (21s.).—Design of miscellaneous structures

"The Design of Ferro-Concrete Chimneys," by Taylor, Glenday and Faber ("Engineering," March 13, 1908).

IX. Concrete.

MATERIALS

"B.S.S. No. 12, 1931."—Latest British Standard Specification for Portland Cement.

"A Hundred Years of Portland Cement (1824-1924)," by A. C. Davis (21s.)

"The Making and Testing of Portland Cement" (published by G. and T. Earle, Ltd.)—Full particulars of modern cement manufacture, with details of testing methods and appliances, and notes on the effect of various factors on the ultimate strength of mortars and concretes.

"Beton-Kalender," Part I.—Gives cement specifications of the leading countries of the world

"B.S.S.," No. 146.—Portland Blast-furnace Cement.

"I.C.C. Regulations"—General requirements for suitable aggregates including list of prohibited materials

"R.I.B.A. Report"—Ditto

"American Joint Committee Report"—Grading of fine and coarse aggregate.

"B.S.S.," No. 63.—Sizes of broken stone and chippings.

"Modern Methods of Concrete Making," by E. S. Andrews and A. E. Wynn (1s.).—A complete account of the use of the water-cement-ratio and grading by fineness modulus

PROPERTIES OF CONCRETE

"B.S. Abstracts"—References to the activities and conclusions of various investigators engaged on research work dealing with the properties of cement and concrete, e.g. Young's modulus, compressive and tensile strengths, etc.

- "L.C.C. Regulations."—Allowable working stresses (conservative practice).
- "German Regulations."—Give working stresses for various conditions of design.
- "Revue des Matériaux de Construction et de Travaux Publics," 1930.—Paper by M. R. Feret with reference to the relation between the compressive and tensile strengths of concrete.
- "Concrete Engineer's Handbook," by G. A. Hool and N. C. Johnson.—Gives results of experiments carried out at Cornell University in 1916 with reference to the coefficient of expansion of concretes.
- "Some Methods of Securing Impermeability in Concrete."—A paper giving a comprehensive study of the question, read by E. S. Andrews before the Concrete Institute
- "Plastic Yield, Shrinkage, and other problems of Concrete," by Oscar Faber (Vol. 225, Proc of I of C.E.).
- "Studies in Reinforced Concrete" (Technical Papers No 11 and 12 (1s net each) of the Building Research Station), Watford—Give latest conclusions arrived at in the course of investigations on shrinkage and plastic flow.
- "Rapid Hardening Cements"—The publications of the Building Research Station contain test results of various types of cement used in their experiments. Also, in "C & C.E.," Mar 1930, Sir Owen Williams gives comparative results of tests made on commercially supplied British rapid hardening cements.

• X. - Reinforcement.

- "Steel for Bridges and General Building Construction" (B.S.S. No. 15, 1930)
- "Hard-Drawn Steel Wire for Concrete Reinforcement" (B.S.S. No. 165, 1929)
- "Tests on High Tension Steels in Reinforced Concrete Design"—A paper read before the Concrete Institute in 1921 by H. Kempton Dyson.
- "Hormigon Armado," Vol III, April and May 1930 —Article by Chas E Reynolds on the relative economy of rolled bars and cold drawn welded mesh in floor construction.
- "L.C.C. Regulations"—Allowable stress in reinforcement and particulars of grip length, hooks and anchorages
- "Strength of Materials," by A. Morley (12s 6d net)—Particulars of methods and appliances for testing steel
- "B.S.S. No. 59"—Definition of yield point and elastic limit

XI. —Beams and Slabs.

- "Elementary Principles of Reinforced Concrete Construction," by Ewart S Andrews (7s 6d)—Derivation of fundamental resistance formulae for rectangular sections in accordance with (a) "straight line no tension" theory, and (b) non-rectilinear stress distribution (e.g. parabolic); also formulae for T beams and beams with compressive reinforcement.
- "Reinforced Concrete Design," by Faber and Bowie (Vol I, Theory, 11s; Vol II, Practice, 18s) Study of elementary principles.

XII. —Shear.

- "R.I.B.A. Report"—Appendices discuss shear action in reinforced concrete beams. Recommendations of the Report allow the concrete to take its full amount of the shear unless extra security is required
- "L.C.C. Regulations"—Regulations referring to stresses and arrangement of reinforcement
- "German Regulations"—Typical of Continental practice, but considered by many foreign engineers as too severe.
- "Reinforcement for Shear in Reinforced Concrete Beams," by Edward Godfrey (paper read before the Institution of Structural Engineers, Feb. 22, 1923, and reprinted in Godfrey's "Engineering Failures").—An interesting, comparative, and controversial discussion.
- "Reinforced Concrete Beams in Bending and Shear," by Oscar Faber (9s)—Gives useful results of tests on beams.

XIII. —Columns.

"L. C. C. Regulations"—Conservative regulations for the design of columns.

"German Regulations"—Slender columns.

"Reglamento para calculo, construccion y prueba de las obras de hormigon armado" (Argentine Regulations)—Give the slender-column formula that was in the earlier German Regulations, and that is referred to in the present book as that used in Continental practice.

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott (16s.).—Comparison of formule for slender columns.

"Arrol's Bridge and Structural Engineer's Handbook," by A. Hunter.—Comprehensive list of sections with radii of gyration, etc.

"Reinforced Concrete" (1909), by C. F. Marsh and W. Dunn (21s.).—Tests on hooped columns.

"Le Génie Civil," Nov. and Dec. 1902 and Jan. 1903.—Articles by M. Considère on his tests on hooped columns, results afterwards incorporated in book form and published in English; abstract of the test results given in "Engineering Record," Dec. 1902 and Jan. 1903, also in Marsh and Dunn's "Reinforced Concrete."

XIV. —Combined Stress.

"Combined Bending and Direct Forces," by Chas. E. Reynolds ("C. & C.E.," July 1931).—Derivation of the formule given in the present volume, with curves.

"Arrol's Bridge and Structural Engineer's Handbook,"—Entirely analytical method contributed by Chas. E. Reynolds.

"Combined Bending and Direct Force," by Chas. E. Reynolds ("C. & C.E.," Dec. 1925).—A practical method of design and analysis of rectangular sections.

"Arrol's Reinforced Concrete Reference Book," by E. A. Scott (16s.).—Tables giving the section moduli of various reinforced concrete column sections.

[Most of the methods advocated in the standard text-books involve the use of complicated formule or the possession of elaborate curves; the treatment is usually restricted to given cover ratios, to equality of tensile and compressive reinforcements, and to a given modular ratio. The derivation of the impracticable general formula is given in Andrews' "Elementary Principles of Reinforced Concrete Construction." In the present author's opinion the most satisfactory treatment of the subject is given by Dr. Oscar Faber in his article "Design of Reinforced Concrete Members for Bending combined with Thrust or Tension," published in "The Builder," Jan. 5, 1923 *et seq.*, and in "Concrete and Constructional Engineering," 1926. This method is applicable to all cover ratios and amounts of compressive reinforcement for checking given sections if the necessary graphs are available; the direct design of sections is limited to stresses of 10,000 and 600 lb. per sq. inch.]

XV. —Specifications, Quantities, and Costing.**SPECIFICATIONS.**

"Civil Engineering Specifications and Quantities," by G. S. Coleman and G. M. Flood.—For general civil engineering contracts, but not particularly reinforced concrete work.

"Specification in Detail," by F. W. Macey.—Comprehensive detailed treatment of the building and allied trades.

"R.I.B.A." Standard Conditions of Contract."

SPECIAL MATERIALS.

"Specification" (issued annually).—Full particulars of proprietary materials used in building and allied trades.

"B.S.S., No. 15"—Steel for bridges and general building construction. Steel for rivets.

"B.S.S., No. 24"—(Part 1).—Cast steel.

"B.S.S., No. 51"—Wrought iron, Grade C.

"B.S.S."—Cast iron; various standard specifications deal with cast iron for different purposes (see list published by British Standards Institution)

"B.S.S., No. 6."—Rolled steel sections for structural purposes (Maker's handbooks also give full particulars of standard joist, angle, tee and channel sections.)

"Design and Construction of Formwork for Concrete Structures," by A. E. Wynn (20s.).

"Building Construction" (Elementary and Advanced), by C. Mitchell—Details of masonry, joinery work and brickwork

"Kempe's Engineer's Yearbook."—Description of and uses for various classes of timber; specification for Dr. Angus Smith's solution

"B.S.S., No. 144."—Creosote for the preservation of timber.

"Proc. of I. of C.E."—Prof. Matthew's report on concrete and timber piles

"Reinforced Concrete Bridges," by W. L. Scott (28s.).—Details of mechanical hinges for concrete bridge work.

"Arrol's Bridge and Structural Engineer's Handbook," by A. Hunter—Complete specifications for structural steel work

QUANTITIES

Draft Report of Committee appointed by Inst. of C.E. on Civil Engineering Quantities (1932). Includes a section on Reinforced Concrete

"Standard Method of Measurement of Building Works"—Useful sections on reinforced concrete work

"Civil Engineering Specifications and Quantities," by G. S. Coleman and G. M. Flood—Gives notes on quantities for structural work, excluding reinforced concrete

"Quantity Surveying for Builders," by W. L. Evershed (10s. 6d.)—Methods of taking-off and billing for general structures

"Scottish Mode of Measurement for Reinforced Concrete Work."

COSTING

"Estimating and Cost-Keeping for Concrete Structures," by A. E. Wynn (15s.)—A complete treatment of modern methods of estimating and costing, with examples from practice.

"Laxton and Lockwood's Builders' Price Book" (issued annually)—Gives current material and labour rules for building and allied trades; recent issues include labour costs for reinforced concrete construction

Current unit rates for labour, materials, and labour and materials combined, with special reference to reinforced concrete, are given in "Concrete and Constructional Engineering" (published monthly); for general building trade rates see "The Builder" (weekly); for engineering rates see "Engineering" (weekly) or "The Engineer" (weekly)

"Molesworth's Engineer's Pocket Book"—Gives particulars of "man-hours" required to complete given amounts of work, e.g. excavation.

INDEX

(N.B. Reference to particular structures or structural members only are included in this index)

- Archlocks, 101
- Arches (see also Bridges), 87-97, 110, 118, 119
 - Foundations for, 62, 98
 - Hinges for, 53, 87, 165
 - Art galleries, Loads for, 175
 - Assembly halls, Loads for, 175
- Beams, Bending moment and shear force on, 26-37, 187-191, 193, 195, 197-199
 - Consistency for, 219
 - Design of, 126, 128-132, 131, 135, 236, 231, 239, 247, 257, 260
 - Economical design of, 9
 - in frame construction, 18, 51
 - Loading examples for, 171, 178
 - Moment of inertia of, 53-56
 - Quantities for, 167
 - Shear resistance of, 136-140, 148, 239
 - Shuttering for, 162, 163
 - Steel weights and areas for, 116, 222
 - Stresses for, 118
 - with thrusts and pulls, 118-161, 214, 245
- Book stores, Loads for, 175
- Bridges (see also Arches), 7
 - Consistency for, 219
 - Dispersion of loads on, 11, 15, 181
 - Examples on, 178, 180
 - Garlet, 97
 - Impact on, 14, 181
 - Loading on, 11-14, 175, 177
 - Moments in deck slabs of, 38
 - Piers and abutments for, 98, 116
 - Stresses for, 117, 124
 - Wind pressure, 11, 181
- Buildings (see also Floors, Columns, Beams, etc.), 98-101
 - Earthquake-resisting, 52
 - Loading for, 8, 9, 13, 14, 175
 - Stresses for, 113
 - Timber for, 163
- Bunkers (see also Silos and Hopper Bottoms), 7, 83-86, 101, 118, 119, 215
 - Coal drawing, 9, 101
 - Consistency for, 219
 - Definition of, 71
 - Hinges for, 53
 - Loads for, 9, 179
 - Moments in columns for, 51, 52
 - Pressures in, 17-23, 183
- Canalovers (see Beams)
- Chimneys, 102, 118, 156, 157
 - Wind pressure on, 10, 11
- Churches, Loads for, 175
- Cinemas, Loads for, 175
- Columns (see also Foundations),
 - Aggregate for, 110
 - Bending in, 18-59, 118-161, 202-205, 244, 245
 - Consistency for, 219
 - Design of, 126, 141-147, 210, 213
 - Economical design of, 5, 117
 - End conditions, 52, 53
 - Moment of inertia of, 53-56
 - Quantities for, 167
 - Shuttering for, 162, 163
 - Steel weights and areas for, 127, 222
 - Stresses for, 118
- Containers, Definitions of, 71
 - Weights of stored materials, 119
- Cooling towers, 101
- Counterforted walls, 75, 76, 190
- Crushing plants, 101
- Culverts, 103-105
- Dance halls, 14, 175
- Drill halls, 14, 175
- Floors (see also Slabs)
 - Aggregate for, 112
 - Costs of, 170, 171, 268-270
 - Design of, 256-264
 - Granolithic finish for, 114, 175
 - Hollow tile, 98, 175
 - Loads on, 8, 9, 175
 - Protection of, 101, 111, 115
- Foundations (see also Piles and R.R. dts.), 57-73, 206-211
 - Aggregate for, 110
 - "Blinding" for, 115, 219
 - Bolder, 101
 - Consistency for, 219
 - for earthquake resisting structures, 52
 - Mass concrete, 116
 - Punching shear in, 110
- Galleries, 62, 102, 178, 201
 - Crane loads for, 179
- Garages, 112, 175
- Gas printers, 101
- Grand-stands, Loads for, 175
- Gymnasiums, Loads for, 175
- Hopper bottoms, 86, 163, 217, 265, 267
 - Conical, 71
 - Definition of, 74
 - Suspended, 71
- Hospitals, Loads for, 175
- Hotels, Loads for, 175
- Intels, 8, 174, 175
- Offices, Loads for, 174, 175
- Pile caps, 53, 72, 73, 120
- Piles (see also Sheet Piles), 67, 70, 126, 153, 163, 164, 110, 211
 - Consistency for, 219
 - Helmets for, 105
 - Quantities for, 167
 - Safe loads on, 67-70
 - Shores for, 70, 105
 - Tests for, 57
 - Timber, 7, 164
- Pithead gears, 101
- Raft foundations, 59-62, 61, 101
- Reading rooms, Loads for, 175
- Reservoirs (see Tanks)
- Restaurants, Loads for, 175-9
- Retaining walls, 74-80, 212, 213
 - Definition of, 74
 - Examples on, 82
 - Joints in, 107, 108
 - Pressures on, 17-23, 181
- Roads, 105-107, 116
 - Aggregate for, 112
 - Consistency for, 219
 - Joints in, 107, 108
 - Surfacing for, 7, 175
- Roofs, 7, 98, 99
 - Consistency for, 219
 - Loading example for, 174
- Roofs, Loads on, 8, 9, 175
 - Surfacing of, 115, 175
 - Weights of, 8, 9, 175
- Schools, Loads for, 175
- Screening plants, 101
- Sheet piles (see also Wharves), 77
 - Passive pressure on, 25
 - Shores for, 77
 - Walls, 75-79, 212, 213
- Shops, Loads for, 175
- Silos, 74, 85, 86, 118
 - Consistency for, 219
 - Foundations for, 52
 - Loads and pressures in, 9, 23-25, 181, 185
 - Moments in columns of, 51, 52
- Slabs (see also Floors)
 - Bending moments in, 37-44, 101
 - Cold-drawn steel for, 122, 123
 - Consistency for, 219
 - Design of, 116, 133, 131, 200, 232-235
 - Flat, 41, 201, 261-264, 270
 - Joints in, 108
 - Openings in, 99
 - Point loads on, 14-16
 - Polygonal and circular, 401
 - Quantities for, 167
 - Rectangular, 38
 - Shuttering for, 162, 163
 - Steel areas and weights for, 127, 222, 224
- Slurry basins, 101
- Stairs, 99-101
 - Consistency of, 219
 - Loads on, 8, 9, 175
 - Non-slip treads for, 114
- Stationery stores, Loads for, 175
- Stores (see Warehouses, Bunkers, Tanks, etc.)
- Swimming baths, 108
- Tanks (see also Water Towers), 6, 81, 82, 116, 125, 212, 213
 - Consistency for, 219
 - Definition of, 71
 - Foundations for, 62
 - Joints in, 107, 108
 - Loads, 9
 - Pipes in, 166
 - Pressures on, 17
 - Stresses in, 120, 123
- Theatres, Loads for, 175
- Walls (see also Retaining Walls, Bunkers, Silos), 101
 - Aggregate for, 110
 - Brickwork, 7, 164
 - Consistency for, 219
 - Foundations for, 65, 208
 - Panel, 7
 - Partition, 175
 - Quantities for, 167
 - Shuttering for, 162, 163
 - Warehouses, 9, 116, 175
- Water towers, 101
 - Columns and braces of, 51
 - Example of, 180
 - Tanks, Bottom of elevated, 82, 83, 216, 217
 - Wind pressure on, 11, 181
- Wharves (see also Sheet Pile Walls), 163, 165
- Workshops, 14, 113, 175

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“ESTIMATING AND COST KEEPING FOR CONCRETE STRUCTURES.” By A. E. Wynn, B.SC., A.M.A.M.SOC.C.E. Published 1930. 272 pages, 92 illustrations, 2 folders. Price 15/-; by post 15/6 anywhere in the world.

A FULL knowledge of costs, and of how to use this knowledge when preparing tenders, will eliminate most of the risks in the contracting business. The author of this volume has had a very wide experience in contracting for all kinds of concrete and reinforced concrete structures, and has closely studied the systems in general use. As a result of this study and practical experience the author has adopted in his own business methods of estimating, cost-keeping, and book-keeping which are particularly suitable for the use of concrete constructors. The system has the advantage of extreme simplicity, and ensures the achievement with a minimum of effort of the accuracy in estimating that is essential to successful contracting. The book is written throughout in readily understandable language, and reproductions are given of all the forms required.

The chapters are as follows :—

- I—Introduction.
- II—Preparing to Tender.
- III—The Quantity Estimate.
- IV—Estimating Unit Costs.
- V—Finishing and General Items.
- VI—Quantity and Cost Estimate for a Building.
- VII—Estimates for Concrete Arch Bridges.
- VIII—Estimate for a Dam.
- IX—The Cost Analysis.
- X—Office Management.
- XI—Cost-Keeping.
- XII—Labour Costs.
- XIII—Labour Distribution Forms.
- XIV—Progress and Cost Charts.
- XV—Plant Costs and Charges.
- XVI—Book-Keeping.

Specimen pages from this book are given on pages 285 and 286.

ESTIMATING AND COST KEEPING.

Below and on page 286 are specimen pages from "Estimating and Cost Keeping for Concrete Structures" (See page 284)

ESTIMATING UNIT COSTS.

39

Considering now the labour cost "*in front of the mixer*," to distribute to the forms either wheelbarrows holding an average of 2 cu. ft. of concrete or concrete carts holding about 5 cu. ft. can be used. Although carts will hold $2\frac{1}{2}$ to 3 times as much concrete as wheelbarrows, they are heavier, harder to dump and turn round, and fewer trips per hour will be made with them.

One labourer with wheelbarrow can receive, wheel not over 50 ft., and dump in forms about 30 cu. ft., or 15 loads per hour, making 1 trip every 4 minutes, and 1 minute should be added for each additional 50 ft. One labourer with cart can receive, wheel not over 50 ft., and dump in forms about 50 cu. ft., or 10 loads per hour, making 1 trip every 6 minutes and $1\frac{1}{2}$ minutes should be added for each additional 50 ft.

To find how many wheelers will be required to maintain the output of one-half cubic yard every 5 minutes, divide the time of wheeling for 1 trip by the period of time allowed between loads to empty the hopper and take the nearest whole number higher than the fraction.

Using wheelbarrows, 6 trips per batch will be necessary, or 50 seconds between trips, so that the number of wheelers = $4 \times 60/50$, say, 5 men.

Using carts, 3 trips per batch give 100 seconds between trips, so that the number of wheelers will be $6 \times 60/100$, say, 4 men.

One man will be required to handle the gate of the receiving hopper, and at least 2 labourers will be required to tamp and puddle the concrete in the forms and move runways. In addition, at least one carpenter will be required inspecting forms, setting screeds, repairing runways, etc.

The cost then "*in front of the mixer*," using carts, will be per batch :

1 labourer on receiving hopper	=	5 minutes	
4 " wheeling each	=	20 minutes	
2 " puddling " 5 "	=	10 "	
			35 minutes
			per hour =
1 carpenter		5 minutes	
		per hour	

Over the whole operation will be a labour or concrete foreman whose time must be included.

The total cost of concreting under these conditions for an output of 48 cu. yds. per day will be

Behind the mixer,	7 labourers each 8 hours	=	56 hours @	per hour	
At " "	1 mixer operator	8 "	@	" "	
	1 hoist "	8 "	@	" "	
In front of " "	7 labourers each 8 hours	=	56 hours @	" "	
	1 carpenter	8 "	@	" "	
1 foreman		8 "	@	" "	
					Total cost of 48 cu. yd. = <u>X</u>

Unit labour cost per cu. yd. = $X/48$.

It will be noticed that there are the same number of labourers behind as in front of the mixer ; this is a common rule to use. • If the operations

Below and on page 285 are specimen pages from "Estimating and Cost Keeping for Concrete Structures." (See page 284.)

ESTIMATE FOR A DAM.

99

plant and for storing aggregate and building forms. Since the whole area of the reservoir had to be cleared of trees there was plenty of rough timber available for building a trestle, no wrought timbers being necessary.

All excavation had to be deposited at the back of and against the dam, so it was decided to build a trestle on this side because, although it would mean the loss of the timber bents, the embankment would add to the stability of the gantry and would afford a convenient working space. Also the concrete could be run out on flat trucks on a narrow-gauge track laid on the trestle and hauled by a cable and hoist, using the derrick as a "dead man" to haul against. The travelling derrick would handle the forms, do the excavating with a grab bucket, and handle the concrete in dump buckets.

The next consideration was the concreting plant. With a minimum of men, it had to be mechanically operating as fast as possible. It was desired to pour 12,000 cu. yd. in eight months, or 200 working days. This gave a rate of 60 cu. yd. per day, or 120 cu. yd. every other day, since forms would have to be handled between and there would be some lost time on account of weather.

A mixer turning out $\frac{3}{4}$ cu. yd. every $3\frac{1}{2}$ minutes was estimated, or at the rate of $121\frac{1}{2}$ cu. yd. per 9-hour day (this rate was maintained, and often increased). An overhead charging-bin fed by a guy-derrick and operating a grab bucket, with batch-boxes for proportioning, would be necessary. If the plant were set in the side of the hill some advantage could be taken of gravity action, so it was decided to build the cement shed on top and chute the cement on to the platform of the mixer (large plum-stones could be handled the same way, chuting on to a flat truck). Owing to lack of space aggregates could not be stored at the high level, so a temporary road was built down the hillside to the mixer level. To transport the concrete to the derrick a narrow-gauge track with flat trucks and dump buckets was considered, using a hoisting engine and cable. The number of trucks, length of track, switches, etc., required to maintain the output had to be estimated.

An examination of the surrounding country showed deposits of rather fine gravel deficient in the larger sizes, so for economy and for better grading it was considered some field stone would have to be crushed. Since this had to be collected from neighbouring fields up to a mile away several schemes were compared for cost, and it was finally decided to estimate on using tractors to haul dump wagons. The crusher could be placed near to the aggregate pile so that the same derrick could handle the stone, gravel, and sand.

Other items of plant needed at the site that had to be estimated were rock-drilling equipment, blacksmith's outfit, saw-table, pumps, and many small items. Each item was priced separately and 50 per cent. of the total cost was charged as plant rental. Some of the larger items were calculated at second-hand prices, at which it was known they could be bought. Since it was estimated the plant would be required for

"CONCRETE AND CONSTRUCTIONAL ENGINEERING"

The monthly Concrete Journal. Price 1/6 per copy.

Annual Subscription, 18/- post free.

Since its inception in 1906, this journal has built up a reputation for its authoritative articles on all phases of the many applications of concrete and reinforced concrete. It is the only journal published in this country dealing solely with reinforced concrete in all its aspects, and counts amongst its regular contributors the leading experts at home and abroad in the theory and practice of reinforced concrete. It is the only medium through which those engaged in the constructional industry can keep up to date with current theory and practice.

In each issue there are valuable articles of interest to consulting engineers, architects, and contractors, to whom this journal is indispensable. Every important new reinforced concrete work is fully illustrated and described. Special features of each issue include the most complete Data for Pricing Concrete and Reinforced Concrete Work published in this country, and a Questions and Answers section in which subscribers' problems and queries are solved or replied to in detail by experts.

Send a postcard for free specimen copies.

“DESIGN AND CONSTRUCTION OF FORMWORK FOR CONCRETE STRUCTURES.” By A. E.

Wynn, B.Sc., A.M.AM.SOC.C.E. 320 pages, 219 illustrations, 12 folders, 11 design tables. Strongly bound in cloth covers. Price 20/-; by post 20/9 anywhere in the world. Revised. 1930.

THIS is the only book published in the English language dealing solely and exhaustively with the subject of Formwork. It is indispensable to anyone engaged in the design or construction of concrete structures. The volume gives complete designs for formwork for every type of plain and reinforced concrete structure, from simple footings to arch bridges, with tables from which the sizes and quantities of timber required for any type or size of structure can be seen at a glance.

The following is a list of the chapters :—

- I—Form Building in General.
- II—Materials, Loads, Pressures and Stresses.
- III—Theoretical Design of Forms.
- IV—Design Tables.
- V—Design Problems.
- VI—Detail Construction of Footing Forms.
- VII—Detail Construction of Column Forms.
- VIII—Wall Forms.
- IX—Detail Construction of Beam and Girder Floor Forms.
- X—Forms for Rib Floors and Structural Steel Fireproofing.
- XI—Miscellaneous Forms in Building Construction.
- XII—Forms for Flat Slab Construction.
- XIII—Forms for Conduits, Sewers and Culverts.
- XIV—Forms for Tanks, Silos, Bins and Standpipes.
- XV—Forms for Dams, Piers and Heavy Walls.
- XVI—Steel Forms in Building and Wall Construction.
- XVII—Steel Forms for Curved Surfaces.
- XVIII—Arch Falsework.
- XIX—Other Bridge Forms.
- XX—Patent Devices.
- XXI—Planning the Work.

LIST OF TABLES.

- 1. Maximum span of floor sheathing for various thicknesses of slab.
- 2. Maximum span of wall sheathing for various heights of wall.
- 3. Maximum span of column sheathing for various heights of column.
- 4. Maximum spacing of floor joists for various spans and thicknesses of slab.
- 5. Maximum concentrated loads carried by ledgers and wales for various spans and spacing of loads.
- 6—9. Spacing of column yokes for various sizes and heights of column.
- 10. Maximum spacing of wales and wall ties for various sizes of studs and heights of wall.
- 11. Safe loads on posts.

(See opposite for specimen page from this book.)

FORMWORK

Specimen page from "Design and Construction of Formwork for Concrete Structures"
(See page 288)

40

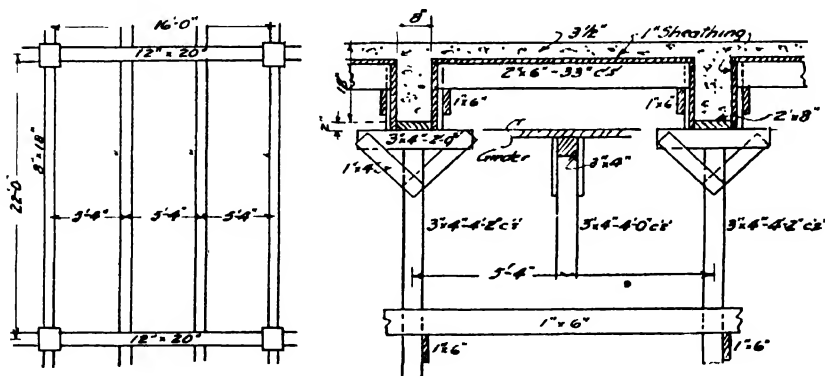
DESIGN OF FORMWORK.

(a) Sheathing	$\frac{14}{12} = 1.17$	(b) $\frac{14}{12} = 1.17$
Joists	$\frac{16 \times 12}{1.44} = 133.33$	$\frac{12 \times 12 \times 14}{1.44 \times 31} = 44.44$
Ledgers	$\frac{64}{1.44} = 44.44$	$\frac{72}{1.44} = 50.00$
Posts	$\frac{16 \times 30}{1.44 \times 6} = 27.78$	$\frac{12 \times 40}{1.44 \times 5.5} = 61.11$
Braces	$\frac{6 \times 32}{1.44 \times 6} = 22.22$	$\frac{6 \times 30}{1.44 \times 5.5} = 22.22$
	3.12 c.f.	3.00 c.f.

There is not much difference between the two designs for amount of material used, so it will be a question of the relative cost of sizes.

Design 3: Beam and Girder Panel, Short Span Slab.

Design forms for typical bay of beam and girder construction, consisting of $3\frac{1}{2}$ -in. slab, 8 in. by 18 in. beams, span 22 ft. and 5 ft. 4 in. on centres and 12 in. by 20 in. girders, span 16 ft.



DESIGN-3

From Table 1, 1-in. sheathing for $3\frac{1}{2}$ -in. slab will span 32 in.

Span of joists will be 4 ft. 6 in. From Table 4, interpolating between 3 in. and 4 in. slabs, we can use 2 in. by 4 in. at $16\frac{1}{2}$ in. on centre, or 2 in. by 6 in. at 39 in. on centre, which, however, must be reduced to $32 + 1 = 33$ in. on centre because of the deflection of the sheathing. The latter will be more economical, so will be chosen.

Load carried to ledger, including 40 lbs. live load = $(42 + 40) \times 2.75 \times 2.25 = 510$ lbs.

“REINFORCED CONCRETE RESERVOIRS AND TANKS.” By W. S. Gray, B.A., M.A.I., A.M.INST.C.E.I.

Published 1931. 200 pages, 119 illustrations. Price 10/-; by post 10/6 anywhere in the world.

CHAPTERS

- I—CIRCULAR TANKS.**
- II—GASHOLDER AND TAR TANKS.**
- III—SHALLOW CIRCULAR TANKS OF LARGE DIAMETER.**
- IV—OPEN RESERVOIRS.**
- V—RESERVOIRS WITH COUNTERFORTED WALLS.**
- VI—OPEN RECTANGULAR TANKS BELOW GROUND.**
- VII—RECTANGULAR TANKS ABOVE GROUND.**
- VIII—FLOOR AND WALL JOINTS AND DRAINAGE.**
- IX—SWIMMING BATHS AND TANKS WITH SLOPING FLOORS.**
- X—TANKS WITH CONICAL OR PYRAMIDICAL FLOORS.**
- XI—COVERED TANKS.**
- XII—LARGE ROOFED TANKS IN WATERLOGGED GROUND.**
- XIII—EXAMPLES OF COMPLETED RESERVOIRS.**
- XIV—NOTES ON DESIGN AND CONSTRUCTION METHODS.**
- XV—TESTS AND REPAIRS.**

THIS book deals in a thoroughly practical manner with the design and construction of plain and reinforced concrete reservoirs and tanks, swimming-baths and pools, and other water-containing structures both above and below ground level. While the book assumes that the reader has a knowledge of the elements of reinforced concrete design and construction, it is written and illustrated so that

RESERVOIRS AND TANKS (*continued*)

its contents will be really useful to an engineer who has not before undertaken this class of work, while the experienced designer will find all the available information on the subject presented in a handy form.

The author has not confined the work to the broad principles of design, but discusses various methods in detail and points out the simplest solution of the various designing problems. Throughout the book the reader is given advantage of the author's considerable practical experience in the design and construction of reservoirs and tanks. Even those experienced in this class of work cannot fail to find valuable ideas for simplifying their work, for in addition to the author's personal experience there are presented here for the first time in the English language useful ideas from Continental practice.

Each chapter deals very thoroughly with its subject, and discusses the small but important details so often neglected in text-books. The book is one in which the reader will be certain of obtaining the information he requires on any problem in reservoir and tank design and construction.

A chapter is included on construction methods, showing how costs can be kept down by the adoption of suitable plant and plant lay-out, formwork, etc.—a subject of very great importance but not always given proper consideration. This chapter should be invaluable to a contractor carrying out this class of work.

The illustrations, 119 in number, are clearly drawn, and include complete designs for reservoirs, tanks, swimming-baths, etc., with details of expansion joints, floor and wall joints, reinforcement for walls and floors, formwork, detail photographs of modern construction methods, etc.

• For the first time in any book, valuable information is included on the design and construction of tanks in ground subjected to subsidence by mining, and on the stability of tanks in water-logged ground.

“CONCRETE CONSTRUCTION MADE EASY.” By Leslie Turner, B.S.C., A.M.INST.C.E., and Albert Lakeman, I.R.I.B.A., M.I.STRUCT.E. Published 1930. 113 pages, 65 illustrations, 16 tables. Price 3/6 net ; by post 4/- anywhere in the world.

THERE are no unsolved problems in the design and erection of any type of concrete structure to the builder who has a copy of this book, by authors well known for their ability to explain reinforced concrete problems in readily understandable language. This book gives full designs with tables and all information necessary to enable builders to erect foundations, walls, columns, floors, roofs, staircases, beams, water tanks, retaining walls, stanchion bases, lintels, formwork, etc., of any size without further assistance.

To those who are fresh to the subject the book contains a mine of information, while those who have had experience in reinforced concrete design and construction will find it a fruitful source of advice, for it is essentially a practical work.

The chapters include :

- I FOUNDATIONS.—(Foundation Pressures—Footings for Reinforced Concrete Columns and Steel Stanchions—Excavation—Reinforcement—Formwork—Quantities—Wall Footings—Raft Foundations).
- II COLUMNS.—(Quantities—Formwork—Erection of Forms—Reinforcement—Concreting).
- III—BEAMS.—(Single or One-span Beams Determination of Dimensions—Wall Bearings Construction—Formwork—Materials—Continuous-Span Beams).
- IV—LINTELS. (Design—Formwork Lifting).
- V—FLOOR AND ROOF SLABS.—(Loading and Supporting Slabs—Formwork—Placing Reinforcement—Concreting—Finishes—Single-Span Slabs—Continuous-Span Slabs—Quantities—Roof Slabs).
- VI—PANEL WALLS.
- VII—STAIRCASES.—(Pre-cast Steps *In Situ* Stairs—Baluster Fixing—Surface Finish—Constructional Details—Reinforcement—Formwork).
- VIII—SAW TOOTH ROOFS.—(Method of Manufacture—Reinforcement—Handling—Design of Truss).
- IX—WATER TANKS.—(Tanks above Ground Level—Tanks below Ground Level—Quantities).
- X—RETAINING WALLS.—(Excavation—Shuttering—Reinforcement—Removal of Shuttering—Expansion Joints—Returns and Angles—Sloping Sites).
- XI—GENERAL SPECIFICATION AND NOTES.

"ELEMENTARY GUIDE TO REINFORCED CONCRETE." By Albert Lakeman, L.R.I.B.A., M.I.STRUCT.E., Honours Medallist Construction; late Lecturer Woolwich Polytechnic. Sixth Edition revised 1930. 94 pages, 79 illustrations. Price 2/-; by post 2/3 anywhere in the world.

FOR the builder, the student, the clerk-of-works, the foreman, or anyone interested in building and wishing to gain a knowledge of the principles and practice of reinforced concrete, there is no better book than this. It is especially written for the beginner, and on the assumption that the reader has no previous knowledge of the subject whatever. The author is well known for his lucid manner of explanation, which he has used in this book to make clear the principles governing the design of reinforced concrete in a way that can be readily grasped without reference to any other text-book.

The chapters include: Introduction and Explanation of Terms; Loads; Reinforcement; Neutral Axis; Reinforced Concrete Beams; Fixed and Supported Beams; Leverage of Beams; Bending Moments; Shearing Stress and Slabs; Reinforced Concrete Columns; Reinforced Concrete Walls; Bases and Foundations.

The simple explanations are illustrated with clearly-drawn diagrams, 79 in number, which are used wherever necessary to make the text readily understandable.

Some Appreciations.

"Every builder carrying out reinforced concrete would do well to give his foreman a copy of this book, and obviate many mistakes"—*The Architect*.

"Explanatory matter is kept as simple as possible, and highly technical language avoided. Will appeal to students"—*Daily Telegraph*.

"Will enable foremen to supervise work intelligently, thus enabling builders to reduce the danger of defective work with its serious consequences"—*Carpenter and Builder*.

"We do not often come across a book dealing with technical matters without mathematics and formulae, but the author of this little work has achieved this difficult task in an admirable manner."—*Architects' Journal*

TWO OF THE 79 EXPLANATORY DIAGRAMS

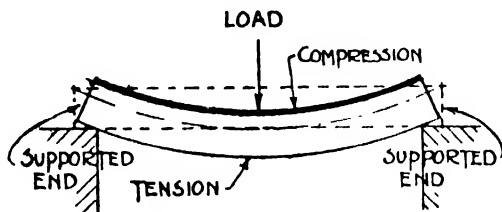


FIG. 30—Effect on Beam of Central Concentrated Load

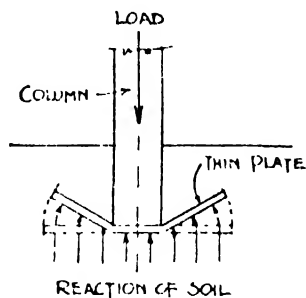


FIG. 73—Effect of Load on Soil

“REINFORCED CONCRETE BEAMS IN BENDING AND SHEAR.”

By Oscar Faber, O.B.E., D.SC.,
M.INST.C.E. 150 pages, 113 illustrations. Published 1924.
Price 9/- net ; by post 9/6 anywhere in the world.

THIS is an exhaustive treatise on the subject of bending and shear in reinforced concrete beams both with and without compression reinforcement, illustrated with 113 drawings and photographs. The volume is a result of exhaustive experimental and research work by the author, and contains new rules and formulæ which have a scientific basis and are of such a form that they may be conveniently used by engineers.

Some Appreciations.

“Dr Faber has not only specially designed beams of practical dimensions in such a way that they must fail under bending and shearing stresses respectively, but he has also analysed the bearing which the results he has obtained from his experiments have upon the methods of design recommended in the report on Reinforced Concrete of the Royal Institute of British Architects, enforced by the London County Council Regulations dealing with Reinforced Concrete, and advocated by himself. Dr Faber is to be congratulated upon the value of his experiments and of the deductions he has derived therefrom. Without doubt the author's conclusions, observations, and analyses are very convincing, while the correctness of the results obtained by the application of his theory is amply demonstrated by comparing them with the ultimate loadings on his experimental beams.”—*Engineering*.

“It is good to have in permanent book form Dr. Faber's very important work upon the strength of reinforced concrete beams in bending and shear. Engineers and students are strongly recommended to read this book with care, and to realise that there is often more to be obtained from original work emanating from this country than the scissors-and-paste compilations that flow so freely from abroad.”—*EWART S. ANDREWS, B.Sc., M.INST.C.E., in Concrete and Constructional Engineering*

“MODERN METHODS OF CONCRETE MAKING.”

By Ewart S. Andrews, B.SC., M.INST.C.E., and A. E. Wynn, B.SC., A.M.AM.SOC.C.E. Second Edition. 1928.
Price 1/- ; by post 1/2 anywhere in the world.

IN this book the authors describe in simple and readily understood language the methods of making concrete at a minimum cost. The cement-water ratio method of predetermining the strength of concrete is fully and simply explained, and tables and curves are given showing the quantity of water necessary to give maximum strength with different concrete mixes and with different size aggregates. It is clearly shown how the use of the correct water content enables the cement to be reduced, with consequent economy.

The first edition of this book was sold in six months. This new edition has been enlarged to include data relating to typical British Portland cements and rapid-hardening Portland cements. It deals especially with the methods of producing strong and uniform concrete at the lowest cost and shows how a 1:9 concrete made in accordance with the author's recommendations can be as strong as a 1:6 concrete proportioned by haphazard methods.

"THE CONCRETE YEAR BOOK." A Handbook, Directory, and Catalogue of Concrete. (Published annually on January 1.) Edited by Oscar Faber, O.B.E., D.Sc., M.INST.C.E., and H. L. Childe, Editor Concrete Publications Ltd. Revised every year. 764 pages. Price 3/6; by post 4/3 anywhere in the world.

HANDBOOK SECTION.

The Handbook Section contains authoritative chapters on practically every aspect of concrete and reinforced concrete design and construction, embracing the latest practice at home and abroad. No attempt is made at giving individual opinions, but to present in concise and convenient form specifications or methods which are either standard practice or recommendations formulated after thorough investigation by competent bodies. This section includes, among other subjects :—

Costs.—Illustrations, constructional details, and costs of a large number of concrete and reinforced concrete structures of every description, from which approximate costs of recently erected structures can be seen at a glance.

Concrete Making.—Notes on the best method of securing strong concrete of uniform strength, water content, coloured concrete, storage of cement, properties of aggregates, concrete in sea water, curing, consolidation, cementation, etc.

Slabs.—Tables for the design of Reinforced Concrete Slabs for various spans and loadings.

Footings.—Tables from which the size and reinforcement of footings necessary for various pressures can be seen at a glance.

Loadings.—Data relating to Floor and Roof Loads, Loads of Walls and Partitions, Wind Pressures, Earth Pressures, Foundation Pressures, etc.

Beams.—Properties of common sizes of reinforced concrete beams.

Floors.—Specifications and Designs for various types of concrete and reinforced concrete floors.

• **Granolithic Floors.**—Specification for granolithic floors.

Water Towers.—Designs and tables for ascertaining at a glance the approximate cost of water towers of different sizes and heights above ground.

• “THE CONCRETE YEAR BOOK” (*continued*)

Bridges.—Design and construction of various types of reinforced concrete bridges, brought up to date with the latest Ministry of Transport Regulations.

Silos.—Design and Construction Data for Reinforced Concrete Silos.

Surface Treatment.—Illustrated specifications for different methods of surface treatment for concrete.

Reservoirs.—Data on design and construction of Reservoirs and Underground Tanks, with typical Design; tables and curves showing at a glance approximate costs of circular and rectangular reservoirs of various capacities.

Piling.—Various driving formulæ, and notes on casting and driving concrete piles.

Formwork.—Timbers and timber sizes. Designs for shuttering for walls, columns, beams, etc., of various sizes. Tables showing timber sizes required for various strengths.

Pre-Cast Concrete.—Specifications for the manufacture of many types of concrete products and cast stone, with drawings of moulds in wood, plaster, gelatine, etc.

Cement.—Notes on strengths of cement, tests for cement, etc.

Concrete Roads.—Up-to-date specifications for concrete road construction.

Memoranda.—The most complete memoranda ever published of everyday use to the concrete user.

Bibliography of every book published in the English language dealing with concrete and cement and allied subjects.

DIRECTORY.

The Directory Section is the only complete directory of the concrete industry published in this country, classified under different headings for ease of reference. A complete list of trade names and brands in use in the industry is a valuable feature.

CATALOGUE.

The Catalogue Section contains full particulars of the businesses or products of nearly 400 firms connected with or catering for the concrete industry, and is invaluable to anyone seeking a firm of contractors to carry out special kinds of work, or a machine or product for a special purpose.

"MANUFACTURE AND USES OF CONCRETE PRODUCTS AND CAST STONE." By H. L.

Childe. 354 pp., 265 illus., colour plates. Completely re-written and enlarged 1930. Price 6/-; by post 6/6 anywhere in the world.

THIS volume deals exhaustively with every phase of the manufacture of pre-cast concrete, concrete products, and cast stone of every description and for all purposes. This is the only volume dealing comprehensively with the subject. Throughout it is written in simple language, illustrated with photographs and clear drawings on practically every page.

All the available information on pre-cast concrete, from tiles to architectural cast stone, is embodied in this practical work, which no products manufacturer, borough surveyor, or builder can afford to be without.

The essentials of selection of materials, grading and proportioning, mixing, curing, etc., are fully covered. All the generally used methods of surface treatment (and many new ones) are described and illustrated in colour and half-tone, and the methods of making coloured concrete are explained. Various surface textures are illustrated, and the methods of obtaining them described.

The design and manufacture of moulds for all types of concrete products and all shapes of cast stone are dealt with at length, the descriptions of the best and simplest moulds for each type of product being illustrated by more than 100 working drawings and illustrations of moulds in wood, plaster, sand, gelatine, concrete, etc.

Working drawings are given of moulds for columns, cornice, posts, paving flags, kerb and channel, blocks and slabs, bricks, roofing tiles, steps, sills, lintels, edging, balustrade, sundials, bird baths, lettered panels, ornamental work, etc., etc.

Suitable proportions of cement and aggregate are given for all products.

“PRE-CAST CONCRETE FACTORY OPERATION AND MANAGEMENT.” By H. L. Childs. 216 pages. 146 illustrations. Published 1930. Price 3/6; by post 4/- anywhere in the world.

THIS new volume gives complete descriptions of the methods and processes used by more than twenty manufacturers of all classes of pre-cast concrete. The works described in this volume include the largest makers of paving flags, roofing tiles, and similar products; concrete and breeze slab and block factories; large and small cast-stone works, etc., covering the whole range of pre-cast concrete units. The works of local authorities, railway companies, etc., are included.

In each case full information is given on the lay-out of the plant; the types of machines and other plant used for different purposes; the materials and proportions favoured by the different manufacturers; the mixing, casting, finishing, and curing processes; the types of moulds used, and how the moulds are made, etc.

Among the factories described and illustrated are:

GREAT WESTERN RAILWAY CONCRETE DEPOT SOUTHERN RAILWAY CONCRETE DEPOT
J. ARNOLD & SON, LTD. ART. PAVEMENTS & DECORATIONS LTD. ATLAS STONE CO., LTD.
CAMBRIDGE ARTIFICIAL STONE CO., LTD. CONCRETE UTILITIES LTD. EMERSON & NORRIS, LTD.
EVERLASTING TILE CO., LTD. GLOUCESTER COUNTY COUNCIL DEPOT. HOLCREFE PRODUCTS LTD.
BALES CONCRETE MANUFACTURING CO. LTD. MARLEY TILE CO., LTD.
MONO CONCRETE CO., LTD. PATENT IMPERVIOUS STONE AND CONSTRUCTION CO., LTD.
SHARP, JONES & CO., LTD. STUART'S GRANOLITHIC CO., LTD. WALLINGFORD CONCRETE BRICK AND TILE WORKS. WHARF LANE CONCRETE CO., LTD.
WADDON CONCRETE CO., LTD.

“MOULDS FOR CAST STONE AND PRE-CAST CONCRETE.” 80 pages, size 11 in. × 7½ in. Published 1930. Price 2/6; by post 3/- anywhere in the world.

THIS new publication contains large-scale clear working drawings, with explanatory notes, for making wood and plaster moulds for:

ARCHES; BALUSTRADING; BIRD BATHS (of several different designs); CHIMNEY CAPS; CHIMNEY POTS; COLUMNS (Round, Square, and Octagonal); COPINGS; PIER CAPS; FENCES (of several different designs); FIREPLACE SURROUNDS; FLOWER BOXES; PEDIMENTS; PERGOLAS; GATE PIERS; GATE POSTS; PILES; PIPES; TENNIS SURROUND POSTS; SILLS; SUNDIALS; TANKS; TROUGHS (Circular and Rectangular); WELL LININGS; TRACERY WINDOWS, ETC.

These large clear working drawings are presented in such a form that they can be followed by any carpenter without previous experience of mould making, with the certainty that the cheapest and best moulds will be made. In each case a good design is given for the article in question, and every detail of its mould explained by drawings and notes.

All the moulds described have been constructed and used by practical mould-makers with many years of experience.

